CORRECTION TO THE PAPER "FOURIER SERIES XI: GIBBS' PHENOMENON"

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In my previous paper "Fourier series XI: Gibbs' phenomenon" (Kōdai Math. Sem. Rep. 8(1956), 181—188), the hypotheses of Theorem 4 are insufficient and may be replaced as follows:

THEOREM 4. *Suppose that*

(1) $$f(x) = l\psi(x-\xi) + g(x)$$

where $\psi(x)$ *is a periodic function with period* 2π *such that*

(2) $$\psi(x) = (\pi - x)/2 \qquad (0 < x < 2\pi)$$

and where

(3) $$\limsup_{x\downarrow\xi} f(x) = l\pi/2, \qquad \liminf_{x\uparrow\xi} f(x) = -l\pi/2,$$
$$\liminf_{x\downarrow\xi} f(x) \geqq -l\pi/2, \qquad \limsup_{x\uparrow\xi} f(x) \leqq l\pi/2.$$

Let the Fourier series of $f(x)$ *be*

(4) $$f(x) \sim \sum_{n=1}^{\infty} \alpha_n \sin n(x-\xi),$$

where

$$\alpha_n = \frac{l}{n} + a(n), \qquad \sum |\varDelta\alpha_n| < \infty,$$
$$\frac{1}{t}\int_0^t ta(t)\,dt = o(1),$$

and

$$m\cdot\max_{0\leqq t\leqq m} \sum_{j=1}^{\infty} |a(t+2jm) - a(t+(2j-1)m)| \to 0 \qquad as \quad m\to\infty,$$

then there exists a number r_0, $0 < r_0 < 1$, *with the following property: the* (C,r) *means of the Fourier series of* $f(x)$ *present Gibbs' phenomenon at* ξ *for* $r < r_0$, *but not for* $r \geqq r_0$, r_0 *being the Cramér number.*

In the proof of Theorem 4 (p. 188) it is sufficient to prove

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$$n\int_0^1 (1-t)^r a(nt) \sin nxt\, dt \to 0$$

as $n \to \infty$, $x \to 0$ independently. Since

$$n\int_0^1 (1-t)^r a(nt) \sin nxt\, dt$$

$$= \int_0^n \left(1 - \frac{t}{n}\right)^r a(t) \sin xt\, dt,$$

it is sufficient to prove

$$\int_0^n a(t) \sin xt\, dt \to 0.$$

Here we write

$$\int_0^n a(t) \sin xt\, dt = \int_0^{\pi/x} + \int_{\pi/x}^{2\pi/x} + \int_{2\pi/x}^{3\pi/x} + \cdots + \int_{(2k+1)\pi/x}^{n}$$

$$= \int_0^{\pi/x} + \sum_{j=1}^{k} \int_0^{\pi/x} \left\{ a\left(t + \frac{2j\pi}{x}\right) - a\left(t + \frac{(2j-1)\pi}{x}\right) \right\} \sin xt\, dt$$

$$+ \int_{(2k+1)\pi/x}^{n} \equiv I_1 + I_2 + I_3, \quad \text{where } \frac{(2k+1)\pi}{x} < n \leqq \frac{(2k+3)\pi}{x}.$$

$I_1 = o(1)$ and $I_3 = o(1)$, since $\frac{1}{t}\int_0^t ta(t)\, dt = o(1)$.

$I_2 = o(1)$, since

$$m \cdot \max_{0 \leqq t \leqq m} \sum_{j=1}^{\infty} | a(t + 2jm) - a(t + (2j-1)m) | \to 0 \qquad \text{as } m \to \infty.$$

This proves Theorem 4.

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