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DECOMPOSITION NUMBERS FOR SPIN CHARACTERS OF EXCEPTIONAL WEYL GROUPS OF TYPE E_n

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Introduction

It is well known that the groups, $W(E_6)\cong S_4(3).2\cong U_4(2).2$; $W(E_7)\cong C_2\times B_3(2)\cong C_2\times D_7(2)$, where C_2 is the cyclic group of order 2 and $W(E_8)\cong 2.\ O_8^+(2).2\cong 2.\ D_4(2).\ 2$ (see, [CCN]),

In this paper, which is a continuation of [Sa], the decomposition matrices for the spin characters of the exceptional Weyl groups of type E_n ($n=6, 7, 8$), for the prime numbers $p=5, 7$ are determined. In all cases except $W(E_8)$ and $p=5$, the relevent prime number divides, the order of the group to the first power only.

First we use the central characters to split the ordinary characters into p-blocks (For general information about p-blocks, see [CR]). Let B be a p-block of a group G and let $\Psi_1, \cdots, \Psi_s$ and $\phi_1, \cdots, \phi_r$, respectively, denote the ordinary and p-modular irreducible characters of G in B. The restriction of each Ψ_i to p-regular classes of G, denoted by $\bar{\Psi}_i$ is a p-modular character of G and $\bar{\Psi}_i=\sum_{j=1}^{r} d_{ij}\phi_j$ $(1\leqq i\leqq s)$ where d_{ij}'s are non-negative ingers, called the decomposition numbers of B for the prime p. The $(s\times r)$ matrix $D_P^B(G)=(d_{ij})$ is called the decomposition matrix of B for the prime p. Furthermore, a principal character $\sum_{i=1}^{s} c_i\Psi_i$ will be identified with the column of integers $c=(c_i)$ and it may be indecomposable or sum of principal indecomposable characters in B.

One way to constructs $D_P^B(G)$ is to find the principal indecomposable characters in B. This method is described by James and Kerber [JK] and is as follows; suppose that the matrix

$$R_P^B(G)=(c^{(0)}, c^{(1)}, \cdots, c^{(n_1)}, \cdots, c_{r-1}^{(0)}, c_{r-1}^{(1)}, \cdots, c_{r-1}^{(n_{r-1})}, c_r)$$

of principal characters has been found such that, with a suitable arrangement of the ordinary irreducible characters, for $0\leqq k_i\leqq n_i$ and $i=1, \cdots, (r-1)$, each of the matrices

$$R_P^B(G)=(c_1^{(k_1)}, \cdots, c_{r-1}^{(k_{r-1})}, c_r)$$

has the triangle of zeros above the main diagonal and 1's on the main diagonal.

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Then $D_p^B(G)$ has the same shape as $R_p^B(G)$; the entries in $R_p^B(G)$ are greater than or equal to the corresponding entries in $D_p^B(G)$; $d_r=c_r$ and for any $i=1, \cdots, r-1$; $k_i=0, 1, \cdots, n_i$

$$d_i=c_j^{(k_i)} \longrightarrow \sum_{j=1+i}^{r}(a_j^{(k_i)})d_j$$

where $a_j^{(k_i)}$ are non-negative integers and d_i ($i=1, \cdots, r$) are the columns of $D_p^B(G)$.

If G is a group, p a prime then the symbols irr(G), irm$_p(G)$, indec$_p(G)$ denote the full set of ordinary irreducible, p-modular irreducible and principal p-indecomposable characters of G, respectively. Similarly, the full set of irreducible spin, p-modular spin and principal p-indecomposable spin characters of G are denoted respectively by sirr(G), sirm$_p(G)$ and sindec$_p(G)$. If there is no confusion about p, then we will omit it. Furthermore $\uparrow G(\downarrow G)$ denote the induction from (restriction to) a subgroup of G and the symbol ϕ (least degree character of corresponding column) denote the p-modular irreducible character corresponding to that column in the matrix.

1. $W(E_6)$, $p=5$

There are 11 elements in sirm(E_6), which are distributed over ten 5-blocks. The elements 40_s, 40_{ss}, 120_s, 60_s, 60_{ss}, 80_s, 80_{ss}, 20_s and 20_{ss} of sirr(E_6), each form a 5-block on its own. There is also 5-block of full deffect containing 8_s, 72_s, 64_s, $64_{ss}\in$sirr(E_6), the later two being exceptional for $p=5$, (see, [Mo, p. 578]) and hence the Brauer tree is as follows, (see, [Fe; Theorem 9.2; p. 307]),

$$8_s \text{——} 72_s \text{——} 64_s=64_{ss}\,.$$

2. $W(E_7)$, $p=5$

In this case, the elements of sirr(E_7) are distributed into ten 5-blocks. The following 280_s, 720_s, 560_s, 120_s, 280_r, 720_r, 560_r and 120_r are those elements of sirr(E_7), whose degrees are divisible by 5, each form its own block are in sindec(E_7) and are also belonging to sirm(E_7).

There remain four blocks (with full defect) the first two blocks each contains four elements of sirr(E_7), whereas the third and fourth blocks each contains five elements of sirr(E_7) namely:

B_1: 48_s, 112_s, 64_s and 64_{ss}

B_2: 48_r, 112_r, 64_r and 64_{rr}

B_3: 8_s, 168_s, 112_{ss}, 512_s and 448_s

B_4: 8_r, 168_r, 112_{rr}, 512_r and 448_r.

On consideration of degrees, the Brauer trees of the block B_1 and B_2 are respectively, therefore as follows;

$$48_s \text{---} 112_s \text{---} 64_s = 64_{ss}$$

$$48_r \text{---} 112_r \text{---} 64_r = 64_{rr}\,.$$

We now determine the decomposition matrices corresponding to the block B_3 and B_4.

Inducing the elements 40_{ss}, 120_s, 40_s of sindec(E_6) to $W(E_7)$, we get the following decompositions.

$$40_{ss}\uparrow E_7 = 112_{ss}+560_s+448_s+112_{rr}+560_r+448_r\,.$$

$$120_s\uparrow E_7 = 280_s+512_s+2.720_s+560_s+448_s+120_s$$

$$+280_r+512_r+2.720_r+560_r+448_r+120_r\,.$$

and

$$40_s\uparrow E_7 = 48_s+168_s+280_s+112_s+512_s$$

$$+48_r+168_r+280_r+112_r+512_r\,.$$

Now considering the block decomposition of characters, the direct summand of elements of sindec(E_7) will imply the connection of some pairs of characters in the Brauer trees corresponding to the block B_3 and B_4 which together with the fact that $8_s(8_r)$ and $168_s(168_r)$ are both congruent to -2(modulo 5) is enough to uniquely determine the Brauer trees for B_3 and B_4 as given by:

$$8_s \text{---} 112_{ss} \text{---} 448_s \text{---} 512_s \text{---} 168_s$$

and

$$8_r \text{---} 112_{rr} \text{---} 448_r \text{---} 512_r \text{---} 168_r\,.$$

3. $W(E_8)$, $p=5$

Here $|\mathrm{sirm}(E_8)|=23$, and they are distributed over 11 5-blocks. Each of the elements 5600_s, 4800_s, 5600_{ss}, 800_s, 2800_{ss}, 2800_s, 5600_{sss}, 11200_s and 8400_s of sirr(E_8) forms a block on its own and the elements 320_s, 1120_s, 6720_s, 6480_s, $1680_s \in$ sirr(E_8) lies in a block of defect 1. Furthermore, the Brauer tree of this block is:

$$320_s \text{---} 1680_s \text{---} 6720_s \text{---} 6480_s \text{---} 1120_s\,.$$

Since the spin principal characters $(112_s+64_s+64_{ss})\uparrow E_8$, $560_s\uparrow E_8$, and $(112_{ss}+448_s)\uparrow E_8$, induced from $W(E_7)$ gives the connection of 320_s with 1680_s; 6720_s with 6480_s and 6480_s with 1120_s and this can be seen from their respective decompositions given below:

$$320_s+448_s+1680_s+2592_s+5600_{ss}+9072_s+7168_s+2.8192_s+2.7168_{ss}\,.$$

$$2592_s+2.5600_s+2.4800_s+2.9072_s+2800_s+7168_s+1120_s+2.8400_s$$

$$+2.11200_s+6480_s+2.8192_s+2016_{ss}+2016_{sss}+2.7168_{ss}+1344_s\,.$$

And

$$448_s+224_s+2592_s+1344_s+5600_s+2.4800_s+2.9072_s+3.8400_s+2.11200_s$$

$$+6720_s+6480_s+2.8192_s+2016_{ss}+2016_{sss}+2.7168_s+896_s.$$

The remaining elements ot $\mathrm{sirr}(E_8)$ belong to a principal block. Since 13 elements of $\mathrm{sirm}(E_8)$ has been determined therefore the principal block contains 10 elements which we have to determined yet.

Let $\mathrm{sindec}(E_7)=\{\phi_1, \cdots, \phi_{20}\}$ and $\mathrm{sindec}(S_9)=\{\S_1, \cdots, \S_6\}$ (see [Ya] for $D_{\langle 9,5\rangle}$), as given in the Appendix. In [KM], it has been shown that 56_P, 28_P, $8_P \in \mathrm{irm}(E_8)$.

Now consider the following principal spin characters of $W(E_8)$ which are formed by inducing from $W(E_7)$, S_9 and taking the products of characters as appropriate and then restricting them to the principal block. We obtain the matrix $R_5^{B_1}(E_8)$; where the columns correspond to $c_1=\phi_1\uparrow E_8$, $c_1'=\S_1\uparrow E_8$, $c_2=\phi_5\uparrow E_8$, $c_3=\phi_6\uparrow E_8$, $c_4=(\S_2\uparrow E_8)/2$, $c_5=8_P\otimes[1680_s+6720_s]$, $c_6=\S_6\uparrow E_8)/2$, $c_6'=28_P\otimes 8400_s$, $c_7=\phi_9\uparrow E_8$, $c_8=8_P\otimes 4800_s$, $c_9=28_P\otimes 2800_s$ and $c_{10}=(\phi_8\uparrow E_8)/2$ respectively, given below:

$R_5^{B_1}(E_8)$

	c_1	c_1'	c_2	c_3	c_4	c_5	c_6	c_6'	c_7	c_8	c_9	c_{10}
16_s	1	1										
112_s	1		1									
448_s	1		1	1								
224_s	1	2			1							
448_{ss}	1	1	1			1						
896_s		2			2		1	1				
1344_s	1	2			1				1			
1344_{ss}	1	2	1							1		
2016_{ss}		2			2		1				1	
9072_s	1	6	1	1	4		3	4	1	1		1
2592_s	1	2	1	1	1	1	1				1	
2016_s		2	1		1		1	1		1		
7168_s		3	1	1	1	1	2	3		1		1
8192_s		4		2	3	1	3	3			1	1
2016_{sss}		2			2		1	1	1			
7168_{ss}		2		2	2		2	3	1			1

The final colum $c_{10}\in\text{sindec}(E_8)$, since no subsum of $9072_s+7168_s+8192_s+7168_{ss}$, is congruent to zero (mod 25) and we denote it by d_{10}. Also by the same argument, c_9, c_8, c_7, $c_5\in\text{sindec}(E_8)$ and therefore $d_i=c_i$, for $i=5, 7, 8, 9$.

Then the possibilities for c_6 are $d_6=c_6-\alpha\cdot d_9-\beta\cdot d_{10}$, where $\alpha=0$ or 1 and $\beta\in[0, 1, 2]$. Now, since $c_{11,6}=1$ and $c'_{11,6}=0$, where $c_{i,j}$ has the usual meaning of matrix entries and therefore $c''_6:=c_6-d_9$ is a principal spin character. We consider later whether any copies of d_{10} can be subtracted from c''_6.

The columns c''_6, d_7, d_9 and d_{10} may be subtracted from c_4 once each. Whatever is the d_4, the possibilities for c_3 are $d_3=c_3$ or c_3-d_{10}. It can be verified that

$$\begin{aligned}28_P\otimes 5600_s=&448_s+1680_s+2592_s+2.1344_s+5.5600_s+8400_s\\&+6480_s+4800_s+2.5600_{sss}+2.11200_s+2.6720_s\\&+2.9072_s+8192_s+2.2016_{sss}+3.7168_{ss}+2800_s,\end{aligned}$$

and hence

$$P_1:=448_s+2592_s+2.1344_s+2.9072_s+8192_s+2.2016_{sss}+3.7168_{ss}$$

as a sum of elements of sindec(E_8) in the principal block of $W(E_8)$. However, $P_1=c_3+2\cdot d_7-d_{10}$ and this implies that d_{10} is a component of the column c_3. Thus we put $d_3=c_3-d_{10}$. Clearly $d_3\in\text{sindec}(E_8)$, by the usual subsum argument. Now the possibilities $d_2=c_2$ or c_2-d_8. It can be checked that

$$\begin{aligned}56_P\otimes[320_s+1680_s]=&16_s+3.112_s+4.320_s+224_s+3.448_{ss}+5.1680_s\\&+4.2592_s+2016_{ss}+1344_s+2.5600_s+4800_s\\&+2.5600_{ss}+9072_s+1120_s+3.2800_s+11200_s\\&+6720_s+1344_{ss}+6480_s+8192_s+5600_{sss}\end{aligned}$$

and hence

$$\begin{aligned}P_2:=&16_s+3.112_s+3.448_s+224_s+3.448_{ss}+2016_{ss}+1344_s\\&+9072_s+8192_s+1344_{ss}+4.2592_s\end{aligned}$$

as a sum of elements in sindec(E_8) in the principal block of $W(E_8)$. But $P_2=c_1+2\cdot c_2-2\cdot d_8+d_9$ implies that c_2-d_8 is a sum of elements in sindec(E_8) as c_1-d_8 is not. However, the usual subsum argument shows that $d_2=c_2-d_8\in\text{sindec}(E_8)$. As $c_{2,1}=1$ and $c'_{2,1}=0$, so $d_1=c_1-d_2$.

Now

$$\begin{aligned}P_3:=(&300, 30, -6, 8, -1, -2, 2, -1, 0, 3, 0, 2, 30, 3,\\&0, 30, 2, 2, 0, 20, 6, 0, 2)\end{aligned}$$

is in indec(E_8) whose values on 5-regular (-1)-regular classes are given. Clearly, $\phi(224_s):=224_s-\phi(16_s)$ is a modular spin character of degree 208 of $W(E_8)$, hence $\phi(224_s)\otimes P_3$, is a principal spin character of $W(E_8)$ and it contains

$$c_4' :=224_s+2592_s+9072_s+2.8192_s+2.2016_{ss}+896_s$$

which is a sum of elements of sindec(E_8) in the principal block, as

$$\phi(244_s\otimes P_3)=224_s+2.8192_s+2.2016_{ss}+2.1120_s+11200_s$$
$$+2800_s+2.6480_s+2592_s+9072_s+896_s\,.$$

Thus at this stage, the matrix $R_5^{B_2}(E_8)$ is our next approximation for the decomposition matrix corresponding to the principal block. It contains all the simplified columns, c_4 and a new column c_4'.

$R_5^{B_2}(E_8)$

	d_1	d_2	d_3	c_4	c_4'	d_5	c_6''	d_7	d_8	d_9	d_{10}
16_s	1										
112_s		1									
448_s		1	1								
224_s	1			1	1						
448_{ss}		1				1					
896_s				2	1		1				
1344_s	1			1				1			
1344_{ss}	1								1		
2016_{ss}				2	2					1	
9072_s	1			4	1		3	1	1		1
2592_s		1	1	1	1	1				1	
2016_s				1			1		1		
7168_s				1		1	2		1		1
8192_s			1	3	2	1	2			1	1
2016_{sss}				2			1	1			
7168_{ss}			1	2			2	1			1

We must still resolve whether the following are possible:

(i) $c_4'-\alpha_9\cdot d_9$ where $\alpha_9\in[0,\,1]$

(ii) $c_6''-\alpha_{10}\cdot d_{10}$ where $\alpha_{10}\in[0,\,1,\,2]$.

We first show that $c_4'-d_9$ is a principal spin character. This is equivalent to showing that there exist $\phi(2016_{ss})\in \mathrm{sirm}(E_8)$, having degree 1808 corresponding to the 9th column. We shall remove this ambiguity by showing that the character $\phi(2016_{ss})$ with degree 1600 arising from the matrix $R_5^{B_2}(E_8)$ not a modular spin character. Now the product character $\phi(16_s)\otimes\phi(2016_{ss})$ is a sum of elements in $\mathrm{irm}(E_8)$. However, these elements also occur without non-negative coefficients, so it follows that $d_4=c_4'-d_9\in \mathrm{sindec}(E_8)$.

We note that in addition to d_4, the column c_4 may contains c_6'', d_7 and d_9 once each. Comparing the coefficient of 9072_s in c_4, with the column d_4, c_6'', d_7 and d_9 implies that d_{10} is a constituent of c_6''.

Hence

$$d_6=c_6''-\alpha_{10}\cdot d_{10}\,, \qquad \text{where } \alpha_{10}\in[1,\,2]\,.$$

If $d_6=c_6''-d_{10}$, then there must exist $\phi(9072_s)\in \mathrm{sirm}(E_8)$, of degree 4816 which is associated with the last column. Now by computing the product $\phi(224_s)\otimes\phi(9072_s)$ and then writing it in term of elements of $\mathrm{irm}(E_8)$, we see that 175_P, $125_n\in \mathrm{irm}(E_8)$ having degrees divisible by 5^2, occur with negative entries which is not true. Hence $c_6''-2\cdot d_{10}\in \mathrm{sindec}(E_8)$ by the usual argument and we denote it by d_6. This completely determines the decomposition matrix $D_5(E_8)$ corresponding to the principal block and as given by:

$D_5(E_8)$

16_s	1									
112_s		1								
448_s		1	1							
224_s	1			1						
448_{ss}		1			1					
896_s				1		1				
1344_s	1						1			
1344_{ss}	1							1		
2016_{ss}				1					1	
9072_s	1			1		1	1	1		1
2592_s		1	1		1				1	
2016_s						1		1		
7168_s					1			1		1
8192_s			1	1	1				1	1
2016_{sss}						1	1			
7168_{ss}			1				1			1
	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

4. $W(E_7)$, $p=7$

Here $|\mathrm{sirm}(E_7)|=24$ and as the prime 7 divides the degrees of each of the following elements 168_s, 280_s, 112_s, 112_{ss}, 560_s, 448_s, 168_r, 280_r, 112_r, 112_{rr}, 560_r and 448_r of $\mathrm{sirr}(E_7)$, so each form its own block and therefore are in $\mathrm{sindec}(E_7)$.

There remains two other blocks (with full defect), each of which contains seven elements of $\mathrm{sirr}(E_7)$, namely:

$$B_1:\ 8_s,\ 48_s,\ 512_s,\ 720_s,\ 120_s,\ 64_s \text{ and } 64_{ss}.$$

$$B_2:\ 8_r,\ 48_r,\ 512_r,\ 720_r,\ 120_r,\ 64_r \text{ and } 64_{rr}.$$

Since 8_s, 40_s, 72_s, 64_s, $64_{ss} \in \mathrm{indec}(E_6)$ and thus

$$8_s \uparrow E_7 = 8_s + 48_s + 168_s + 8_r + 48_r + 168_r$$

$$40_s \uparrow E_7 = 48_s + 168_s + 280_s + 112_s + 512_s + 48_r + 168_r + 280_r + 112_r + 512_r$$

$$72_s \uparrow E_7 = 112_s + 112_{ss} + 512_s + 720_s + 560_s + 112_r + 112_{rr} + 512_r + 720_r + 560_r$$

$$64_s \uparrow E_7 = 720_s + 560_s + 448_s + 64_s + 720_r + 560_r + 448_r + 64_r$$

$$64_{ss} \uparrow E_7 = 720_s + 560_s + 448_s + 64_{ss} + 720_r + 560_r + 448_r + 64_{rr}.$$

Now blocks distribution of characters and characters degrees argument implies that the Brauer tree for B_1 is as follows:

$$\begin{tikzcd}[arrows=-] & & & 64_s \arrow[d] & \\ 8_s \arrow[r] & 48_s \arrow[r] & 512_s \arrow[r] & 720_s \arrow[r] \arrow[d] & 120_s \\ & & & 64_{ss} & \end{tikzcd}$$

Similarly, the Brauer tree corresponding to the other block is obtained and is given below:

$$\begin{tikzcd}[arrows=-] & & & 64_r \arrow[d] & \\ 8_r \arrow[r] & 48_r \arrow[r] & 512_r \arrow[r] & 720_r \arrow[r] \arrow[d] & 120_r \\ & & & 64_{rr} & \end{tikzcd}$$

5. $W(E_8)$, $p=7$

There are twentynine 7-regular α-regular classes in $W(E_8)$ so there are twentynine 7-modular irreducible spin characters.

Each of the irreducible spin character of $W(E_8)$, namely: 112_s, 448_s, 224_s, 448_{ss}, 1680_s, 1344_s, 5600_s, 2016_s, 5600_{ss}, 9072_s, 2800_{ss}, 5600_{sss}, 7168_s, 1120_s, 8400_s, 11200_s, 6720_s, 2800_s, 1344_{ss}, 2016_{ss}, 2016_{sss}, 7168_{ss} and 896_s, whose degrees are divisible by 7, from its own block.

All the remaining spin characters namely: 16_s, 320_s, 2592_s, 800_s, 4800_s, 6480_s and 8192_s of the group $W(E_8)$ belong to a single block of defect one.

From above section, we have that (8_s+48_s), 112_s, 112_{ss}, 448_s and (8_s+512_s) are in sindec(E_7). Then

$$(8_s+48_s)\uparrow E_8=16_s+2.112_s+320_s+2.448_{ss}+1680_s+2016_s+5600_{ss}+2.1344_s.$$

$$112_s\uparrow E_8=320_s+448_s+1680_s+2592_s+5600_s+9072_s+7168_s.$$

$$112_{ss}\uparrow E_8=448_s+224_s+2592_s+1344_s+4800_s+8400_s+9072_s.$$

$$448_s\uparrow E_8=5600_s+4800_s+9072_s+898_s+2.8400_s+2.11200_s+6720_s$$
$$+6480_s+2.8192_s+2016_{ss}+2016_{sss}+2.7168_{ss}.$$

$$280_s\uparrow E_8=2016_s+2.5600_{ss}+9072_s+800_s+2.5600_{sss}+7168_s$$
$$+11200_s+6720_s+1344_{ss}+6480_s.$$

and

$$(48_s+512_s)\uparrow E_8=112_s+320_s+448_s+2.1680_s+2592_s+5600_s+2.2016_s$$
$$+3.5600_{ss}+2.9072_s+5600_{sss}+2.7168_s+8400_s$$
$$+2.11200_s+2.6720_s+2800_s+1344_{ss}+6480_s+8192_s.$$

Now restricting these six decomposition to a block of nonzero defect of $W(E_8)$, we get the following matrix of principal characters:

$$R_7(E_8):=\begin{array}{r|cccccc|}
16_s & 1 & & & & & \\
320_s & 1 & 1 & & & & 1 \\
2592_s & & 1 & 1 & & & 1 \\
4800_s & & & 1 & 1 & & \\
800_s & & & & & 1 & \\
6480_s & & & & 1 & 1 & 1 \\
8192 & & & & 2 & & 1
\end{array}$$

Since the characters 320 and 6480 are not 7-conjugates and belong to the same block therefore they have no modular constituent in common. Hence we must subtract the second column from the last column. The same arguments implies that the last column must also be subtracted from the fourth column, since 4800_s and 6480_s are not 7-conjugates.

Hence the Brauer tree for this block is as follows:

$$16_s \text{——} 320_s \text{——} 2592_s \text{——} 4800_s \text{——} 8192_s \text{——} 6480_s \text{——} 800_s\,.$$

Appendix

$D_5(E_7)$

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	ϕ_{12}	ϕ_{13}	ϕ_{14}	ϕ_{15}	ϕ_{16}	ϕ_{17}	ϕ_{18}	ϕ_{19}	ϕ_{20}
8_s	1																			
112_{ss}	1	1																		
448_s		1	1																	
512_s			1	1																
168_s				1																
48_s					1															
112_s					1	1														
64_s						1														
64_{ss}						1														
280_s							1													
720_s								1												
560_s									1											
120_s										1										
8_r											1									
112_{rr}											1	1								
448_r												1	1							
512_r													1	1						
168_r														1						
48_r															1					
112															1	1				
64_r																1				
64_{rr}																1				
280_r																	1			
720_r																		1		
560_r																			1	
120_r																				1

$D_{\langle 9,5\rangle}$

$\langle 9\rangle$	1					
$\langle 54\rangle$	1	1				
$\langle 54\rangle$	1	1				
$\langle 432\rangle$		1				
$\langle 81\rangle$			1			
$\langle 81\rangle$				1		
$\langle 63\rangle$			1		1	
$\langle 63\rangle$				1		1
$\langle 531\rangle$					1	1
	$\S_1$	$\S_2$	$\S_3$	$\S_4$	$\S_5$	$\S_6$

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