M. TEZUKA
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THE COHOMOLOGY OF $SL_2(F_p)$ AND THE HECKE ALGEBRA ACTIONS

BY MICHISHIGE TAZUKA

Dedicated to the Memory of Dr. Takehiko Miyata

Introduction.

Let p be a prime number and F_p be the prime field with p elements. Our first main purpose of this paper is to calculate $H^n(SL_2(F_p), S(V))$ where $S(V)$ is the symmetric algebra of a two dimentional vector space V over F_p. The cohomology group $H^n(SL_2(F_p), S^t(V))$ is obtained by Y. H. Rhie and G. Whaples [11] under the condition $t=1$ and by M. Kuga, W. Parry and C. H. Sah [7] under the condition $t \leqq p-1$, $n=1, 2$, $p \geqq 5$. We shall give the complete description of mentioned above without any condition in section 2 and 3 (Theorem 3 and 4).

Our second purpose is to determine the Hecke algebra actions on $H^n(SL_2(F_p), S(V))$ (Theorem 5) and determine $H^n(GL_2(F_p), S(V))$ (Theorem 6) in section 4. These results are also extentions of [7].

The Hecke algebra actions on a cohomology group are defined by Y. H. Rhie and G. Whaples [11] and a brief account on these properties are found in [7] and [4] by E. Cline, B. Parshall and L. Scott. We summarize their results for completeness in section 1.

Furthermore we consider cohomologies with coefficients attached to one-dimensional representations of $GL_2(F_p)$ in section 5.

In section 6, we determine the Hecke algebra actions on $H^*(B)_{(p)}$ by the Weyl group S_3 of $GL_3(F_p)$ where B is a Borel subgroup of $GL_3(F_p)$.

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§ 1. Preliminaries.

In this section, we collect some of the basic definitions and results on homological algebra [2], [3], [9] and Hecke algebra [4], [7], [11].

1.1. *Conjugate homomorphisms*

Let G and G' be finite groups and $f: G' \to G$ be a homomorphism. When

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A is a G-module, we can regard A as a G'-module by

$$g'\cdot a=f(g')a \qquad \text{for} \quad g'\in G,\ a\in A.$$

We use Eilenberg-MacLane cohomology group and recall a basic fact [2, 31.1, 31.2].

LEMMA 1. *Let $\{C_i, \partial_i\}$ and $\{C'_i, \partial'_i\}$ be a free resolution of G and G' respectably. Then there is a chain map $\{\phi_i\}: \{C'_i, \partial'_i\}\rightarrow\{C_i, \partial_i\}$ such that $\phi_i(g'c')=f(g')\phi_i(c')$ for i-chain $c'\in C'_i$ and $g'\in G'$. Furthermore let $\{\phi_i\}$ and $\{\phi'_i\}$ be any two chain map induced by f. In case f is injective, we have a chain homotopy $\{\Phi_i\}: \{C'_i\}\rightarrow\{C_{i+1}\}$. i.e.*

$$\partial_{i+1}\Phi_i-\Phi_{i-1}\partial_i=\phi_i-\phi'_i \qquad \textit{for} \quad i>0$$

and

$$\partial_1\Phi_0=\phi_0-\phi'_0.$$

We consider pairs (G, A) where G is a finite group and A is a G-module. We say $(f, \theta): (G, A)\rightarrow(G', A')$ is a morphism if $f: G'\rightarrow G$ is injective and $\theta: A\rightarrow A'$ is a homomorphism such that $\theta(g'\cdot a)=g'\theta(a)$ for $g'\in G'$ and $a\in A$. Given such a morphism (f, θ), we can define the homomorphism

$$(f, \theta)^*: H^*(G, A)\longrightarrow H^*(G', A')$$

as follows. Take an n-cochain $t\in \mathrm{Hom}_G(C_n, A)$ and put $(f, \theta)t=ft\phi_i$, where ϕ_i is a chain map induced by f. From Lemma 1, we see that $(f, \theta)^*$ is well defined.

We restrict our attention to the following case. Let H be a subgroup of G. Then we define an adjoint homomorphism $ad_g: H\rightarrow H^g$ by $ad_g(h)=ghg^{-1}$ and $H^g=gHg^{-1}$. We also denote by $ad_{g^{-1}}$ an n-th chain map induced by $ad_{g^{-1}}$: $H^g\rightarrow H$. When we define $\theta: A\rightarrow A$ by $\theta(a)=ga$ and $f: H^g\rightarrow H$ by $f=ad_{g^{-1}}$, we have for $h'\in H^g$

$$\theta(h'\cdot a)=\theta(ad_{g^{-1}}(h')a)=gad_{g^{-1}}(h')a=h'ga=h'\theta(a).$$

Therefore $(ad_{g^{-1}}, \theta): (H, A)\rightarrow(H^g, A)$ is a morphism and it induces the homomorphism

$$(ad_{g^{-1}}, \theta)^*: H^*(H, A)\longrightarrow H^*(H^g, A).$$

According to [3], we will denote $(ad_{g^{-1}}, \theta)^*$ by c_g hereafter. We call it the conjugate homomorphism associated with $ad_{g^{-1}}$.

LEMMA 2. (1) *Let $\{C_i, \partial_i\}$ be a free resolution of G, that is also a free resolution of H and H^g. Then conjugate homomorphism c_g is given by $c_g t=gtg^{-1}$ for n-cochain $t\in\mathrm{Hom}_G(C_n, A)$.*

(2) [9]. *In terms of the homogeneous resolution, we have*

$$(c_g t)(h'_0, \cdots, h'_n)=gt(ad_{g^{-1}}h'_0, \cdots, ad_{g^{-1}}h'_n) \qquad \textit{for} \quad h'_i\in H^g.$$

Proof. For $c \in C_n$ and $h' \in H^g$, we have $g^{-1}(h'c) = ad_{g^{-1}}(h')g^{-1}c$. Therefore we can take $ad_{g^{-1}}(c) = g^{-1}c$ for a chain map induced by $ad_{g^{-1}}$. The results follow from the definition of the conjugate homomorphism. q. e. d.

Remark 1. In [2], [3], the part (1) of the lemma above shows that our definition of c_g agrees with that of [2], [3].

We determine a chain map of the standard resolution of a cyclic group induced by an adjoint homomorphism. Recall that the standard resolution of a cyclic group C of order n generated by x is given by

$$C_i = Z[C]e_i \quad \text{and} \quad \partial_{2i}(e_{2i}) = Ne_{2i-1},\ \partial_{2i+1}(e_{2i+1}) = Te_{2i},$$

where we put $T = x-1$ and $N = 1+x+\cdots+x^{n-1}$, and the augmentation $\varepsilon: C_0 \to Z$ is $\varepsilon(x^i e_0) = 1$.

PROPOSITION 1. *Let C be a cyclic subgroup of G of order n with generator x. $N_G(C)$ denotes the normalizer of C. Let an element g of $N_G(C)$ act as $ad_{g^{-1}}(x) = x^a$, $(a, n) = 1$. Then a chain map $ad_{g^{-1}}$ on the standard resolution of C is given by*

$$ad_{g^{-1}}(x^j e_{2i}) = a^i ad_{g^{-1}}(x^j)e_{2i}$$

and

$$ad_{g^{-1}}(x^j e_{2i+1}) = a^i \sum_{0 \leq \alpha \leq a-1} x^\alpha ad_{g^{-1}}(x^j)e_{2i+1}.$$

Proof. Since we see that $ad_{g^{-1}}(x^j e_i) = ad_{g^{-1}}(x^j)ad_{g^{-1}}(e_i)$, it is enough to check that $\{ad_{g^{-1}}\}$ commute with the boundary maps. First $ad_{g^{-1}}$ commutes with the augmentation. For $n=2i$, $i>0$, we have

$$\begin{aligned}
ad_{g^{-1}}(\partial_{2i}(e_{2i})) &= \sum_{j=0}^{n-1} ad_{g^{-1}}(x^j e_{2i-1}) \\
&= \sum_{j=0}^{n-1} a^{i-1} \sum_{\alpha=0}^{a-1} x^\alpha ad_{g^{-1}}(x^j)e_{2i-1} = a^{i-1} \sum_{\alpha=0}^{a-1} \sum_{j=0}^{n-1} x^{\alpha+j} e_{2i-1} \\
&= a^i N e_{2i-1} = \partial_{2i}(ad_{g^{-1}}(e_{2i})).
\end{aligned}$$

For $n=2i+1$, we have

$$\begin{aligned}
ad_{g^{-1}}(\partial_{2i+1}(e_{2i+1})) &= ad_{g^{-1}}(xe_{2i} - e_{2i}) \\
&= a^i(ad_{g^{-1}}(x)e_{2i} - e_{2i}) = a^i(x^a e_{2i} - e_{2i}) \\
&= a^i(x-1)\left(\sum_{\alpha=0}^{a-1} x^\alpha e_{2i}\right) = \partial_{2i+1}\left(a^i \sum_{\alpha=0}^{a-1} x^\alpha e_{2i+1}\right) \\
&= \partial_{2i+1}(ad_{g^{-1}}(e_{2i+1})).
\end{aligned}$$

q. e. d.

1.2. *The Hecke algebra.*

In the following paragraphes, we recall the basic properties of the Hecke algebra defined in [4], [7], [11]. Let H be a subgroup of G. We consider the free Z-module $H\backslash G/H$ generated by the elements $\frac{1}{|H|}HaH$, where $G=\coprod HaH$ and HaH means $\sum_{s\in HaH} s$. When we regard $H\backslash G/H$ as a submodule of the $\boldsymbol{Q}$-group ring $\boldsymbol{Q}[G]$, $H\backslash G/H$ has a natural ring structure induced from $\boldsymbol{Q}[G]$ and so is called the Hecke algebra with respect to G and H. We denote by $|^G$ and $|_H$ the corestriction map

$$\mathrm{Cor}: H^n(H, A) \longrightarrow H^n(G, A)$$

and the restriction map

$$\mathrm{Res}: H^n(G, A) \longrightarrow H^n(H, A)$$

defined in [2], [3].

We define the right actions of the Hecke algebra $H\backslash G/H$ on an element $\mu\in H^n(H, A)$ by

$$\mu\Big(\frac{1}{|H|}HaH\Big)=c_{a^{-1}}\mu|_{H^{a^{-1}}\cap H}|^H.$$

Particularly, in the case of $H\lhd G$, we have

$$\mu\Big(\frac{1}{|H|}HaH\Big)=c_{a^{-1}}\mu.$$

We can rewrite the stability theorem of Cartan-Eilenberg in term of the Hecke algebra. Recall that an element $\mu\in H^n(H, A)$ is stable if $\mu|_{H^a\cap H}=c_a\mu|_{H^a\cap H}$ for all $a\in G$.

THEOREM 1. [3, *Theorem* 10, 1. *Chap.* XII]. *Let H be a p-Sylow subgroup of G. Then the restriction map $H^n(G, A)H^n(H, A)$ is injective on the set of all the stable elements.*

We define the augmentation $\varepsilon: H\backslash G/H\to Z$ by $\varepsilon\Big(\frac{1}{|H|}HaH\Big)=[H: H^{a^{-1}}\cap H]$. We call an element $\mu\in H^n(H, A)$ a fixed point for $H\backslash G/H$ when $\mu\Big(\frac{1}{|H|}HaH\Big)=\varepsilon\Big(\frac{1}{|H|}HaH\Big)$ are satisfied for all $a\in G$.

THEOREM 2. [4]. *Let H be a p-Sylow subgroup of G. Then μ is stable if and only if μ is a fixed point for $H\backslash G/H$.*

§2. The cohomology of U.

Let $S(V)=\bigoplus_{t\geq 0} S^t(V)$ be the symmetric algebra of a two-dimensional vector space V over F_p and $S^t(V)$ be the homogeneous part of degree t. Hereafter we fix a basis of V and identify $S(V)$ with the polynomial algebra $F_p[x_1, x_2]$. We define the action of $SL_2(F_p)$ on V by $(Ax_1, Ax_2)=(x_1, x_2)A$ for $A\in SL_2(F_p)$ and extend this action on $S(V)$ as a ring homomorphism. L et U be the cyclic subgroup of $SL_2(F_p)$ which consists of the upper triangular matrices with diagonals one.

PROPOSITION 2.

(1) $H^0(U, S(V))\cong F_p[x_1, v]$ *where* $v=\prod_{\lambda\in F_p}(x_2+\lambda x_1)$.

(2) *The multiplication by* v *induces the isomorphism*

$$H^n(U, S^t(V))\cong H^n(U, S^{t+p}(V)) \quad \textit{for} \quad n>0.$$

(3) $$H^n\Big(U, \bigoplus_{t=0}^{p-1} S^t(V)\Big)\cong\begin{cases}\sum_{j=0}^{p-2} F_p x_1^j & \textit{if} \quad n=2i,\ i>0\\ \sum_{j=0}^{p-2} F_p x_2^j & \textit{if} \quad n=2i+1.\end{cases}$$

Proof. (1) It is easy to see that $v=\Pi(x_2+\lambda x_1)$ is invariant under the action of U and so $S(V)^U$ contains $F_p[x_1, v]$. Let $F\in S(V)$ be invariant. Then we can suppose that x_1 does not devide F and that F does not contain the monomial x_1^k. Regard F as a polynomial with one variable x_2, and we can write

$$F=vL+R,\ \deg_{x_2}(R)<p \quad \text{and} \quad L,\ R\in S(V).$$

Since R is invariant and is devided by x_2, R must be devided by v. Therefore R must be zero. Repeating this argument, F is shown to be a power of v. This implies that $S(V)^U$ is exactly the polynomial ring $F_p[x_1, v]$.

(2), (3) We put $y=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $N=\sum_{i=0}^{p-1} y^i$, $T=y-1$ in the group ring $Z[U]$. We consider N or T as a homomorphism on $S(V)$ and set $N(S(V))=\mathrm{Im}\,N$, ${}_NS(V)=\mathrm{Ker}\,N$ and $T(S(V))=\mathrm{Im}\,T$, $S(V)^U=\mathrm{Ker}\,T$ respectively. From the standard resolution, we have

$$H^n(U, S(V))=\begin{cases} S(V)^U/N(S(V)) & \text{if} \quad n=2i,\ i>0\\ {}_NS(V)/T(S(V)) & \text{if} \quad n=2i+1.\end{cases}$$

From (1), we can write $f\in S(V)$ in the form $f(x_1, x_2)=\sum_{j=1}^{p-1} g_j(x_1, v)x_2^j+h(x_1, v)$. Since $N(g_jx_2^j)=g_jN(x_2^j)$ and $N(h)=0$, we only need to calculate $N(x_2^j)$, $1\leqq j\leqq p-1$. Then we calculate $N(x_2^j)=\sum_{0\leq\lambda\leq p-1}(x_2+\lambda x_1)^j=\sum_{0\leq k\leq j}\binom{j}{k}\sum_{0\leq\lambda\leq p-1}\lambda^k x_1^k x_2^{j-k}$ and use the formulas

$$\sum_{0\leq\lambda\leq p-1}\lambda^k=\begin{cases}0 & \text{if} \quad k\not\equiv 0 \mod (p-1)\\ -1 & \text{if} \quad k\equiv 0 \mod (p-1).\end{cases}$$

We have

$$N(x_2^j)=\begin{cases}0 & \text{if} \quad 1\leqq j\leqq p-2\\ -x_1^{p-1} & \text{if} \quad j=p-1.\end{cases}$$

Therefore we obtain $N(S(V))=x_1^{p-1}S(V)^U$ and ${}_NS(V)=\sum_{j=0}^{p-1} x_2^jS(V)^U$.

Similarly we obtain $T(S(V))=x_1(S(V)^U)$. The results are immediate.

q. e. d.

Remark 3. The invariant algebras of $S(V)^U$ is well known. See [13].

§ 3. The cohomology of $SL_2(F_p)$.

Let B the subgroup of $SL_2(F_p)$ which is generated by the matrices (a_{ij}) $1 \leqq i, j \leqq 2$ with $a_{12}=0$ and M be a $F_p[SL_2(F_p)]$-module.

LEMMA 4. *The inclusion map from B to $SL_2(F_p)$ induces an isomorphism $H^n(SL_2(F_p), M) \simeq H^n(B, M)$ for $n>0$.*

Proof. Since B contains a p-Sylow subgroup U, the induced map is injective from Theorem 1. We have the decomposition: $SL_2(F_p)=B \amalg BwB$, where w is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then we see $B \cap wBw=T$, where T is the group of the diagonal matrices in $SL_2(F_p)$. Since p does not devide the order of T, we have $H^n(T, M)=0$ for $n>0$. Therefore the Hecke algebra acts on $\mu \in H^n(B, M)$ trivially, *i.e.* $\mu\left(\frac{1}{|B|}BwB\right)=0$. From Theorem 2, any element of $H^n(B, M)$ is stable.

q. e. d.

LEMMA 5. *The inclusion map from U to B induces an isomorphism $H^n(B, M) \simeq H^n(U, M)^B$.*

Proof. Since U is the mormal p-Sylow subgroup of B, the stable elements of $H^n(U, M)$ are just the fixed parts for B. q. e. d.

THEOREM 3. $H^0(SL_2(F_2), S(V)) \cong F_2[x_1v, x_1^2+v]$

and

$$H^n(SL_2(F_2), S(V)) \cong H^n(U, S(V)) \quad \textit{for} \quad n>0.$$

THEOREM 4. *Let p be an odd prime. Then we have*

(1) $H^0(SL_2(F_p), S(V)) \cong F_p[x_1v, x^{p(p-1)}+v^{p-1}]$.

(2) *$H^n(SL_2(F_p), S(V))$ has the p-period $p-1$ within the positive degrees with respect to the homological degree n.*

(3) *The multiplication $(x_1^p+v)^{p-1}$ induces the isomorphism*

$$H^n(SL_2(F_p), S^t(V) \cong H^n(SL_2(F_p), S^{t+p(p-1)}) \quad \textit{for} \quad n>0.$$

(4) $H^n(SL_2(F_p), \bigoplus_{0 \leq t < p(p-1)} S^t(V))$

$$\cong\begin{cases}\bigoplus_{0\leq s<p-1-2i} F_p x_1^{s+2i} v^s \bigoplus_{0\leq s<2i} F_p x_1^s v^{p-1-2i+s}, & if \quad n=2i,\ 0<i<\dfrac{p-1}{2}\\ \bigoplus_{0\leq s\leq p-3-2i} F_p x_2^s v^{p-3-2i-s} \bigoplus_{p-3-2i<s\leq p-2} F_p x_2^s v^{2p-4-2i-s}, & if \quad n=2i+1,\ 0\leqq i\leqq\dfrac{p-3}{2}\\ \bigoplus_{0\leq s\leq p-2} F_p(x_1 v)^s, & if \quad n=p-1.\end{cases}$$

To prove the theorems, we prepare a lemma.

LEMMA 6. *If we put* $t=\begin{bmatrix}\lambda & 0\\ 0 & \lambda^{-1}\end{bmatrix}$, $\lambda\in F_p^\times$, *the action by* c_t *on* $H^n(U, S(V))$ *is given by*

(1) $c_t(x_1^{s_1}v^{s_2})=\lambda^{-2i+s_1-s_2}x_1^{s_1}v^{s_2}$, *if* $n=2i$, $i>0$

and

(2) $c_t(x_2^{s_1}v^{s_2})=\lambda^{-2(i+1)-s_1-s_2}x_2^{s_1}v^{s_2}$, *if* $n=2i+1$,

where we identify $x_1^{s_1}x_2^{s_2}\in S(V)$ *with a cochain of* $\mathrm{Hom}_U(Z[U]e_n, S(V))$ *under the correspondence* θ: $S(V)\simeq\mathrm{Hom}_U(Z[U]e_n, S(V))$ *given by*

$$\theta(x_1^{s_1}x_2^{s_2})(e_n)=x_1^{s_1}x_2^{s_2}.$$

Proof. (1) From Proposition 1, the action by c_t induced by $ad_{t^{-1}}$ on the cochain group is

$$\begin{aligned}(*)\quad c_t(x_1^{s_1}x_2^{s_2})&=t(x_1^s x_2^s)(ad_{t^{-1}}(e_{2i}))\\ &=((tx_1)^{s_1}(tx_2)^{s_2})(ad_{t^{-1}}(e_{2i}))\\ &=((tx_1)^{s_1}(tx_2)^{s_2}(\lambda^{-2i}e_{2i})\\ &=\lambda^{-2i+s_1-s_2}x_1^{s_1}x_2^{s_2}(e_{2i}).\end{aligned}$$

Reducing to the cohomology $H^{\mathrm{even}}(U, S(V))$, we have

$$c_t(x_1^{s_1}v^{s_2})=\lambda^{-2i+s_1-s_2}x_1^{s_1}v^{s_2}.$$

(2) We take $y=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ for a generator of U. Then we note that a cochain $x_1^{s_1}$, $x_2^{s_2}\in\mathrm{Hom}_U(Z[U]e_{2i+1}, S(V))$ has the following property:

$$x_1^s(ye_{2i+1})=(yx_1)^s(e_{2i+1})=x_1^s(e_{2i+1})$$

and

$$\begin{aligned}x_2^s(ye_{2i+1})=(yx_1)^s(e_{2i+1})&=x_2^s(e_{2i+1})-(y-1)x_2^s(e_{2i+1})\\ &\equiv x_2^s(e_{2i+1}) \bmod \text{coboundaries}.\end{aligned}$$

This implies

$$(*)\quad (c_t(x_1^{s_1}x_2^{s_2}))(e_{2i+1})=t(x_1^{s_1}x_2^{s})(ad_{t-1}(e_{2i+1}))$$
$$=((tx_1)^{s_1}(tx_2)^{s_2}(ad_{t-1}(e_{2i+1}))$$
$$=\lambda^{s_1-s_2}x_1^{s_1}x_2^{s_2}\left(\lambda^{-2i}\sum_{j=0}^{\lambda-2-1}y^j e_{2i+1}\right)$$
$$=\lambda^{-2i+s_1-s_2}\sum_{j=0}^{\lambda-2-1}(y^jx_1)^{s_1}(y^jx_2)^{s_2}(e_{2i+1})$$
$$\equiv\lambda^{-2(i+1)+s_1-s_2}(x_1^{s_1}x_2^{s_2})(e_{2i+1}) \text{ mod coboundaries.}$$

Reducing to the cohomology, we obtain

$$c_t(x_2^{s_1}v^{s_2})=\lambda^{-2(i+1)-s_1-s_2}x_2^{s_1}v^{s_2}. \qquad \text{q. e. d.}$$

Remark 3. Y. H. Rhie and G. Whaples have proved this lemma in the case of $s_1=1$ and $s_2=0$.

Proof of the theorem.

The 0-dimensional cohomology is the subring of $S(V)$ which is invariant by $SL_2(F_p)$. The generators of this ring are determined by Dickson [5]. In the case of $SL_2(F_2)$, we note that $B=U$ in $SL_2(F_2)$. From Lemma 4 and 5, we have the isomorphism

$$H^n(SL_2(F_2), S(V))\cong H^n(U, S(V)) \qquad \text{for} \quad n>0.$$

Let p be an odd prime. From Lemma 4 and 5, it is enough to determine the elements of $H^n(U, S(V))$ invariant under the actions by the diagonal matrices. These explicit formulas are given in Lemma 6. Immediately we find that the p-period is $p-1$.

If $x_1^{s_1}v^{s_2}\in H^{2i}(U, S(V))$ is invariant under B, the relations between s_1 and s_2 are $-2i+s_1-s_2\equiv 0 \mod (p-1)$ and $0\leqq s_1\leqq p-2$, $0\leqq s_2$, $0<i<\frac{p-1}{2}$. We solve this congruence equation. And the solutions are

$$\begin{cases} s_1=s+2i,\ 0\leqq s<p-1-2i \\ s_2\equiv s \mod (p-1),\ s_2>0 \end{cases}$$

and

$$\begin{cases} s_1=s,\ 0\leqq s<2i \\ s_2=p-1-2i \mod (p-1),\ s_2\geqq 0. \end{cases}$$

We can do the same arguments on $H^n(U, S(V))$ and $H^{p-1}(U, S(V))$. q. e. d.

Remark 4. In [10], R. G. Swan shows that the p-period of cohomology group whose p-Sylow subgroup is cyclic is given by $2|N_G(U)|/|C_G(U)|$. Then Theorem 4, (2) is straightforward.

§4. The Hecke algebra actions and the cohomology of $GL_2(F_p)$.

In this secition, we suppose that p is an odd prime. We denote $SL_2(F_p)$, $GL_2(F_p)$ by Γ, $\tilde{\Gamma}$ respectibly. We define two distinct $\tilde{\Gamma}$-actions on $S(V)$. The one, $\rho=\bigoplus_{m\geq 0}\rho_m$ of $\tilde{\Gamma}$ on $S(V)=\bigoplus_{m\geq 0}S^m(V)$, is defined by ; $(\rho_1(A)x_1, \rho_1(A)x_2)=(x_1, x_2)A$ for $A\in\tilde{\Gamma}$ and $\rho_m(A)f(x_1, x_2)=f(\rho_1(A)x_1, \rho_1(A)x_2)$. The other, $\rho^*=\bigoplus_{m\geq 0}\rho_m^*$, is defined as $\rho_m^*(g)=\det(g)^{-m}\rho_m(g)$. $H^n(\tilde{\Gamma}, \rho)$ (resp. $H^n(\tilde{\Gamma}, \rho^*)$) denote the cohomology with the coefficient $S(V)$ through the action ρ (resp. ρ^*).

Since Γ is a normal subgroup of $\tilde{\Gamma}$, we can write $\Gamma\alpha\Gamma=\Gamma\alpha=\alpha\Gamma$ and $\Gamma\backslash\tilde{\Gamma}/\Gamma$ $=\left\{\sum a_\alpha\Gamma\alpha\Gamma:\ \alpha=\begin{bmatrix} a & 0\\ 0 & 1\end{bmatrix}\ a\in F_p^\times,\ a_\alpha\in Z\right\}\cong Z[F_p^\times]$. From Lemma 3, the Hecke algebra action by $\Gamma\backslash\tilde{\Gamma}/\Gamma$ on $\mu\in H^n(\Gamma, \rho_m)$ (resp. $H^n(\Gamma, \rho^*)$) is given by $\mu\left(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\right)=c_{\alpha^{-1}}\mu\left(\text{resp. } \mu\left(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\right)=\det(\alpha)^m c_{\alpha^{-1}}\mu\right)$, taking account of the asterisque in the proof of Lemma 6. By definition, we see that $\rho|\Gamma$ and $\rho^*|\Gamma$ and so $H^*(\Gamma, \rho)=H^*(\Gamma, \rho^*)$. But, as the next theorem shows, the corresponding Hecke algebra actions are not equivalent.

THEOREM 5. *The Hecke algebra actions $\Gamma\backslash\tilde{\Gamma}/\Gamma$ on $H^n(\Gamma, \rho)$ (resp. $H^n(\Gamma, \rho^*)$) are as follows*:

(1) *If $n=2i+1$, $\frac{1}{|\Gamma|}\Gamma\alpha\Gamma$ is a scalar multiplication by $\det(\alpha)^{i+1}$ (resp. $\det(\alpha)^{-i+1}$).*

(2) *If $n=k(p-1)+2i$, $0<i<\frac{p-1}{2}$, we have*

$$(x_1^{s+2i}v^s)\left(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\right)=(-1)^k\det(\alpha)^{-(s+i)}x_1^{s+2i}v^s$$
$$(\textit{resp. } (-1)^k\det(\alpha)^{s+i}x_1^{s+2i}v^s)$$

and

$$(x_1^s v^{p-1-2i+s})\left(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\right)=(-1)^k\det(\alpha)^{i-s}x_1^s v^{p-1-2i+s}$$
$$(\textit{resp. } (-1)^k\det(\alpha)^{s-1}x_1^s v^{p-1-2i+s}).$$

(3) *If $n=k(p-1)$ and $k>0$, we have*

$$(x_1v)^s\left(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\right)=(-1)^k\det(\alpha)^{-s}(x_1v)^s$$
$$(\textit{resp. } (-1)^k\det(\alpha)^s(x_1v)^s).$$

Proof. We prove in the case of the coefficient ρ^*. Since $p\nmid[\tilde{\Gamma}, \Gamma]$ and Γ acts trivially on $H^n(\Gamma, \rho^*)$, it is sufficient to consider the action of $\Delta=\left\{\begin{bmatrix} a & 0\\ 0 & 1\end{bmatrix}:\ a\in F_p^\times\right\}$ on $H^n(U, S(V))$.

From Proposition 1 and the remark preceding Theorem 5, we obtain

$$\Big((x_1^{t_1}v^{s_2})\Big(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\Big)\Big)(e_{2i})=\det(\alpha)^{s_1+ps_2}(c_{\alpha^{-1}}x_1^{s_1}v^{s_2})(e_{2i})$$
$$=\det(\alpha)^{s_1+s_2}(\alpha^{-1}x_1)^{s_1}(\alpha v)^{s_2}(\det(\alpha)^{i}e_{2i})$$
$$=\det(\alpha)^{s_2+i}(x_1^{s_1}v^{s_2})(e_{2i})$$

and

$$\Big((x_2^{s_1}v^{s_2})\Big(\frac{1}{|\Gamma|}\Gamma\alpha\Gamma\Big)\Big)(e_{2i+1})=\det(\alpha)^{s_1+ps_2}(c_{\alpha^{-1}}x_2^{s_1}v^{s_2})(e_{2i+1})$$
$$=\det(\alpha)^{s_1+s_2}(\alpha^{-1}x_2)^{s_1}(\alpha^{-1}v)^{s_2}\Big(\det(\alpha)^{i}\sum_{j=0}^{\det(\alpha)-1}y^{j}e_{2i+1}\Big)$$
$$=\det(\alpha)^{s_1+s_2+i+1}(x_2^{s_1}v^{s_2})(e_{2i+1}) \text{ mod coboundaries.}$$

Then the theorem follows from Theorems 3 and 4. q. e. d.

Remark 5. M. Kuga, W. Parry and C. H. Sah [7, Theorem 1.5.4] have shown the theorem in the case of $n=1, 2$ and $t\leqq p-1$.

We can calculate the cohomotogy of $GL_2(F_p)$ with coefficients ρ and ρ^*.

THEOREM 6.

(1) $H^0(\tilde{\Gamma}, \rho)\cong F_p[(x_1v)^{p-1}, x_1^{p(p-1)}+v^{p-1}]$ *and* $H^0(\tilde{\Gamma}, \rho^*)\cong H^0(\Gamma, S(V))\cong F_p[x_1v, x_1^{p(p-1)}+v^{p-1}]$.

(2) $H^n(\tilde{\Gamma}, \rho)\cong H^n(\tilde{\Gamma}, \rho^*)$ *for* $n>0$.

(3) $H^n(\tilde{\Gamma}, \rho)$ *has the p-period* $2(p-1)$ *within the positive degrees.*

(4) *The multiplication by* $x_1^{p(p-1)}+v^{p-1}$ *induces the isomorphism*

$$H^n(\tilde{\Gamma}, S^t(V))\cong H^n(\tilde{\Gamma}, S^{t+p(p-1)}(V)) \qquad \textit{for} \quad n>0.$$

(5) $H^n(\tilde{\Gamma}, \bigoplus_{0\leq m<p(p-1)}\rho_m)$

$$\cong\begin{cases} F_p x_1^{i}v^{p-1-2i} & \textit{if} \quad n=2i,\ 0<i<p-1 \\ 0 & \textit{if} \quad n=2i+1,\ n\neq 2p-3 \\ F_p\oplus\bigoplus_{1\leq s\leq p-2}F_p x_2^{s}v^{p-2-s} & \textit{if} \quad n=2p-3 \\ \bigoplus_{0\leq s\leq p-2}F_p(x_1v)^{s} & \textit{if} \quad n=2(p-1). \end{cases}$$

Proof. From Theorem 1, we have $H^n(\tilde{\Gamma}, \rho)=H^n(\Gamma, \rho)^{\Delta}$. For example in the case of $n=k(p-1)+2i$, $0<i<\frac{p-1}{2}$, the invariant elements of $H^n(\Gamma, S(V))$ under the action ρ satisfy the conditions by Theorems 4 and 5.

(1) $x_1^{s+2i}v^{s}=(-1)^{k}\det(\alpha)^{-(s+i)}x_1^{s+2i}v^{s}$ for any $\alpha\in\Delta$, $0\leqq s<p-1-2i$

and

(2) $x_1^s v^{p-1-2i+s} = (-1)^k \det(\alpha)^{i-s} x_1^s v^{p-1-2i+s}$ for any $\alpha \in \Delta$, $0 \leqq s < 2i$.

We can determine the values s from the above equations and obtain the solutions from (1) and (2): If $k \equiv 1 \mod(2)$, $s = \frac{p+1}{2} - i$ and, if $k \equiv 0 \mod(2)$, $s = i$ respectively. Therefore we have shown that

$$H^n(\tilde{\Gamma}, \bigoplus_{0 \leq m < p(p-1)} \rho_m) = F_p x_1^i v^{p-1-2i}, \quad \text{if} \quad n = 2i,\ 0 < i < \frac{p-1}{2}.$$

We can prove the others similarly. q. e. d.

§ 5. The cohomology of one-dimensional representations.

In section 4, we determined the cohomology of the polynomial representation of $\tilde{\Gamma}$. Here we consider the cohomology derived from one-dimensional representations, and use the some notations as in section 4.

Let M and M' be $\tilde{\Gamma}$-module. Then we define $M \otimes M'$ as follows.

$$M \otimes M' \cong M \underset{Z}{\otimes} M' \quad \text{as an abelian group}$$

and the left $\tilde{\Gamma}$-action is defined by

$$\sigma(m_1 \otimes m_2) = \sigma(m_1) \otimes \sigma(m_2).$$

If M and M' are as above, we have a pairing called the cup product

$$H^p(\tilde{\Gamma}, M) \otimes H^q(\tilde{\Gamma}, M') \longrightarrow H^{p+q}(\tilde{\Gamma}, M \otimes M').$$

We consider one-dimensional representations $\phi^i : \tilde{\Gamma} \to F_p^{\times}$ where $\phi^i(g) = \det(g)^i$ We regard Z/pZ as a $\tilde{\Gamma}$-module $Z/pZ(i)$ through ϕ^i. Then we have isomorphisms

$$Z/pZ(i) \cong Z/pZ(x_1^i v^i)$$

$$Z/pZ(i) \otimes Z/pZ(j) \cong Z/pZ(i+j)$$

and

$$Z/pZ(i+p-1) \cong Z/pZ(i).$$

PROPOSITION 3. (Aguadé [1]). *$H^*(\Gamma, Z/pZ) \cong Z[u_1] \otimes \Lambda(v_1)$, $|u_1| = p-1$, $|v_1| = p-2$ and $H^*(\tilde{\Gamma}, Z/pZ) \cong Z/pZ[u_2] \otimes \Lambda(v_2)$, $|u_2| = 2(p-1)$, $|v_2| = 2p-3$.*

Proof. We only show the cohomology of Γ. Use the split exact sequence

$$0 \longrightarrow F_p \longrightarrow S(V) \longrightarrow S(V)^+ \longrightarrow 0$$

and we have $H^*(\Gamma, S(V)) = H^*(\Gamma, Z/pZ) \oplus H^*(\Gamma, S(V)^+)$. From Theorem 4, we have an additive isomorphism

$$H^*(\Gamma, Z/pZ) \cong Z/pZ[u_1] \otimes \Lambda(v_1), \ |u_1| = p-1 \quad \text{and} \quad |v_1| = p-2.$$

Since $i^*: H^n(\Gamma, Z/pZ) \to H^n(U, Z/pZ)$ is injective and $H^*(U, Z) \cong F_p[x] \otimes \Lambda(y)$, $|x|=2$ and $|y|=1$, as a ring isomorphism, we can choose u_1(resp. v_1) so as to satisfy $i^*(u_1)=x^{p-1/2}$(resp. $i^*(v_1)=x^{p-3/2}y$). q. e. d.

THEOREM 7.

(1) $H^n(\tilde{\Gamma}, Z/pZ(i))=0$, *if* $i \not\equiv \frac{p-1}{2} \bmod (p-1)$ *and* $i \not\equiv 0 \bmod (p-1)$, *and*

$$H^n\left(\tilde{\Gamma}, Z/pZ\left(\frac{p-1}{2}\right)\right) \cong \begin{cases} Z/pZ & \text{if } n \equiv p-2 \text{ or } p-1 \bmod 2(p-1) \\ 0 & \text{otherwise.} \end{cases}$$

(2) *If we put* α *(resp.* β*) for a generator of* $H^{p-1}\left(\tilde{\Gamma}, Z/pZ\left(\frac{p-1}{2}\right)\right)$ $\left(\right.$*resp.* $\left.H^{p-2}\left(\tilde{\Gamma}, Z/pZ\left(\frac{p-1}{2}\right)\right)\right)$, *the cup product of* $\bigoplus_i H^*(\tilde{\Gamma}, Z/pZ(i))$ *is given by*

$$\alpha^2=u_2, \ \alpha\beta=v_2 \quad \text{and} \quad \beta^2=0.$$

Proof. We have $H^n(\tilde{\Gamma}, Z/pZ(i)) \to H^n(\Gamma, Z/pZ(i))^T$, where T is the group of the diagonal matrices. Because $Z/pZ(i)=Z/pZ$ on U, non zero cohomology groups can appear only when $n \equiv p-2$ or $p-1 \bmod 2(p-1)$. Then the conjugate actions c_t, $t \in T$ on a n-cochain $y^i v^i$ are

$$c_t(y^i v^i)=\det(t)^{i-(p-1/2)} y^i v^i.$$

We see that there is a non zero invariant space for the actions if and only if $i \equiv 0$ or $i \equiv \frac{p-1}{2} \bmod 2(p-1)$.

(2) From [3, Chap. XII, 7], the restriction map

$$i^*; \bigoplus_i H^*(\tilde{\Gamma}, Z/pZ(i)) \longrightarrow \bigoplus_i H^*(U, Z/pZ)$$

preserves the cup products. Noticing that

$$i^*(u_2)=x^{p-1}, \ i^*(v_2)=x^{p-2}y$$

and

$$i^*(\alpha)=x^{p-1/2}, \qquad i^*(\beta)=x_1^{p-3/2}y,$$

we get the formulas of the Theorem. q. e. d.

§6. The Hecke algebra $B \backslash GL_3(F_p)/B$ actions.

In this section, we adapt the notations to those of [12] so as to simplify the computations. Let U be a p-Sylow subgroup of $GL_3(F_p)$ which is generated by

$$a=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, b=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}, \quad c=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

THEOREM 8. [6], [8].

(1) *If $p=2$, the algebra $H^*(U, Z)$ are generated by elements y_1, y_2, e, v, where $|y_i|=2, 1\leqq i\leqq 2, |e|=3$ and $|v|=4$. The relations are submitted to the relations*

$$2y_1=2y_2=2e=4v=0 \quad and \quad y_1y_2=0,\ e^2=(y_1+y_2)v.$$

(2) *If p is an odd prime, the algebra $H^*(U, Z)/\sqrt{0}$ are generated by elements y_1, y_2, b_{p-2}, v where $|y_i|=2, 1\leqq i\leqq 2, |b_{p-2}|=2(p-1)$ and $|v|=2p$. The relations are submitted to the relations*

$$py_1=py_2=pb_{p-2}=p^2v=0 \quad and \quad b_{p-2}^2=y_1^{p-1}y_2^{p-1},$$

$$y_ib_{p-2}=y_iy_j^{p-1},\ y_1^py_2=y_1y_2^p.$$

Next we recall some facts about the elements of $H^*(U, Z)$ which are in the images by the correstriction map. If p is an odd prime, we have the ring isomorphism

$$H(\langle a, c\rangle, Z)\cong Z/pZ[y_1, u]\otimes\Lambda(\beta(xz))$$

$$\text{for} \quad *>0,\ |y_1|=|u|=2,\ |\beta(xz)|=3$$

and $y_1=\beta(x_1)$, $u=\beta(z)$, where x_1 and z are the dual elements of a and b with the identification $H^1(\langle a, c\rangle, Z/pZ)\cong\mathrm{Hom}(\langle a, c\rangle, Z/pZ)$ and β is the mod p Bockstein operation. If $p=2$, the above isomorphism is only additive. Let $i: \langle a, c\rangle\to U$ be the inclusion map and $i_!$ and i^* be the correstriction and the restriction maps respectibly. Then we define

$$b_0=i_!(u), \qquad \text{if} \quad p=2$$

and

$$b_i=i_!(u^{i+1}) \quad \text{for} \quad 1\leqq i\leqq p-3 \quad \text{and} \quad b_{p-2}=i_!(u^{p-1})+y_1^{p-1}, \quad \text{if } p \text{ odd}.$$

PROPOSITION 4. *In case of $p=2$, we have*

$$b_0=y_1 \text{ and } i^*(u)=u^2+uy_1,\ i^*(y_1)=y_1,\ i^*(e)=\beta(xz).$$

Proof. The formulas above are proved in [6] except $b_0=y_1$. To prove this, we apply the cor-res exact sequence of Lewis [8] to $H=\langle a, c\rangle$ and $G=U$. Then we have

$$0 \longrightarrow H^3(U, Z) \xrightarrow{\rho} T^3 \xrightarrow{\tau} H^2(U, Z) \xrightarrow{\cup y_2} H^4(U, Z)$$

and

$$0 \longrightarrow H^2(a, c, Z)_{\langle b\rangle} \xrightarrow{\mu} T^3 \xrightarrow{\varepsilon} H^3(a, c, Z)^{\langle b\rangle} \longrightarrow 0$$

where $\varepsilon\cdot\rho=\mathrm{Res}$ and $\tau\cdot\mu=\mathrm{Cor}$. We see that $H^2(\langle a, c\rangle, Z)_{\langle b\rangle}=Z/2Zu$ and $H^3(\langle a, c\rangle, Z)^{\langle b\rangle}=Z/2Z\beta(xz)$. From the cohomology of U and $\mathrm{Res}(e)=\beta(xz)$, we see that $i_!(u)$ is y_1. q. e. d.

THEOREM 8. [8]. *Let p be an odd prime. Then the elements b_i defined above are non-zero and they are submitted to the relations in $H^*(U, Z)$*

$$y_i b_j = 0, \quad \text{if} \quad 1 \leqq i \leqq 2, \quad 1 \leqq j \leqq p-3$$

and

$$y_i b_{p-2} = y_i y_j^{p-1}, \quad i \neq j, \quad b_{p-2}^2 = y_1^{p-1} y_2^{p-1}.$$

Furthermore we have

$$i^*(v) = u^p - u y_1^{p-1} \quad \text{and} \quad i^*(y_1) = y_1.$$

PROPOSITION 5. *Let $n = kp + r$, $0 \leqq r < p$ be a positive integer. Then we have*

$$i_!(u^n) = \sum_{i=0}^{k} \binom{k}{i} v^{k-i} y_1^{i(p-1)} i_!(u^{r+i}), \; i_!(0) = p.$$

Proof, We write u^{n+p} as $u^n(u^p - u y_1^{p-1}) + u^{n+1} y_1^{p-1}$. From the formulas $i_!(a \cdot i^*(b)) = a \cdot b$ and $i_! i^*(a) = pa$, we have

$$i_!(u^{n+p}) = v i_!(u^n) + y_1^{p-1} i_!(u^{n+1}).$$

Repeating this argument, we obtain the proposition. q. e. d.

COROLLARY 1. $i_!(u^{p(p-1)}) = p v^{p-1} + y_1^{(p-1)^2} y_2^{p-1} - y_1^{p(p-1)}$.

Proof. Since $\binom{p-1}{i} \equiv (-1)^i \bmod p$, it is immediate from the proposition.

THEOREM 9. [12].

(1) *In the case of $p=2$, we have*

$$H^*(B, Z)_{(2)} = H^*(U, Z)_{(2)}.$$

(2) *If p is an odd prime, the cohomology ring $H^*(B, Z)_{(p)}/\sqrt{0}$ are isomorphic to the subalgebra of $H^*(U, Z)_{(p)}/\sqrt{0}$ which is generated by*

$$y_1^{p-1}, \; y_2^{p-1}, \; v^{p-1}, \; b_{p-2} \quad \text{and} \quad (y_1 y_2)^i v^{p-1-i}, \; 1 \leqq i \leqq p-1.$$

Since $GL_3(F_p) = \coprod_{w \in S_3} BwB$, we have $B \backslash GL_3(F_p)/B = Z\left[\frac{1}{|B|} BwB\right]$. We can identify BwB with $w \in S_3$.

THEOREM 10. *The Hecke algebra actions on the generators of $H^*(B, Z)_{(p)}/\sqrt{0}$ are written as in the following tables.*

(1) *In the case of $p=2$,*

w	(12)	(23)	the others
y_1	0	y_1	0
y_2	y_2	0	0
e	0	0	0
v	$2v$	$2v$	0

(2) *In the case of an odd prime p,*

w	(12)	(23)	the others
y_1^{p-1}	0	$b_{p-2}-y_1^{p-1}$	0
y_2^{p-1}	$b_{p-1}-y_2^{p-1}$	0	0
v^{p-1}	$y_2^{p(p-1)}-y_1^{p-1}y_2^{(p-1)^2}$	$y_1^{p(p-1)}-y_1^{(p-1)^2}y_2^{p-1}$	0
the others	0	0	0

Proof. We prove the theorem for $w=(23)$ and $v^{p-1}\in H^*(B, Z)_{(p)}$. It is sufficient to consider the Hecke algebra actions on $H^*(U, Z)$ instead of $H^*(B, Z)_{(p)}$. So we have

$$\begin{aligned} v^{p-1}\left(\frac{1}{|B|}BwB\right) &= i_!(i^*c_w(v^{p-1}))=i_!(c_w(i^*v^{p-1})) \\ &= i_!\Big(\prod_{\lambda\in F_p}(y_1-\lambda u)^{p-1}\Big)=i_!(y_1^{p(p-1)}+\Pi(u-\lambda y_1)^{p-1}-u^{p(p-1)}) \\ &= i_!(i^*y_1^{p(p-1)}+i^*v^{p-1}-u^{p(p-1)}) \\ &= pv^{p-1}-pv^{p-1}+y_1^{p(p-1)}-y_1^{(p-1)^2}y_2^{p-1} \\ &= y_1^{p(p-1)}-y_1^{(p-1)^2}y_2^{p-1}. \end{aligned}$$

q. e. d.

Remark 6. This shows that the Hecke algebra $B\backslash GL_3(F_p)/B$ does not act as a scalar multiplication different from [7] and [11]. It seems to be occured in the action of the Hecke algebra $B\backslash GL_n(F_q)/B$ on $H^*(B)$ generally. In the sence, the tables would give us a non trivial example of a theorem [4].

References

[1] J. Aguadé, The cohomology of the GL_2 of a finite field, Arch. Math., **34** (1980), 509-516.

[2] A. Babakhanian, Cohomological methods in group theory, Marcel Dekker, Inc. 1972.

[3] H. CARTAN AND S. EILENBERG, Homological algebra, Princeton Univ. Press, Princeton, 1956.
[4] E. CLINE, B. PARSHALL AND L. SCOTT, Cohomology of finite groups of Lie type, Publ. Math. I.H.E.S., **45** (1975), 169-191.
[5] L. DICKSON, A fundamental system of invariants of the general linear group with a solution of the form problem, Trans. Amer. Math. Soc., **12** (1911), 75-98.
[6] L. EVENS, On the Chern classes of representation of finite group, Trans. Amer. Math. Soc., **115** (1965), 180-193.
[7] M. KUGA, W. PARRY AND C.H. SAH, Group cohomology and Hecke operators, Manifolds and Lie groups, Birkhauser (1981), 223-266.
[8] G. LEWIS, The integral cohomology rings of groups of order p^3, Trans. Amer. Math. Soc., **132** (1968), 501-529.
[9] S. MAC LANE, Homology, Springer 1963.
[10] R.G. SWAN, The p-period of a finite group, Illinois J. Math., **4** (1960), 341-346.
[11] Y.H. RHIE AND G. WHAPLES, Hecke operators in cohomology of groups, J. Math. Japan, **22** (1970), 431-442.
[12] M. TEZUKA AND N. YAGITA, The mod p cohomology ring of $GL_3(F_p)$, J. Algebra, **81** (1983), 295-303.
[13] M.-J. DUMAS, Notes des membres et correspondants et notes présentées ou transmises par leurs soins, C.R. Acod. Sci. Paris, **260** (1965), 5655-5658.

DEPARTMENT OF MATHEMATICS
TOKYO INSTITUTE OF TECHNOLOGY
OHOKAYAMA, MEGURO-KU, TOKYO