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MARGOLIS HOMOLOGY AND MORAVA K-THEORY FOR COHOMOLOGY OF THE DIHEDRAL GROUP

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Abstract

In this paper, we note that the Margolis homology $H(H^*(BG;\boldsymbol{Z}/p), Q_n)$ relates deeply the Morava K-theory $K(n)^*(BG)$. In particular we compute $K(n)^*(BD)$ for the dihedral group D by using Atiyah-Hirzebruch spectral sequence.

§ 0. Introduction.

Let G be a finite group and $H^*(BG;\boldsymbol{Z}/p)$ be the cohomology of G with the coefficient $\boldsymbol{Z}/p$ for a prime number p. Since the restriction map to a sylow p-group S of G is injective, it is important to know the cohomology of p-groups. However it seems a very difficult problem to compute $H^*(BS;\boldsymbol{Z}/p)$ when S is a nonabelian p-group. In this paper we consider the case $p=2$. The smallest nonabelian 2-groups S have the order 2^3, which have two types D and Q; the dihedral and the quaternion groups. The cohomology $H^*(BG;\boldsymbol{Z}/p)$, $G=D, Q$ are determined by Atiyah, Evens respectively [A], [E].

In this paper we first study the Margolis homology $H(H^*(BD;\boldsymbol{Z}/2), Q_n)$ for the dihedral group D and next study Morava K-theory $K(n)^*(BD)$ where $K(n)^*(-)$ is the cohomology theory with the coefficient $K(n)^*=\boldsymbol{Z}/p[v_n, v_n^{-1}]$. Such $K(n)^*(BD)$ are given by Tezuka—Yagita [T-Y2] using BP-theory. However we use here only Atiyah—Hizebruch spectral sequence for $K(n)^*$ theory. In particular we correct some inaccuracy of results in Tezuka—Yagita [T-Y2].

Quite recently I. J. Leary decided the muliplicative structure of $H^*(BG;\boldsymbol{Z}/p)$ for groups of order p^3 [Ly2] by using the cohomology of group $\tilde{G}$ which is the central product of G and 1-dimensional sphere S^1. The cohomology ring $H^*(BD;\boldsymbol{Z}/2)$ is very easy. But its Margolis homology seems not so easy. Hence we first study Margolis homology of $H^*(B\tilde{D};\boldsymbol{Z}/2)$ and next consider that of $H^*(BD;\boldsymbol{Z}/2)$. I thank Nobuaki Yagita who introduced me to these problems.

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§ 1. The nonabelian p-group of the order 8.

Let G be a nonabelian group of $|G|=8$. Then G is one of the following groups

$$D=\langle a, b \mid a^4=b^2=1, [a, b]=a^2), \text{ dihedral group,}$$

$$Q=\langle a, b \mid [a^4=b^4=1, [a, b]=a^2=b^2\rangle, \text{ quaternion group.}$$

For each group G, there is a central extension

(1.1) $$1 \longrightarrow \boldsymbol{Z}/2 \longrightarrow G \longrightarrow \boldsymbol{Z}/2 \oplus \boldsymbol{Z}/2 \longrightarrow 1$$

which induces the spectral sequence

$$E_2^{*,*}=H^*(B(\boldsymbol{Z}/2\oplus\boldsymbol{Z}/2\,;\,\boldsymbol{Z}/2),\ H^*(B(\boldsymbol{Z}/2\,;\,\boldsymbol{Z}/2))) \Longrightarrow H^*(BG\,;\,\boldsymbol{Z}/2)\,.$$

where $E_2^{*,*}=S_2\otimes\boldsymbol{Z}/2[z]$ and $S_2=\boldsymbol{Z}/2[x_1, x_2]$.

It is known that ([Ls], [Q]) that

$$d_2 z=\begin{cases} x_1x_2 & \text{for } G=D \\ x_1,\ x_2+x_1^2+x_2^2 & \text{for } G=Q \end{cases}$$

Then by the Cartan-Serre transgression theorem

$$d_3 z^2=x_1^2x_2+x_1x_2^2\,.$$

Now we consider the case of the dehedral group.

LEMMA 1.2. *When* $G=D$, $H^*(BG\,;\,\boldsymbol{Z}/2)\cong E_3\cong S_2/(x_1x_2)\otimes\boldsymbol{Z}/2[z^2]$

Proof. We know that $d_2z=x_1x_2$ and $E_2^{*,*}=\boldsymbol{Z}/2[x_1, x_2]\otimes\boldsymbol{Z}/2[z]$. Let $a\in \boldsymbol{Z}/2[x_1, x_2]$. Now $d_2(az)=d_2a\cdot z+(-1)^{|a|}a\cdot d_2z=(-1)^{|a|}a\cdot x_1x_2$ and $d_2(az^2)=0$. Therefore $\operatorname{Ker} d_2(E_2^{1,*})=0$ and $\operatorname{Im} d_2(E_2^{1,*})=\text{Ideal}(x_1x_2)$. Hence $E_3^{*,*}=H(E_2^{*,*}, d_2)$ $=\boldsymbol{Z}/2[x_1, x_2]/(x_1x_2)\otimes\boldsymbol{Z}/2[z^2]$. Since $d_3z^2=x_1^2x_2+x_1x_2^2=0 \bmod (x_1x_2)$, we have $E_3^{*,*}\cong E_\infty^{*,*}$. q. e. d.

§ 2. $H^*(BD\,;\,\boldsymbol{Z}/2)$.

In this section we calculate the cohomology of the dehedral group D by the another way. Given a finite group G and a central cyclic subgroup C, we fix an embedding of C into S^1, and define $\tilde{G}=G\times_{\langle c\rangle}S^1$. Then we have the exact sequence

$$1 \longrightarrow S^1 \longrightarrow \tilde{D} \longrightarrow \boldsymbol{Z}/2\oplus\boldsymbol{Z}/2 \longrightarrow 1$$

which induces the spectral sequence.

$$E_2^{*,*}=H^*(B(\boldsymbol{Z}/2\oplus\boldsymbol{Z}/2\,;\,\boldsymbol{Z}/2),\ H^*(BS^1\,;\,\boldsymbol{Z}/2)) \Longrightarrow H^*(B\tilde{D}\,;\,\boldsymbol{Z}/2),$$

where $E_2^{*,*}=\boldsymbol{Z}/2[x_1, x_2]\otimes\boldsymbol{Z}/2[u]$ and $d_3u=x_1^2x_2+x_1x_2^2$. The E_2-term is given by

$$E_3^{*,2j}\cong\begin{cases}\boldsymbol{Z}/2[x_1, x_2]/(d_3u) & j=0 \bmod 2\\ \mathrm{Ker}\,(d_3u) & j=1 \bmod 2\end{cases}$$

In this paper, let us write $\mathrm{gr}\,A=F$ if $F=\bigoplus_{i=0}^{s} F_i/F_{i+1}$ for some filtration $A=F_0\supset F_1\supset\cdots\supset F_s$.

THEOREM 2.1. $H^*(B\tilde{D}\,;\boldsymbol{Z}/2)\cong E_3^{*,*}\cong\boldsymbol{Z}/2[x_1, x_2]/(x_1^2x_2+x_1x_2^2)\otimes\boldsymbol{Z}/2[u^2]$.

Proof. If $d_3(au)=ad_3u=a(x_1^2x_2+x_1x_2^2)=0$ in $\boldsymbol{Z}/2[x_1, x_2]$, where $a\in\boldsymbol{Z}/2[x_1, x_2]$, then $a=0$. Hence $\mathrm{Ker}\,(d_3u)=0$. Now $d_5u^2=d_5Sq^2u=Sq^2(x_1^2x_2+x_1x_2^2)=x_1x_2(x_1^3+x_2^3)=0 \bmod (x_1^2x_2+x_1x_2^2)$. Hence $E_3^{*,*}\cong E_\infty^{*,*}$. q. e. d.

To find $H^*(BD\,;\boldsymbol{Z}/2)$, given $H^*(B\tilde{D}\,;\boldsymbol{Z}/2)$, we use the Serre spectral of the fibration

$$S^1\longrightarrow BD\longrightarrow B\tilde{D}\,.$$

This induces the spectral sequence

$$E_2^{*,*}=H^*(B\tilde{D}\,;\boldsymbol{Z}/2)\otimes H^*(S^1\,;\boldsymbol{Z}/2)\Longrightarrow H^*(BD\,;\boldsymbol{Z}/2)\,.$$

THEOREM 2.2. *Let $z\in H^1(S^1\,;\boldsymbol{Z}/2)$ be a generator. Then*

$$\mathrm{gr}\,H^*(BD\,;\boldsymbol{Z}/2)\cong H^*(B\tilde{D}\,;\boldsymbol{Z}/2)/(d_2z)\oplus(\mathrm{Ker}\,d_2z)\cdot z$$

$$\cong S_2\otimes\boldsymbol{Z}/2[u^2]/(x_1x_2)\oplus S_2\otimes\boldsymbol{Z}/2[u^2]/(x_1x_2)\{(x_1+x_2)z\}$$

Proof. First note $d_2z=x_1x_2$. Since $x_1^2x_2+x_1x_2^2=x_1x_2(x_1+x_2)$, $\mathrm{Ker}\,d_2$ is generated by $\{(x_1+x_2)\}$. q. e. d.

In section §1 we know already $H^*(BD\,;\boldsymbol{Z}/2)\cong S_2\otimes\boldsymbol{Z}/2[u]/(x_1x_2)$. From Theorem 2.2, a filtration of $C=H^*(BD)$ is given

$$F_1=H^*(B\tilde{D}\,;\boldsymbol{Z}/2)/(x_1x_2)\cong S_2\otimes\boldsymbol{Z}/2[u^2]/(x_1x_2)$$

$$C/F_1\cong\mathrm{Ker}d_2z\cong S_2\otimes\boldsymbol{Z}/2[u^2]/(x_1x_2)\{(x_1+x_2)z\}$$

with identifying $(x_1+x_2)z$ by u.

§3. Margolis homology of $H^*(BD\,;Z/2)$.

We consider the Margolis homology defined by the Milnor primitive derivation Q_n, $H(H^*(BD\,;\boldsymbol{Z}/2), Q_n)$. Here Q_n is defined by $Q_n(x_1)=x_1^{2^{n+1}}$, $Q_n(x_2)=x_2^{2^{n+1}}$. It is known that $u^2\in H^*(B\tilde{D}\,;\boldsymbol{Z}/2)$ is represent by Chern class. Hence $Q_n(u^2)=0$.

Let us denote u^2(resp. x_1^2, x_2^2) by c(resp. y_1, y_2).

THEOREM 3.1. $H(H^*(B\tilde{D}; \boldsymbol{Z}/2), Q_n) \cong \boldsymbol{Z}/2[y_1, y_2, c]/(y_1^2y_2+y_1y_2^2, y_1^{2^n}, y_2^{2^n})$

$\oplus(\boldsymbol{Z}/2[y_1=y_2, c]/(y_1^{2^n}))\{x_1x_2\}$.

Proof. If $f \in \boldsymbol{Z}/2[x_1, x_2]/(x_1^2x_2+x_1x_2^2) \otimes \boldsymbol{Z}/2[u^2]$, then we can write $f = a+bx_1+cx_2+dx_1x_2+ex_1^2x_2$, where $a \in \boldsymbol{Z}/2[x_1^2, x_2^2, u^2]/(x_1^4x_2^2+x_1^2x_2^4)$, $b \in \boldsymbol{Z}/2[x_1^2, u^2]$, $c \in \boldsymbol{Z}/2[x_2^2, u^2]$, $d \in \boldsymbol{Z}/2[x_1^2=x_2^2, u^2]$, $e \in \boldsymbol{Z}/2[x_1^2=x_2^2, u^2]$. Then $Q_nf = bx_1^{2^{n+1}}+cx_2^{2^{n+1}}+dx_1^{2^{n+1}}x_2+dx_1x_2^{2^{n+1}}+ex_1^2x_2^{2^{n+1}}$. Here $dx_1^{2^{n+1}}x_2+dx_1x_2^{2^{n+1}} = d(x_1^{2^{n+1}}x_2+x_1x_2^{2^{n+1}})=0 \bmod (x_1^2x_2+x_1x_2^2)$.

Therefore Ker $Q_n = \{a+dx_1x_2\}$ and

$$\operatorname{Im} Q_n = \{bx_1^{2^{n+1}}+cx_2^{2^{n+1}}+ex_1^2x_2^{2^{n+1}} = bx_1^{2^{n+1}}+cx_2^{2^{n+1}}+e(x_1x_2)x_2^{2^{n+1}}\}.$$

Hence we get $H(H^*(B\tilde{D}; \boldsymbol{Z}/2), Q_n) = \boldsymbol{Z}/2[y_1, y_2, c]/(y_1^2y_2+y_1y_2^2, y_1^{2^n}, y_2^{2^n})$
$\oplus(\boldsymbol{Z}/2[y_1=y_2, c]/(y_1^{2^n}))\{x_1, x_2\}$. q. e. d.

THEOREM 3.2. $\operatorname{gr} H(H^*(BD; \boldsymbol{Z}/2), Q_n) \cong (\boldsymbol{Z}/2[y_1, y_2]/(y_1, y_2, y_1^{2^n}, y_2^{2^n}) \otimes \boldsymbol{Z}/2[c]/(y_1c^{2^{n-1}}, y_2c^{2^{n-1}})) \oplus \boldsymbol{Z}/2[c]\{y_1^{2^n-1}e_1 = y_2^{2^n-1}e_2\}$, *where* $e_i = x_i(x_1+x_2)z$.

Proof. From Theorem 2.2, we already know $\operatorname{gr} H^*(BD; \boldsymbol{Z}/2) = H^*(B\tilde{D}; \boldsymbol{Z}/2)/(d_2z) \oplus (\operatorname{Ker} d_2z)z$. First we compute $H(H^*(BD; \boldsymbol{Z}/2)/(x_1x_2), Q_n)$ and secondary compute $H((\operatorname{Ker} x_1x_2)z, Q_n)$. Using the spectral sequence, we get $H(H^*(B\tilde{D}; \boldsymbol{Z}/2), Q_n)$ at last.

Let $C = \operatorname{gr} H^*(BD; \boldsymbol{Z}/2)$ and $F_1 = H^*(B\tilde{D}; \boldsymbol{Z}/2)/(x_1x_2)$. Then we will prove

(3.3) $$H(F_1, Q_n) \cong \boldsymbol{Z}/2[y_1, y_2, c]/(y_1y_2, y_1^{2^n}, y_2^{2^n})$$

(3.4) $$H(C/F_1, Q_n) \cong (\boldsymbol{Z}/2[y_1, y_2, c]/(y_1y_2, y_1^{2^n-1}, y_2^{2^n-1}))\{y_1z, y_2z\}.$$

First we will prove (3.3).
If $f \in \boldsymbol{Z}/2[x_1, x_2, u^2]/(x_1x_2)$, then we can write $f = a+bx_1+cx_2$ where $a \in \boldsymbol{Z}/2[x_1^2, x_2^2, u^2]/(x_1^2x_2^2)$, $b \in \boldsymbol{Z}/2[x_1^2, u^2]$, $c \in \boldsymbol{Z}/2[x_2^2, u^2]$. Operate Q_n to f, then $Q_nf = bx_1^{2^{n+1}}+cx_2^{2^{n+1}}$. Therefore Ker $Q_n = \{a\}$ and Im $Q_n = \{bx_1^{2^{n+1}}+cx_2^{2^{n+1}}\}$. Hence we get (3.3).

Next we will prove (3.4).
If $f \in (\boldsymbol{Z}/2[x_1, x_2, u^2]/(x_1x_2))\{x_1+x_2\}$, then $f = a(x_1+x_2)+bx_1(x_1+x_2)+cx_2(x_1+x_2) = a(x_1+x_2)+bx_1^2+cx_2^2$, where $a \in \boldsymbol{Z}/2[x_1^2+x_2^2, u^2]/(x_1^2x_2^2)$, $b \in \boldsymbol{Z}/2[x_1^2, u^2]$, $c \in \boldsymbol{Z}/2[x_2^2, u^2]$. Then $Q_nf = a(x_1^{2^{n+1}}+x_2^{2^{n+1}})$. Therefore Ker$Q_n = \{(bx_1+cx_2)\{x_1+x_2\}\}$, Im $Q_n = \{a(x_1^{2^{n+1}}+x_2^{2^{n+1}})\}$. Hence we get (3.4).

At least we consider the spectral sequence

$$E_1 = H(F_1, Q_n) \oplus H(C/F_1, Q_n) \Longrightarrow H(C, Q_n).$$

Now we can prove $Q_n(y_iz) = y_iu_1^{2^n}, = y_ic^{2^{n-1}}$, for $i=1, 2$. So we can prove $\operatorname{gr} H(C, Q_n) \cong (\boldsymbol{Z}/2[y_1, y_2]/(y_1y_2, y_1^{2^n}, y_2^{2^n}) \otimes \boldsymbol{Z}/2[c]/(y_1c^{2^n}, y_2c^{2^n})) \oplus \boldsymbol{Z}/2[c]\{y_1^{2^n-1}e_1 = y_2^{2^n-1}e_2\}$. q. e. d.

§ 4. Morava K-theory.

The Morava K-theory $K(n)^*(-)$ is generalized cohomology theory with the coefficient $K(n)^*=\mathbf{Z}/2[v_n, v_n^{-1}]$, $|v_n|=-2^{n+1}+2$.

We consider the Atiyah-Hirzebruch spectral sequence for Morava K-theory

$$E_2^{*,*}=(H^*(X; K(n)^*) \Longrightarrow K(n)^*(X).$$

It is known [Hu], [T-Y] that the differential $d_{2^{n+1}-1}(x)=v_n \otimes Q_n x$. Hence we get

$$E_{2^{n+1}}^{*,*} \cong K(n)^* \otimes H(H^*(X; \mathbf{Z}/2), Q_n)$$

THEOREM 4.1. $\mathrm{gr}\, K(n)^*(B\tilde{D}) \cong K(n)^* \otimes H(H^*(B\tilde{D}; \mathbf{Z}/2), Q_n)$

Proof. $H(H^*(B\tilde{D}; \mathbf{Z}/2), Q_n)$ is generated by even dimensional elements, hence $E_{2^{n+1}}^{*,*} \cong E_\infty^{*,*}$. q.e.d.

Ravenel [R] showed that $\dim_{K(n)_*} K(n)^*(BG)$ is finite for each finite group G. Hopkins-Kuhn-Ravenel [H-K-R] defined $K(n)$-theory Euler character χ_n by

(4.2) $$\chi_n(G)=\dim_{K(n)_*} K(n)^{\mathrm{even}}(BG)-\dim_{K(n)_*} K(n)^{\mathrm{odd}}(BG).$$

For p-groups G, this Euler character can be described in terms of conjugacy classes of commuting n-tuples of elements in G,

$\chi_n(G)=$number of $\{(g_1, \cdots, g_n) | [g_i, g_j]=1, g_i \in G\}/G$ with the conjugate action $g \cdot (g_1, \cdots, g_n) \sim (gg_1g^{-1}, \cdots, gg_ng^{-1})$. They also showed (Lemma 5.3.6 in [H-K-R]) that χ_n is computed inductively

(4.3) $$\chi_n(G)=\sum_{\langle g \rangle} \chi_{n-1}(C_G(g))$$

where $\langle g \rangle$ runs over conjugate classes in G and $C_G(g)=\{h \in G | [h, g]=1\}$ is the centralizer of g in G.

Now we consider $K(n)^*(BD)$. Recall $H(H^*(BD; \mathbf{Z}/2), Q_n)$ in Theorem 3.3. If $d_r\{y_1^{2^n-1}e_1\}=0$ for all r, then $E_4^{*,*} \cong E_\infty^{*,*}$. Hence $\dim_{K(n)_*} K(n)^*(BD)$ is infinite since $c^s \neq 0$.

This contradicts the results of Ravenel, therefore we know

(4.4) $$d_r\{y_1^{2^n-1}e_1\}=v_n^k c^s \text{ for some } s \text{ with } 2(2^n-1)(k+1)+4=4s.$$

From Theorem 3.2, $E_{r+1}^{*,*}$ is generated by even dimensional elements. Hence $E_{r+1}^{*,*} \cong E_\infty^{*,*}$.

LEMMA 4.5. $\dim_{K(n)_*} K(n)^*(BD)=2^{2n}-2^n+s$.

Proof. From Theorem 3.2, $K(n)^*(BD)$ has $K(n)^*$-basis $\{y_1^k, y_2^k\} \otimes c^j \oplus c^h$ $(1 \leq k < 2^n, 0 \leq j < 2^{n-1}, 0 \leq h < s)$. Hence we see $\dim_{K(n)_*} K(n)^*(BD)=2(2^n-1) \times 2^{n-1}+s$. q.e.d.

LEMMA 4.6. $\chi_n(D)=2^{2n}+2^{2n-1}-2^{n-1}$.

Proof. The conjugacy classes of D are $\langle 1\rangle$, $\langle a^2\rangle$, $\langle a^i b^j | 0\leqq i\ j\leqq 1\ (i, j)\neq (0, 0)\rangle$ and their centralizer are D, D, $\boldsymbol{Z}/2\oplus\boldsymbol{Z}/2$ respectively. So from (4.3)

$$\begin{aligned}\chi_n(D)&=\textstyle\sum_{\langle g\rangle}\chi_{n-1}(C_G(g))\\ &=\chi_{n-1}(C_G(1))+\chi_{n-1}(C_G(a^2))+\chi_{n-1}(C_G(a))+\chi_{n-1}(C_G(b))+\chi_{n-1}(C_G(ab))\\ &=\chi_{n-1}(D)+\chi_{n-1}(D)+\chi_{n-1}(\boldsymbol{Z}/4)+\chi_{n-1}(\boldsymbol{Z}/2\otimes\boldsymbol{Z}/2)+\chi_{n-1}(\boldsymbol{Z}/2\otimes\boldsymbol{Z}/2)\\ &=2\chi_{n-1}(D)+3\cdot 2^{2n-2}.\end{aligned}$$

We put $\chi_{n-1}(D)=2^{2n-2}+2^{2n-3}-2^{n-2}$. Then $2\chi_{n-1}(D)+3\cdot 2^{2n-2}=2(2^{2n-2}+2^{2n-3}-2^{n-2})+3\cdot 2^{2n-2}=2^{2n}+2^{2n-1}-2^{n-1}$. Hence we get this Lemma. q.e.d.

From Lemma 4.5 and Lemma 4.6, we know $s=2^{2n-1}+2^{n-1}$, hence $k=2^n+1$.

THEOREM 4.7. $\mathrm{gr}\, K(n)^*(BD)\cong K(n)^*(S_2'/(y_1y_2,\ y_1^{2^n},\ y_2^{2^n}))$
$\otimes\boldsymbol{Z}/2[c]/(y_1c^{2^n-1},\ y_2c^{2^n-1},\ c^{2^{2n-1}+2^{n-1}})$ with $c=u^2$.

Remark. The multiplicative structure of $K(n)^*(BD)$ was given in Theorem 4.2 in [T-Y]. There were some errors, which were corrected in [T-Y3]. The ring structure is

(4.8) $$\begin{aligned}&K(n)^*(BD)\\ &\cong K(n)^*(S'\otimes\boldsymbol{Z}/2[c]/(y_1^{2^n},\ y_2^{2^n},\ v_n^2c^{2^n}=v_nc^{2^n-1}y_1=v_nc^{2^n-1}y_2=y_1y_2).\end{aligned}$$

This consists with ours as following and from (4.8) we deduce

$$\begin{aligned}0&=y_1^{2^n}y_2=v_ny_1^{2^n-1}c^{2^n-1}y_2=\cdots=v_n^{2^n}(c^{2^n-1})^{2^n}y_2=v_n^{2^n}c^{2^{n-1}(2^n-1)}c^{2^n-1}y_2\\ &=v_n^{2^n-1}c^{2^{n-1}(2^n-1)}y_1y_2=v_n^{2^n+1}c^{(2^n-1)2^{n-1}}c^{2^n}=v_n^{2^n+1}c^{2^{2n-1}+2^{n-1}}.\end{aligned}$$

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