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ON SPECTRAL CHARACTERIZATIONS OF MINIMAL HYPERSURFACES IN A SPHERE

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Abstract

Let M be a closed minimal hypersurface in an Euclidean sphere $S^{n+1}(1)$. We first prove that a minimal isoparametric hypersurface M in a 4-dimensional sphere is completely determined by its spectrum $\mathrm{Spec}^p(M)$, here $p \in \{0, 1, 2, 3\}$. In higher dimensional sphere, we prove that if $\mathrm{Spec}^p(M) = \mathrm{Spec}^p(M_{m,n-m})$ for $p=0, 1$, where

$$M_{m,n-m} = S^m\left(\sqrt{\frac{m}{n}}\right) \times S^{n-m}\left(\sqrt{\frac{n-m}{n}}\right)$$

is a Clifford torus, then M is $M_{m,n-m}$. Furthermore, we prove that $M_{n,n} \to S^{2n+1}(1)(n \geqq 4)$ is also characterized by $\mathrm{Spec}^p(M_{n,n})$ for some $p=p(n)$.

§ 1. Introduction

For a smooth compact, oriented Riemannian manifold M of dimension n, let $\Lambda^p(M)$ denote the space of C^∞ differential forms of degree $p=0, 1, \cdots, n$ with real coefficients. The Laplace operator Δ of M acting on functions has a natural generalization to $\Lambda^p(M)$. In the theory of spectrum of Laplace operator on $\Lambda^p(M)$, one can see that the interplay among analysis, topology and geometry is even striking (e.g., see [6]). We denote by $\mathrm{Spec}^p(M)$ the spectrum of Laplace operator on $\Lambda^p(M)$.

It is interesting to see the relation of $\mathrm{Spec}^p(M)$ and the geometry on M, which gives rise to the following old question: Does $\mathrm{Spec}^p(M)$ determine the geometry of Riemannian manifold M? The answer to this problem in general case is negative. This is a consequence of the counter example which is given by Milnor in [10]. So the problem is divided into two directions. One direction is to find new counter examples. A series studies along this line have been done by Vigneras [13], Ikeda [8] and others. Another direction is to give an affirmative answer for a special Riemannian manifold. The studies of this direction have also been done by Berger [1], Patodi [11], Tanno [12] and many others.

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In this paper, we will deal with the latter problem on minimal hypersurfaces in an Euclidean sphere. In his paper [5], Donnelly gave a spectral characterization of the totally geodesic minimal submanifold in a sphere. A further study of this aspect was done by Hasegawa, he characterized some concrete minimal submanifolds in a sphere by the spectrum, particularly Veronese manifolds. And he also characterized Clifford tori by their spectrum with some additional geometric conditions (see [7] for details). On the other hand, from the recent work of Chang [2] (or of Cheng and Wan [3]), we know that the totally geodesic 3-sphere, Clifford torus and Cartan's minimal hypersurface are the only closed minimal hypersurfaces of 4-sphere $S^4(1)$ with constant scalar curvature. For these minimal isoparametric hypersurfaces in $S^4(1)$, we can give a spectral characterization as follows.

THEOREM 1. *Let M be a closed minimal hypersurface in $S^4(1)$. If* $\mathrm{Spec}^p(M)=\mathrm{Spec}^p(M_0)$ *for a given $p(0\leqq p\leqq 3)$, where M_0 is the totally geodesic 3-space, or Clifford torus $S^1(\sqrt{1/3})\times S^2(\sqrt{2/3})$, or Cartan's minimal hypersurface. Then M is nothing but M_0.*

We also know that the Clifford tori $M_{m,n-m}=S^m(\sqrt{m/n})\times S^{n-m}(\sqrt{n-m/n})$ $(1\leqq m\leqq n-1)$ are the only closed minimal hypersurfaces of $S^{n+1}(1)$ with the scalar curvature $=n(n-1)-n$ (see [4]). For these minimal hypersurfaces, we like to give a spectral characterization without any additional geometric conditions. Namely, we have

THEOREM 2. *Let M be a closed minimal hypersurface in $S^{n+1}(1)$. If* $\mathrm{Spec}^p(M)=\mathrm{Spec}^p(M_{m,n-m})$ *for $p=0$ and 1, then M is $M_{m,n-m}$.*

Among the all Clifford tori, we will pay a special attention to $S^n(\sqrt{1/2})\times S^n(\sqrt{1/2})=M_{n,n}$ in $S^{2n+1}(1)$. Berger et al. [1] proved that $S^1\times S^1$ is completely determined by $\mathrm{Spec}^0(S^1\times S^1)$ or $\mathrm{Spec}^1(S^1\times S^1)$. Hasegawa [7] proved that if M is a minimal hypersurface in $S^5(1)$ satisfying $\mathrm{Spec}^0(M)=\mathrm{Spec}^0(M_{2,2})$ and its Euler number $\chi(M)\leqq 4=\chi(M_{2,2})$, then $M=M_{2,2}$. Tanno and Masuda [12] proved that if $\mathrm{Spec}^0(M\times N)=\mathrm{Spec}^0(S^3\times S^3)$, then M (or N) is isometric to S^3. For $n\geqq 4$, we obtain the following.

THEOREM 3. *Let M be a closed minimal hypersurface in $S^{2n+1}(1)$ $(n\geqq 4)$ with* $\mathrm{Spec}^p(M)=\mathrm{Spec}^p(M_{n,n})$ *for some $p=p(n)$ (e.g., p is chosen in* (3.23), (3.24) *below). Then M is $M_{n,n}$.*

We will first set up notations and present some formulas and basic results of minimal hypersurfaces in a sphere in § 2, and the proofs of the above theorems will be given in § 3.

§ 2. Preliminaries

Throughout this paper unless otherwise stated, let M be an n dimensional hypersurface in an Euclidean sphere $S^{n+1}(1)$ to have no boundary and to be compact, connected, and of class C^∞. Let R, $\hat{R}$ and ρ be respectively the Riemann curvature tensor, Ricci curvature tensor and scalar curvature of M. We denote by R_{ijkl} (or a similar way to $\hat{R}$) the components of R. The Gauss equation asserts that:

$$R_{ijkl}=\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}+h_{ik}h_{jl}-h_{il}h_{jk} \tag{2.1}$$

where δ_{ij} is the Kronecker symbol and (h_{ij}) the components of the second fundamental form of M in $S^{n+1}(1)$.

For any fixed point $x_0 \in M$, we can choose a frame field $e_1, \cdots, e_n$ such that (h_{ij}) is diagonalized at that point, say

$$h_{ij}=\lambda_i\delta_{ij}.$$

Let $h=\sum_{i=1}^n h_{ii}=\sum_{i=1}^n \lambda_i$ be the mean curvature of M and $S=\sum_{i,j}^n h_{ij}^2=\sum_{i=1}^n \lambda_i^2$ the square of the length of the second fundamental form. Then we have

$$R_{ijkl}=(1+\lambda_i\lambda_j)(\delta_{ik}\delta_{jl}-\delta_{il}\delta_{jk}), \tag{2.2}$$

$$\hat{R}_{ij}=[(n-1)+h\lambda_i-\lambda_i\lambda_j]\delta_{ij}, \tag{2.3}$$

$$\rho=n(n-1)+h^2-S. \tag{2.4}$$

Therefore, the squares of the length of R and $\tilde{R}$ are

$$|R|^2=2S^2-2\sum_{i=1}^n \lambda_i^4+4h^2-4S+2n(n-1), \tag{2.5}$$

$$|\tilde{R}|^2=h^2S+\sum_{i=1}^n \lambda_i^4+n(n-1)^2-2h\sum_{i=1}^n \lambda_i^3+2(n-1)h^2-2(n-1)S \tag{2.6}$$

where $\sum_{i=1}^n \lambda_i^3$ and $\sum_{i=1}^n \lambda_i^4$ are globally defined functions on M.

Since M is compact, for $p=0, 1, \cdots, n$, we set

$$\mathrm{Spec}^p(M)=\{0\leqq\lambda_{0,p}\leqq\lambda_{1,p}\leqq\cdots\uparrow+\infty\}.$$

For those discrete eigenvalues, we have the Minakshisundaram-Pleijel's asymptotic expansion formula as follows:

$$\sum_{i=0}^{\infty} e^{-\lambda_{i,p}t}\sim(4\pi t)^{-(n/2)}(a_{0,p}+a_{1,p}t+a_{2,p}t^2+\cdots), \qquad (t\to 0^+)$$

here the coefficients $a_{k,p}$, $k=0, 1, 2$ were calculated by Patodi in [11] as follows:

$$a_{0,p}=\binom{n}{p}\mathrm{vol}(M), \tag{2.7}$$

(2.8) $$a_{1,p}=\left(\frac{1}{6}\binom{n}{p}-\binom{n-2}{p-1}\right)\int_M \rho dv,$$

(2.9) $$a_{2,p}=\int_M (c_1(n, p)\rho^2+c_2(n, p)|\tilde{R}|^2+c_3(n, p)|R|^2)dv.$$

where dv denotes the volume element of M and

$$c_1(n, p)=\frac{1}{72}\binom{n}{p}-\frac{1}{6}\binom{n-2}{p-1}+\frac{1}{2}\binom{n-4}{p-2};$$

(2.10) $$c_2(n, p)=-\frac{1}{180}\binom{n}{p}+\frac{1}{2}\binom{n-2}{p-1}-2\binom{n-4}{p-2};$$

$$c_3(n, p)=\frac{1}{180}\binom{n}{p}-\frac{1}{12}\binom{n-2}{p-1}+\frac{1}{2}\binom{n-4}{p-2},$$

here $\binom{l}{q}$ is understood to be zero when $l<0$ or $q<0$ or $l<q$.

Now, we are going to recall some fundamental results in the theory of minimal hypersurfaces in an Euclidean Sphere.

THEOREM A (Chern, Do Carmo and Kobayashi [4] or Lawson [9]). *The Clifford tori* $M_{m,n-m}=S^m(\sqrt{m/n})\times S^{n-m}(\sqrt{n-m/n})$, $m=1, \cdots, n-1$ *are the only closed minimal hypersurfaces in* $S^{n+1}(1)$ *satisfying* $S=n$.

THEOREM B (Chang [2] or also Cheng and Wan [3]). *A closed minimal hypersurface with constant scalar curvature in* $S^4(1)$ *is either an equatorial* 3-*sphere, a Clifford torus, or a Cartan's minimal hypersurface.*

§ 3. Proof of the theorems

In this section, we turn to prove the theorems.

Proof of Theorem 1. Since M is a minimal hypersurface (i.e., $h=0$) in $S^4(1)$, thus from (2.4)-(2.6) we have

(3.1) $$\rho=6-S,$$

(3.2) $$|R|^2=2S^2-2\sum\lambda_i^4-4S+12,$$

(3.3) $$|\tilde{R}|^2=\sum\lambda_i^4+12-4S.$$

Let M_0 denote either the totally geodesic 3-sphere, or Clifford torus $S^1(\sqrt{1/3})\times S^2(\sqrt{2/3})$, or Cartan's minimal hypersurface in $S^4(1)$. We know that M_0 has the constant principal curvatures $\lambda_i^0(1\leq i\leq 3)$. Let ρ_0, $\tilde{R}_0$, R_0 and S_0 denote respectively the scalar curvature, Ricci curvature tensor, Curvature tensor and the square of the length of the second fundamental form of M_0. Then $\rho_0=6-S_0$, $|\tilde{R}_0|^2=\sum(\lambda_i^0)^4+12-4S_0$, $|R|^2=2S_0^2-2\sum(\lambda_i^0)^4-4S_0+12$ and $S_0=\sum(\lambda_i^0)^2$. Let

$a_{k,p}$ and $a^0_{k,p}$ be the coefficients of the asymptotic expansion of Minakshisundaram-Pleijel corresponding to M and M_0 respectively. Since $\mathrm{Spec}^p(M)=\mathrm{Spec}^p(M_0)$ for a given $p(0\leqq p\leqq 3)$, we have $a_{k,p}=a^0_{k,p}$ for $k=0, 1, 2$ from the asymptotic expansion formula. Thus, by (2.7)-(2.9), we have

$$\mathrm{vol}(M)=\mathrm{vol}(M_0), \tag{3.4}$$

$$\int_M \rho dv=\int_{M_0}\rho_0 dv_0, \tag{3.5}$$

$$\begin{aligned}&\int_M (c_1(3, p)\rho^2+c_2(3, p)|\tilde{R}|^2+c_3(3, p)|R|^2)dv\\ &=\int_{M_0}(c_1(3, p)\rho_0{}^2+c_2(3, p)|\tilde{R}_0|^2+c_3(3, p)|R_0|^2)dv_0.\end{aligned} \tag{3.6}$$

Here we have used $1/6\binom{3}{p}\neq\binom{1}{p-1}$ for any $p=0, 1, 2, 3$ in (3.5). Substituting (3.1)-(3.3) into (3.6) and making use of (3.4), (3.5), we have

$$\begin{aligned}&\int_M ((c_1(3, p)+2c_3(3, p))S^2+(c_2(3, p)-2c_3(3, p))\sum\lambda_i^4)dv\\ &=\int_{M_0}((c_1(3, p)+2c_3(3, p))S_0^2+(c_2(3, p)-2c_3(3, p))\sum(\lambda_i^0)^4)dv_0.\end{aligned} \tag{3.7}$$

Since M and M_0 are 3-dimensional minimal hypersurfaces in $S^4(1)$, we have $\sum\lambda_i^4=(1/2)S^2$ and $\sum(\lambda_i^0)^4=(1/2)S_0^2$. Hence (3.7) becomes

$$\left(c_1(3, p)+\frac{1}{2}c_2(3, p)+c_3(3, p)\right)\left(\int_M S^2 dv-\int_{M_0}S_0^2 dv_0\right)=0. \tag{3.8}$$

Because for any $p\in\{0, 1, 2, 3\}$, $c_1(3, p)+(1/2)c_2(3, p)+c_3(3, p)\neq 0$. So (3.8) implies that

$$\int_M S^2 dv=\int_{M_0}S_0^2 dv_0=S_0^2\,\mathrm{vol}(M_0). \tag{3.9}$$

On the other hand, (3.5) implies that

$$\int_M S dv=S_0\,\mathrm{vol}(M_0). \tag{3.10}$$

Thus, by Schwarz inequality, we get

$$S_0\,\mathrm{vol}(M_0)=\int_M S dv\leqq\left(\int_M S^2 dv\right)^{1/2}\left(\int_M dv\right)^{1/2}=S_0\,\mathrm{vol}(M_0).$$

Hence

$$S=S_0,$$

i.e. M is a minimal hypersurface in $S^4(1)$ with constant scalar curvature. From Theorem B, we obtain that M is either the totally geodesic 3-space (when $S=$

$S_0=0$), or Clifford torus (when $S=S_0=3$), and or Cartan's minimal hypersurface (when $S=S_0=6$). This proves $M=M_0$. □

Proof of Theorem 2. Since M is a minimal hypersurface in $S^{n+1}(1)$, we have

$$\rho=n(n-1)-S, \tag{3.1'}$$

$$|R|^2=2S^2-2\sum\lambda_i^4-4S+2n(n-1), \tag{3.2'}$$

$$|\tilde{R}|^2=\sum\lambda_i^4+n(n-1)^2-2(n-1)S. \tag{3.3'}$$

Specially, for $M_{m,n-m}=S^m(\sqrt{m/n})\times S^{n-m}(\sqrt{n-m/n})(1\leqq m\leqq n-1)$, it is well known that $S_0=n$ and $\sum(\lambda_i^0)^4=(n-m)^3+m^3/n(n-m)$, where λ_i^0 are the principal curvature of $M_{m,n-m}$. Since $\mathrm{Spec}^p(M)=\mathrm{Spec}^p(M_{m,n-m})$ for $p=0, 1$. By the same arguments as in the proof of Theorem 1, we have

$$\mathrm{vol}(M)=\mathrm{vol}(M_{m,n-m}), \tag{3.11}$$

$$\int_M \rho dv=\int_{M_{m,n-m}}\rho_0 dv_0, \tag{3.12}$$

and for $p=0, 1$,

$$\int_M (c_1(n, p)\rho^2+c_2(n, p)|\tilde{R}|^2+c_3(n, p)|R|^2)dv \tag{3.13}$$

$$=\int_{M_{m,n-m}}(c_1(n, p)\rho_0{}^2+c_2(n, p)|\tilde{R}_0|^2+c_3(n, p)|R_0|^2)dv_0.$$

Substituting (3.1')-(3.3') and (3.11) (3.12) into (3.13), we get that, for $p=0, 1$,

$$\int_M [(c_1(n, p)+2c_3(n, p))S^2+(c_2(n, p)-2c_3(n, p))\sum\lambda_i^4]dv \tag{3.14}$$

$$=(c_1(n, p)+2c_3(n, p))n^2\,\mathrm{vol}(M_{m,n-m})$$

$$+(c_2(n, p)-2c_3(n, p))\frac{(n-m)^3+m^3}{n(n-m)}\mathrm{vol}(M_{m,n-m}).$$

We regard (3.14) as the linear equations $\int_M S^2 dv$ and $\int_M \sum\lambda_i^4 dv$. Since

$$\det\begin{pmatrix} c_1(n, 0)+2c_3(n, 0) & c_2(n, 0)-2c_3(n, 0) \\ c_1(n, 1)+2c_3(n, 1) & c_2(n, 1)-2c_3(n, 1)\end{pmatrix}=\frac{1}{90}\neq 0,$$

(3.14) has the unique solutions:

$$\int_M S^2 dv=n^2\,\mathrm{vol}(M_{m,n-m}), \tag{3.15}$$

$$\int_M \sum\lambda_i^4 dv=\frac{(n-m)^3+m^3}{n(n-m)}\mathrm{vol}(M_{m,n-m}).$$

On the other hand, from (3.12) we have

(3.16) $$\int_M S dv = n \operatorname{vol}(M_{m,n-m}).$$

Thus, from Schwarz inequality, the first equation of (3.15) and (3.16) imply that

$$S=n.$$

i.e. M is a closed minimal hypersurface in $S^{n+1}(1)$ satisfying $S=n$. From Theorem A, we obtain that M is one of $\{S^k(\sqrt{k/n})\times S^{n-k}(\sqrt{n-k/n})\}_{k=1}^{n-1}$. Among those Clifford tori only $M_{m,n-m}=S^m(\sqrt{m/n})\times S^{n-m}(\sqrt{n-m/n})$ satisfying the second equation of (3.15). Therefore M is nothing but $M_{m,n-m}$. □

Proof of Theorem 3. Let M be a minimal hypersurface in $S^{2n+1}(1)$ $(n\geqq 4)$, and $\operatorname{Spec}^p(M)=\operatorname{Spec}^p(M_{n,m})$ for some p. For Clifford torus $M_{n,n}=S^n(\sqrt{1/2})\times S^n(\sqrt{1/2})\to S^{2n+1}(1)$, by a direct calculation, we know that the square of the length of the second fundamental form equals to $2n$ and the principal curvatures $\lambda_i^0=1$ for $1\leqq i\leqq n$, $\lambda_i^0=-1$ for $n+1\leqq i\leqq 2n$, which lead to $\sum_{i=1}^{2n}(\lambda_i^0)^4=2n$. Therefore, with the same arguments as in the proof of Theorem 2, we have:

(3.17) $$\operatorname{vol}(M)=\operatorname{vol}(M_{n,n})$$

(3.18) $$\left(\frac{1}{6}\binom{2n}{p}-\binom{2n-2}{p-1}\right)\int_M S dv=\left(\frac{1}{6}\binom{2n}{p}-\binom{2n-2}{p-1}\right)\int_{M_{n,n}} 2n\, dv_0$$

and

(3.19) $$\int_M (c_1(2n,p)\rho^2+c_2(2n,p)|\tilde{R}|^2+c_3(2n,p)|R|^2)dv$$
$$=\int_{M_{n,n}}(c_1(2n,p)\rho_0^2+c_2(2n,p)|\tilde{R}_0|^2+c_3(2n,p)|R_0|^2)dv_0.$$

The crucial point in this case is to show that there is at least a $p=p(n)$ such that

$$c_1(2n,p)+2c_3(2n,p)+\frac{1}{2n}(c_2(2n,p)-2c_3(2n,p))<0,$$
(3.20) $$c_2(2n,p)-2c_3(2n,p)\geqq 0,$$
$$\frac{1}{6}\binom{2n}{p}\neq\binom{2n-2}{p-1}.$$

If (3.20) holds for some p, we get, from (3.18)-(3.20),

(3.18′) $$\int_M S dv=2n \operatorname{vol}(M_{n,n})$$

and

(3.21)
$$(c_1+2c_3)\int_M (S^2-4n^2)dv=-(c_2-2c_3)\int_M (\sum \lambda_i^4-2n)dv$$
$$\leqq -(c_2-2c_3)\int_M \left(\frac{S^2}{2n}-2n\right)dv.$$

Here we have used the inequality $\sum_{i=1}^{2n}\lambda_i^4 \geqq ((\sum\lambda_i^2)^2/2n)=(S^2/2n)$ in (3.21). It is easy to see that (3.21) is equivalent to

(3.22)
$$\int_M S^2 dv \leqq \int_M 4n^2 dv,$$

from the first equation of (3.20). Making use of (3.22), (3.18′) and Schwarz inequality, we obtain $S=2n$. By Theorem A, we know that M must be one of the Clifford tori $\{M_{m,2n-m}\}_{1\leq m\leq 2n-1}$. But from (3.21) we also know that M must satisfy $\int_M \sum\lambda_i^4 dv=2n\,\mathrm{vol}(M)$. Among the Clifford tori $\{M_{m,2n-m}\}$, only $M_{n,n}=S^n(\sqrt{1/2})\times S^n(\sqrt{1/2})$ satisfies the restriction. This proves $M=M_{n,n}$.

It remains to indicate that there exists at least a $p=p(n)$ satisfying (3.20). When $8\leqq l=2n\leqq 40$, we put

(3.23)
$$p=p(n)=\begin{cases} 2 & \text{when } l=8,\ 10, \\ 3 & \text{when } l=12, \\ 1 & \text{when } 14\leqq l\leqq 40. \end{cases}$$

Obviously, for a pair (l, p) given in (3.23) the third equation of (3.20) is satisfied. However, it is interesting to note that $c_1(l, p)+2c_3(l, p)>0$, $c_2(l, p)-2c_3(l, p)>0$ except for $l=10$. For $l=10$, there is only $p=2$ (or 8) satisfying $c_1(10, 2)+2c_3(10, 2)=-0.0416\cdots$, $c_2(10, 2)-2c_3(10, 2)=1.583\cdots$, which imply that $c_1(10, 2)+2c_3(10, 2)+(1/10)(c_2(10, 2)-2c_3(10, 2))=0.116\cdots>0$. Therefore, any p given in (3.23) satisfies (3.20) in this case. When $l=2n\geqq 40$, from (2.10) we have

$$c_1+2c_3=\left(\frac{3l^2-3l-40pl+40p^2}{120p(l-p)}\frac{(l-2)(l-3)}{(p-1)(l-p-1)}+\frac{3}{2}\right)\binom{l-4}{p-2},$$
$$c_2-2c_3=\left(\frac{-l^2+l+40pl-40p^2}{60p(l-p)}\frac{(l-2)(l-3)}{(p-1)(l-p-1)}-3\right)\binom{l-4}{p-2}.$$

Taking

(3.24)
$$p=\left[\frac{l-\sqrt{(7/10)l^2+(3/10)l}}{2}\right]\left(\text{or } \left[\frac{l+\sqrt{(7/10)l^2+(3/10)l}}{2}\right]+1\right),$$

where $[x]$ denotes the biggest integer which is not larger than x, by a direct calculation, we conclude

$$c_1(l, p)+2c_3(l, p)>0, \qquad c_2(l, p)-2c_3(l, p)>0, \qquad \frac{1}{6}\binom{l}{p}\neq\binom{l-2}{p-1},$$

i.e., (3.20) holds for the p. The proof of Theorem 3 is completed. □

Remark. We can also give an analogue discussion in complex version.

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