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GROWTH OF COMPOSITE ENTIRE FUNCTIONS

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1. Introduction.

Singh had proved the following result in his paper [2]. Let $f(z)$ and $g(z)$ be entire functions of finite order satisfying $\rho_g<\lambda_f\leqq\rho_f$ and $g(0)=0$. Here ρ and λ indicate the order and the lower order of indexed entire function, respectively. Then

$$\limsup_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}\leqq\rho_f.$$

In this paper we shall prove the following

THEOREM. *under the same assumptions as in Singh's result*

$$\lim_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}=0.$$

This Theorem shows that corollary 1 of Singh's paper [2] does not have any meaning at all. In fact Singh assumed

$$\liminf_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}\geqq\rho_f.$$

and $\rho_g<\lambda_f\leqq\rho_f$. Hence ρ_f should be positive, But Theorem shows that

$$\lim_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}=0.$$

This is a contradiction. This shows that

$$\liminf_{r\to\infty}\frac{\log T(r, f(g))}{T(r, f)}\geqq\rho_f>0$$

does not occur.

2. Proof of Theorem.

Firstly it is possible to choose $\varepsilon>0$ so small that

$$\rho_g+2\varepsilon<\lambda_f-\varepsilon,$$

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since $\rho_g < \lambda_f$. Then there exists $r_0(\varepsilon)$ so that

$$r^{\lambda_f-(\varepsilon/2)} < T(r, f) < r^{\rho_f+\varepsilon}, \qquad \log M(r, g) < r^{\rho_g+\varepsilon}$$

for all $r > r_0(\varepsilon)$ Further it is possible to assume that $\rho_f + \varepsilon < r^{\varepsilon}$. Niino and Suita [1] showed that

$$\log T(r, f(g(z))) \leqq \log T(M(r, g), f)$$

Hence for $r > r_0(\varepsilon)$

$$\begin{aligned}\log T(r, f(g(z))) &< (\rho_f + \varepsilon) \log M(r, g)\\ &< (\rho_f+\varepsilon) r^{\rho_g+\varepsilon}\\ &< r^{\rho_g+2\varepsilon}\\ &< r^{\lambda_f-\varepsilon}.\end{aligned}$$

Hence

$$\frac{\log T(r, f(g(z)))}{T(r, f)} < \frac{r^{\lambda_f-\varepsilon}}{r^{\lambda_f-(\varepsilon/2)}} = r^{-(\varepsilon/2)}.$$

This gives the desired result:

$$\lim_{r\to\infty} \frac{\log T(r, f(g(z)))}{T(r, f)} = 0$$

References

[1] K. Niino and N. Suita, Growth of a composite function of entire functions, Kodai Math. J., **3** (1980), 374-379.

[2] A. P. Singh, Growth of composite entire functions, Kodai Math. J., **8** (1985), 99-102.

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