ON FUBINIAN AND C-FUBINIAN MANIFOLDS

By Yoshihiro Tashiro and Shun-ichi Tachibana

In his previous papers[1)], one of the present authors proved that an orientable hypersurface in an almost complex manifold has an almost contact structure and obtained a condition[2)] in order that a hypersurface in a Kählerian manifold is Sasakian. In the present paper, a hypersurface satisfying the condition will be called a C-umbilical hypersurface. A manifold having the same Sasakian structure as a C-umbilical hypersurface in a locally Fubinian manifold will be said to be locally C-Fubinian. The purpose of the present paper is to show some characteristic properties of Fubinian and C-Fubinian manifolds.

§1. Preliminaries.

Let M be a $2n$-dimensional almost Hermitian manifold with almost complex structure $F=(F_\lambda{}^\kappa)$ and metric tensor $G=(G_{\mu\lambda})$. We shall denote the curvature tensor by $K_{\nu\mu\lambda}{}^\kappa$, the Ricci tensor by $K_{\mu\lambda}$, the scalar curvature by $\kappa=K_{\mu\lambda}G^{\mu\lambda}/2n(2n-1)$, and the covariant differentiation with respect to the Riemannian connection of the metric G by ∇_μ.

If M is Kählerian, we know the identities

$$K_{\nu\mu\lambda\kappa}F_\pi{}^\lambda F_\omega{}^\kappa=K_{\nu\mu\pi\omega}, \tag{1.1}$$

$$F^{\nu\mu}K_{\nu\mu\lambda\kappa}=-2K_\lambda{}^\omega F_{\omega\kappa}=2K_\kappa{}^\omega F_{\omega\lambda}. \tag{1.2}$$

For a vector $V=(V^\kappa)$, we put $\|V\|^2=G_{\mu\lambda}V^\mu V^\lambda$, $\tilde{V}^\kappa=-V^\lambda F_\lambda{}^\kappa$ and

$$K(V)=-K_{\nu\mu\lambda\kappa}\tilde{V}^\nu V^\mu \tilde{V}^\lambda V^\kappa/\|V\|^4, \tag{1.3}$$

$$R(V)=K_{\mu\lambda}V^\mu V^\lambda/\|V\|^2. \tag{1.4}$$

These quantities $K(V)$ and $R(V)$ are the so-called holomorphic sectional curvature and the Ricci curvature (belonging to the direction) of the vector V, respectively.

On the other hand, let $\bar{M}$ be a $(2n-1)$-dimensional almost Grayan manifold with structure (f, g) consisting of an almost contact structure

---

Received April 20, 1963.

1) Tashiro [5]. Terminologies and notations of the papers will be taken over in the present paper. The various kinds of indices run on the following ranges respectively:

$$\kappa, \lambda, \mu, \nu, \omega=1, \dots, 2n;$$
$$h, i, j, k, l=1, \dots, 2n-1;$$
$$A, B, C=1, \dots, 2n-1, \infty.$$

2) See Theorem 8 in [5], or the equation (1.12) in the below.

$$f=(f_B{}^A)=\begin{pmatrix} f_i{}^h & f_i \\ -f^h & 0 \end{pmatrix}$$ [3]

and its associated metric C-tensor

$$g=(G_{CB})=\begin{pmatrix} g_{ji} & 0 \\ 0 & 1 \end{pmatrix},$$

where g_{ji} is a metric tensor associated with f. The structure (f, g) possesses the following properties: The rank of the matrix $(f_i{}^h)$ is equal to $2n-2$,

$$(1.5)\qquad ff=-E:\ f_j{}^i f_i{}^h - f_j f^h = -\delta_j^h,\quad f_j{}^i f_i = 0,\quad f^i f_i{}^h = 0,\quad f^i f_i = 1,$$

$$(1.6)\qquad fgf^t = g:\ g_{lk} f_j{}^l f_i{}^k + f_j f_i = g_{ji},\quad g_{ih} f^h = f_i,\quad g_{ji} f^j f^i = 1,$$

where E indicates the unit matrix (δ_B^A) of degree $2n$. The covariant tensor $f_{ji}=f_j{}^h g_{hi}$ is skew symmetric. We denote the curvature tensor of $\bar{M}$ by $\bar{K}_{kji}{}^h$ and the Ricci tensor by $\bar{K}_{ji}$. The covariant differentiation with respect to the Riemannian connection of g_{ji} in $\bar{M}$ will be denoted by ∇_j, too, which we distinguish by affixing a Latin index from that in an almost Hermitian manifold M with Greek index.

If $\bar{M}$ is Sasakian, then the structure satisfies the equations

$$(1.7)\qquad \nabla_j f_i = f_{ji},\qquad \nabla_j f_{ih} = f_i g_{hj} - f_h g_{ij}$$

in addition to (1.5) and (1.6). Moreover we have the identities

$$(1.8)\qquad \bar{K}_{kji}{}^h f_h = f_k g_{ji} - f_j g_{ki},$$

$$(1.9)\qquad \bar{K}_{ji} f^j f^i = 2n-2,$$

which will be used later. As is seen by (1.7) and the skew-symmetry of f_{ji}, the vector field f^h is a Killing one and its trajectories are geodesics.

Now, in an almost Hermitian manifold M, we consider an orientable hypersurface, which is also denoted by $\bar{M}$ for the following reason. When $\bar{M}$ is represented by $x^\kappa = x^\kappa(u^h)$ by use of local coordinate systems (x^κ) in M and (u^h) in $\bar{M}$, we denote the tangent vectors $\partial_i x^\kappa$ of $\bar{M}$ by $B_i{}^\kappa$, the unit normal vector by C^κ, or sometimes by $B_\infty{}^\kappa$, and put

$$B=(B_B{}^\kappa)=\begin{pmatrix} B_i{}^\kappa \\ C^\kappa \end{pmatrix},\quad B^{-1}=(B_\lambda{}^A)=(B_\lambda{}^h,\ C_\lambda).$$

Then the induced structure (f, g) in $\bar{M}$ defined by

$$(1.10)\qquad f=BFB^{-1}:\ \begin{array}{l} f_i{}^h = B_i{}^\lambda F_\lambda{}^\kappa B_\kappa{}^h,\ f_i = B_i{}^\lambda F_\lambda{}^\kappa C_\kappa, \\ -f^h = C^\lambda F_\lambda{}^\kappa B_\kappa{}^h,\quad 0 = C^\lambda F_\lambda{}^\kappa C_\kappa \end{array}$$

$$(1.11)\qquad g=BGB^t:\ g_{ji} = G_{\mu\lambda} B_j{}^\mu B_i{}^\lambda,\quad \begin{array}{l} G_{\mu\lambda} B_j{}^\mu C^\lambda = 0, \\ G_{\mu\lambda} C^\mu C^\lambda = 1, \end{array}$$

3) Here we put $f_\infty{}^h = -f^h$. This is different from that in [5] in sign. In an almost Grayan manifold, f^h coincide with the contravariant components of the vector $f_i = f_i{}^\infty$.

is an almost Grayan structure.

In a Kählerian manifold M, the induced structure (f, g) in $\bar{M}$ is Sasakian if and only if the second fundamental tensor h_{ji} of $\bar{M}$ has the form

$$h_{ji}=g_{ji}+\mu f_j f_i, \tag{1.12}$$

where μ is a scalar field in $\bar{M}$. Such a hypersurface will be said to be *C-umbilical.* The factor μ is related to the mean curvature $h=h_{ji}g^{ji}/(2n-1)$ by

$$\mu=(2n-1)(h-1) \tag{1.13}$$

and we call it the *C-mean curvature* of $\bar{M}$. Let us seek for a formula used later. Since $B^t g^{-1} f B=GF$, we have

$$f^{kj}B_k{}^\nu B_j{}^\mu - C^\nu \tilde{C}^\mu + \tilde{C}^\nu C^\mu = F^{\nu\mu}. \tag{1.14}$$

Substituting (1.12) into Codazzi's equation[4)]

$$\nabla_k h_{ji} - \nabla_j h_{ki} = B_k{}^\nu B_j{}^\mu B_i{}^\lambda C^\kappa K_{\nu\mu\lambda\kappa} \tag{1.15}$$

and using (1.7), we obtain

$$(\nabla_k \mu) f_j f_i - (\nabla_j \mu) f_k f_i + \mu (2 f_{kj} f_i + f_j f_{ki} - f_k f_{ji}) = B_k{}^\nu B_j{}^\mu B_i{}^\lambda C^\kappa K_{\nu\mu\lambda\kappa}. \tag{1.16}$$

Contracting this equation with $f^{kj} f^i$, taking account of (1.5), (1.10), (1.14) and $f^i B_i{}^\lambda = -C^\mu F_\mu{}^\lambda = \tilde{C}^\lambda$, we obtain the inquired equation

$$2(n-1)\mu = -R(C) + K(C), \tag{1.17}$$

which means that the C-mean curvature μ is the difference of the holomorphic sectional curvature from the Ricci curvature of the normal direction of $\bar{M}$ at each point, to within a constant factor.

If M is in particular an Einstein manifold, then by transvection of (1.16) with g^{ji}, we have $\nabla_k \mu = f_k g^{ji} (\nabla_j \mu) f_i$. Applying $f^{lk}\nabla_l$ to the last equation and using (1.5) and (1.7), we can easily see that $g^{ji}(\nabla_j \mu) f_i = 0$ and hence μ is constant in $\bar{M}$. Thus we have

THEOREM 1. *In an Einstein Kählerian manifold, the mean curvature h and the C-mean curvature μ of a C-umbilical hypersurface are constant.*

§ 2. Fubinian and *C*-Fubinian manifolds.

A Kählerian manifold M is called a locally Fubinian manifold or a manifold of constant holomorphic sectional curvature, if the holomorphic sectional curvature at every point is independent of directions at the point, and its curvature tensor is given by[5)]

$$K_{\nu\mu\lambda\kappa} = k(G_{\nu\kappa}G_{\mu\lambda} - G_{\mu\kappa}G_{\nu\lambda} + F_{\nu\kappa}F_{\mu\lambda} - F_{\mu\kappa}F_{\nu\lambda} - 2F_{\nu\mu}F_{\lambda\kappa}), \tag{2.1}$$

k being a constant and equal to $(2n-1)\kappa/2(n+1)$. A locally Fubinian manifold is an Einstein one:

---

4) See, for instance, Schouten [3], p. 242.

5) Tashiro [4], Yano and Mogi [6].

$$K_{\mu\lambda}=2(n+1)kG_{\mu\lambda} \tag{2.2}$$

and for a unit vector V, we have

$$K(V)=4k, \qquad R(V)=2(n+1)k. \tag{2.3}$$

Now we consider a C-umbilical hypersurface $\bar{M}$ in a locally Fubinian manifold M. By (1.17) and (2.3), we know that $\mu=-k$. Substituting (1.12) and (2.1) into Gauss' equation[6)]

$$\bar{K}_{kjih}=B_k{}^\nu B_j{}^\mu B_i{}^\lambda B_h{}^\kappa K_{\nu\mu\lambda\kappa}+h_{kh}h_{ji}-h_{jh}h_{ki} \tag{2.4}$$

we have

$$\begin{aligned}\bar{K}_{kjih}=&(k+1)(g_{kh}g_{ji}-g_{jh}g_{ki})+k(f_{kh}f_{ji}-f_{jh}f_{ki}-2f_{kj}f_{ih})\\ &-k(g_{kh}f_jf_i+g_{ji}f_kf_h-g_{jh}f_kf_i-g_{ki}f_jf_h).\end{aligned} \tag{2.5}$$

Transvecting this equation with g^{kh}, we have

$$\bar{K}_{ji}=2(n-1)\,[(k+1)g_{ji}-kf_jf_i]. \tag{2.6}$$

In general, a Sasakian manifold, whose curvature tensor possesses the properties (2.5) or (2.6), will be called a *locally C-Fubinian* or *C-Einstein* manifold[7)] respectively. Then we can state that

THEOREM 2. *A C-umbilical hypersurface in a locally Fubinian manifold is a locally C-Fubinian manifold.*

If $k=0$, then $\mu=0$ and hence we have

COROLLARY. *In a 2n-dimensional Euclidean manifold M with natural Kählerian structure, the induced almost Grayan structure of a hypersurface $\bar{M}$ is Sasakian if and only if $\bar{M}$ is a portion of a unit hypersphere in M.*

Suppose now there is an umbilical hypersurface in a locally Fubinian manifold. Put the second fundamental tensor in $h_{ji}=\rho g_{ji}$ and substitute it into (1.15). Then by the same method as that of obtaining (1.17) and by (2.3) one can see that $0=K(C)-R(C)=2(n-1)k$ and hence $k=0$. Thus we have

THEOREM 3. *There is no umbilical hypersurface in a non-Euclidean locally Fubinian manifold.*

§3. A characterization of a locally Fubinian manifold.

It is well known[8)] that an n-dimensional Riemannian manifold is projectively flat, i.e. of constant curvature, if and only if there exists an umbilical hypersurface with constant mean curvature through every point with every $(n-1)$-direction at

6) See, for instance, Schouten [3], p. 242.
7) A C-Einstein manifold is called an η-Einstein one by Okumura [2].
8) Schouten [3], p. 309 and p. 311.

the point, and that it is conformally flat if and only if there exists an umbilical hypersurface through every point with every $(n-1)$-direction at the point. Since a Fubinian manifold is holomorphically projectively flat[9], an analogous proposition for a locally Fubinian manifold may be expected. In fact, we shall establish the following

THEOREM 4. *A $2n$-dimensional Kählerian manifold M is locally Fubinian if and only if there exists a C-umbilical hypersurface with C-mean curvature equal to a constant $-k$ through every point with every $(2n-1)$-direction at the point.*

Proof. Sufficiency. Assume that there exist C-umbilical hypersurfaces stated in the theorem. Every C-umbilical hypersurface in a Kählerian manifold is Sasakian, and from (1.16) we have easily $K_{\mu\lambda}B_j{}^{\mu}C^{\lambda}=0$. In order that this equation holds for arbitrary vectors $B_j{}^{\mu}$ and C^{λ} such that $G_{\mu\lambda}B_j{}^{\mu}C^{\lambda}=0$, $K_{\mu\lambda}$ should be proportional to $G_{\mu\lambda}$ and hence M is an Einstein manifold, $K_{\mu\lambda}=(2n-1)\kappa G_{\mu\lambda}$. From (1.17) and our assumptions, we have

$$K(C)=-2(n-1)k+(2n-1) \tag{3.1}$$

for any unit vector C^{κ}. Therefore the holomorphic sectional curvature $K(C)$ is independent of directions at every point, and hence M is locally Fubinian.

Necessity. Let M be a locally Fubinian manifold whose curvature tensor is given by (2.1). The theorem is true in the case of locally Euclidean manifold, so that we shall only concern with the case $k\neq 0$. We consider the system of partial differential equations

$$\nabla_{\mu}U_{\lambda}=kG_{\mu\lambda}+U_{\mu}U_{\lambda}-\tilde{U}_{\mu}\tilde{U}_{\lambda} \tag{3.2}$$

in an unknown vector field U_{λ}. It is seen that the integrability condition

$$\nabla_{\nu}\nabla_{\mu}U_{\lambda}-\nabla_{\mu}\nabla_{\nu}U_{\lambda}=-K_{\nu\mu\lambda}{}^{\kappa}U_{\kappa} \tag{3.3}$$

of the system (3.2) is identically satisfied by (2.1) and (3.2) itself, and consequently the system is completely integrable. Let P be an arbitrary point and consider a solution of (3.2) with initial value $(U_{\lambda})_{\mathrm{P}}$ at P satisfying $(G^{\mu\lambda}U_{\mu}U_{\lambda})_{\mathrm{P}}=k^2$. Since $\nabla_{\mu}U_{\lambda}=\nabla_{\lambda}U_{\mu}$, the family of $(2n-1)$-directions given by U_{λ} constitutes an involutive distribution in a neighborhood of P. Let $\bar{M}$ be the integral manifold of the distribution through P. Since U^{κ} is normal to $\bar{M}$, we can put $U_{\lambda}=\sigma C_{\lambda}$ where σ is a scalar in $\bar{M}$. Substituting (3.2) into $\nabla_j U_{\lambda}=B_j{}^{\mu}\nabla_{\mu}U_{\lambda}=\nabla_j(\sigma C_{\lambda})$ and noticing $B_j{}^{\mu}\tilde{U}_{\mu}=\sigma B_j{}^{\mu}F_{\mu}{}^{\lambda}C_{\lambda}=\sigma f_j{}^{A}B_A{}^{\lambda}C_{\lambda}=\sigma f_j$, we have

$$kB_{j\lambda}-\sigma f_j\tilde{U}_{\lambda}=\sigma_j C_{\lambda}-\sigma h_j{}^{i}B_{i\lambda}, \tag{3.4}$$

where $\sigma_j=\partial_j\sigma$. Since $B_{i\lambda}$ and $\tilde{U}_{\lambda}$ are tangent to $\bar{M}$, we see that $\sigma_j=0$ and hence $\sigma^2=k^2$ by means of the initial condition. We may suppose that $\sigma=-k$. Transvecting $B_i{}^{\lambda}$ with (3.4), we have

$$h_{ji}=g_{ji}-kf_jf_i. \tag{3.5}$$

As P and the value $(U_{\lambda})_{\mathrm{P}}$ of direction at P are arbitrary, we complete the proof of the theorem.

9) See Tashiro [4].

§4. A construction of a compact C-Fubinian manifold.

We have seen in §2 that a C-umbilical hypersurface in a locally Fubinian manifold is locally C-Fubinian. Now we are going to construct a compact C-Fubinian manifold in a Fubinian manifold with $k\neq -1$ in a concrete way.

Let X be a complex number space of dimension n, and denote its coordinates by z^α and their conjugates by $z^{\alpha*}$ [10]. Putting

$$S=1+2k\sum z^\alpha z^{\alpha*} \tag{4.1}$$

and

$$\Phi=(\log S)/2k, \tag{4.2}$$

a Fubinian manifold M is by definition[11] a maximal connected domain in X, where S does not vanish, and its Kählerian metric is given by

$$\begin{aligned} &G_{\beta^*\alpha}=G_{\alpha\beta^*}=\partial_{\beta^*}\partial_\alpha\Phi=(S\delta_{\beta\alpha}-2kz^\beta z^{\alpha*})/S^2, \\ &G_{\beta\alpha}=G_{\beta^*\alpha^*}=0. \end{aligned} \tag{4.3}$$

The non-trivial components of the contravariant metric tensor are

$$G^{\beta^*\alpha}=S(\delta^{\beta\alpha}+2kz^{\beta*}z^\alpha) \tag{4.4}$$

and those of the Riemannian connection are

$$\left\{ {\alpha \atop \gamma\beta} \right\}=G^{\alpha\varepsilon^*}\partial_\gamma G_{\beta\varepsilon^*}=-2k(\delta_\gamma{}^\alpha z^{\beta*}+\delta_\beta{}^\alpha z^{\gamma*})/S \tag{4.5}$$

and their conjugates.

Then the equation (3.2) is separated into

$$\nabla_\beta U_\alpha=2U_\beta U_\alpha,\ \nabla_{\beta^*}U_\alpha=kG_{\beta^*\alpha}. \tag{4.6}$$

If we define a vector field $U_\lambda=(U_\alpha, U_{\alpha^*})$ by

$$U_\alpha=k\partial_\alpha\Phi=kz^{\alpha*}/S \quad \text{and conj.,} \tag{4.7}$$

then it is easily seen that the vector field U_λ satisfies (4.6). Since $\|U\|^2=2g^{\beta^*\alpha}U_{\beta^*}U_\alpha =2k^2\sum z^{\alpha*}z^\alpha$, the hypersurface $\bar{M}$ defined by $\sum z^{\alpha*}z^\alpha=1/2$ is C-umbilical in M, because U_λ is a vector normal to $\bar{M}$ and its length is equal to k on $\bar{M}$. Therefore $\bar{M}$ is a C-Fubinian manifold, which is diffeomorphic to a $(2n-1)$-dimensional sphere.

§5. C-loxodromes.

A locally Fubinian manifold is characterized by local flatness under a holomorphically projective transformation, a transformation between affine connections

10) In this paragraph, the first Greek indices α, β, γ run over $1, \dots, n$ and we write $\alpha^*=\alpha+n$.

11) See, for instance, Bochner [1].

preserving holomorphic plane curves. An analogue may be expected for a locally C-Fubinian manifold, and for this object we first introduce the notion of C-loxodromes.

In a Sasakian manifold, we consider a curve L: $u^h=u^h(s)$ parameterized with its arc-length s and satisfying the differential equation

$$\frac{\delta^2 u^h}{ds^2}=af_j f_i{}^h \frac{du^j}{ds}\frac{du^i}{ds}, \tag{5.1}$$

where δ indicates covariant differentiation along curves and a is a constant. Putting $\xi^h=du^h/ds$, we can see that $f_i\xi^i$ is constant along L and put $b=f_i\xi^i$. By expanding Frenet formulas for the curve, we can verify that the first principal normal vector of L is given by $(1-b^2)^{-1/2}f_i{}^h\xi^i$ and the second by $(1-b^2)^{-1/2}(b\xi^h+f^h)$, and the first principal curvature is equal to $ab(1-b^2)^{1/2}$, the second equal to $1-ab^2$ and the successives vanish identically. Therefore the curve L is a loxodrome cutting geodesic trajectories of f^h with constant angle. It is reduced to a Riemannian circle if $ab^2=1$ and to a geodesic if $b=0$.

By use of an arbitrary parameter t of L, the equation (5.1) turns into

$$\frac{\delta^2 u^h}{dt^2}=\alpha\frac{du^h}{dt}+af_j f_i{}^h\frac{du^j}{dt}\frac{du^i}{dt}, \tag{5.2}$$

α being a function of t. However, in an almost contact manifold with affine connection, we may also consider the equation (5.2) and call its integral curves *C-loxodromes.*

§6. A characterization of locally C-Fubinian manifolds.

Let Γ_{ji}^h and $\Gamma'{}_{ji}^h$ be symmetric affine connections in an almost contact manifold $\bar{M}$. A correspondence between them will be called a *CL-transformation* if it carries C-loxodromes to C-loxodromes. By standard arguments, it follows from (5.2) that a CL-transformation is expressed by the relation

$$\Gamma_{ji}'^h=\Gamma_{ji}^h+\delta_j^h p_i+\delta_i^h p_j+c(f_j f_i{}^h+f_i f_j{}^h), \tag{6.1}$$

where p_i is a vector field and c is a constant. Then the curvature tensors are related to each other by

$$\begin{aligned} K'{}_{kji}{}^h=&K_{kji}{}^h-\delta_k^h P_{ji}+\delta_j^h P_{ki}+(P_{kj}-P_{jk})\delta_i^h\\ &-c[f_k{}^h\nabla_j f_i-f_j{}^h\nabla_k f_i-(\nabla_k f_j-\nabla_j f_k)f_i{}^h]\\ &+c[(\nabla_k f_i{}^h)f_j-(\nabla_j f_i{}^h)f_k+(\nabla_k f_j{}^h-\nabla_j f_k{}^h)f_i], \end{aligned} \tag{6.2}$$

where we have put

$$P_{ji}=\nabla_j p_i-p_j p_i-c(f_j f_i{}^l+f_i f_j{}^l)p_l-c^2 f_j f_i. \tag{6.3}$$

Now we consider a Sasakian manifold related to a locally Euclidean manifold[12] under a CL-transformation. By (6.2), (1.5), (1.6) and (1.7), the curvature tensor is equal to

12) A locally Euclidean manifold is not Sasakian.

$$
\begin{aligned}
K_{kji}{}^{h}=&\delta_k^h P_{ji}-\delta_j^h P_{ki}-(P_{kj}-P_{jk})\delta_i^h \\
&+c[f_k{}^h f_{ji}-f_j{}^h f_{ki}-2f_{kj}f_i{}^h] \\
&-c[2\delta_k^h f_j f_i+g_{ki}f_j f^h-2\delta_j^h f_k f_i-g_{ji}f_k f^h].
\end{aligned} \tag{6.4}
$$

Contracting h and k in this equation, we have

$$K_{ji}=2(n-1)P_{ji}+(P_{ji}-P_{ij})-c[3f_j{}^k f_{ki}+(4n-5)f_j f_i+g_{ji}]. \tag{6.5}$$

The symmetry of K_{ji} implies that of P_{ji} and consequently p_i is the gradient of a scalar field, say p. Transvecting (6.4) with f_h and by (1.5), we have

$$f_k[P_{ji}-(c+1)g_{ji}]=f_j[P_{ki}-(c+1)g_{ki}].$$

Hence we may put

$$P_{ji}-(c+1)g_{ji}=\nu f_j f_i,$$

ν being a proportional factor. Substituting this into (6.5), we obtain

$$K_{ji}=2[(n-1)(c+1)+c]g_{ji}+2[(n-1)\nu-c(2n-1)]f_j f_i.$$

Transvecting $f^j f^i$ and comparing the result with (1.9), we have $\nu=c$ and therefore

$$P_{ji}=(c+1)g_{ji}+cf_j f_i. \tag{6.6}$$

Substituting this into (6.4), we see that the curvature tensor is equal to the expression (2.5) with $k=c$.

Conversely, it is verified that in a locally C-Fubinian manifold with $k=c$ the integrability condition of the equations

$$\nabla_j p=p_j,$$

$$\nabla_j p_i=(c+1)g_{ji}+p_j p_i+c(f_j f_i{}^h+f_i f_j{}^h)p_h+cf_j f_i$$

is satisfied. Therefore we obtain the

THEOREM 5. *A Sasakian manifold related to a locally Euclidean manifold under a CL-transformation is locally C-Fubinian, and vice-versa.*

BIBLIOGRAPHY

[1] BOCHNER, S., Curvature in Hermitian metric. Bull. Amer Math. Soc. **53** (1947), 179–195.

[2] OKUMURA, M., On infinitesimal conformal and projective transformations of normal contact spaces. Tôhoku Math. J. **14** (1962), 398–412.

[3] SCHOUTEN, J. A., Ricci Calculus. 2nd ed. Berlin, 1954.

[4] TASHIRO, Y., On a holomorphically projective correspondence in an almost complex space. Math. J. Okayama Univ. **6** (1957), 147–152.

[5] TASHIRO, Y., On contact structure of hypersurfaces in complex manifolds, I; II. Tôhoku Math. J. **15** (1963), 62–78; 167–175.

[6] YANO, K. AND I. MOGI, On real representations of Kählerian manifolds. Ann. of Math. **61** (1955), 170–189.

OKAYAMA UNIVERSITY AND
OCHANOMIZU UNIVERSITY.