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SECOND ORDER LINEAR ORDINARY DIFFERENTIAL EQUATIONS WITH TURNING POINTS AND SINGULARITIES II

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§ 1. Introduction.

WKB approximations are asymptotic approximations for solutions of linear ordinary differential equations, in some domain of an independent complex variable, as a parameter tends to the prescribed value. The largest of such domains of an independent variable is called a canonical region for the differential equation considered. In the part I [3] we considered the differential equation of the type

$$\varepsilon^2 \frac{d^2 y}{d x^2} - p(x) y = 0,$$

where x is a complex variable and ε is a positive small parameter, and we constructed canonical regions for the cases where $p(x)=(x-1)^2/x^3$ and $p(x)=(x-1)^2/x$. In this part we treat two cases one of which is given by $p(x)=(x-1)^2/x^2$, and the other $p(x)=x^\nu-1/x^\mu$ (ν is a positive integer, $\mu=2$). The case $\mu=1$ is treated in [4]. The case $\mu\geqq 3$ is treated elsewhere. In the case where $p(x)$ is of the form $c(x-1)^2/x^2$ or $c(x^\nu-1/x^\mu)$ (c is constant) we can also discuss in the same way. For example, if c is positive, we must only replace ε by $\varepsilon/\sqrt{c}$.

As for the first case we construct canonical regions (§ **2**). As for the second case we construct canonical regions (§ **3**) and get two different types of asymptotic approximations (§ **4** and § **5**) of solutions of the differential equation. Between them there is a relation, and we get the relation, i. e., a matching matrix (§ **6**).

§ 2. Canonical regions for the case $p(x)=(x-1)^2/x^2$.

The differential equation corresponding to this case has a second order turning point at $x=1$, a regular singular point at the origin and an irregular singular point at $x=\infty$. In order to decide Stokes curves for the case, let us consider the integral

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$$\xi(1,\ x)=\int_1^x \frac{x-1}{x}\,dx=x-\log x-1$$
$$=\nu-1-\frac{1}{2}\log(\nu^2+\mu^2)+i\Big(\mu-\tan^{-1}\frac{\mu}{\nu}\Big),$$

where $x=\nu+i\mu$ varies on the x-plane such that $-\pi<\arg x\leqq\pi$. Stokes curves and anti-Stokes curves proceed from the turning points and they are defined respectively by equations $\mathrm{Re}\,\xi=\mathrm{const}$, and $\mathrm{Im}\,\xi=\mathrm{const}$. Also other level curves are decided by the above equations. These curves are illustrated in Fig. 1. In Fig. 1, l_j $(j=1, \cdots, 4)$ denotes Stokes curves. The complex conjugates of points on l_1 (l_3) are on l_2 (l_4). L_j $(j=0, \cdots, 3)$ denotes anti-Stokes curves. $L_0=\{x : 0<x\leqq 1\}$ and $L_1=\{x : 1\leqq x\}$. $L_4\cup L_6(L_4\cap L_6=\emptyset)$ and $L_5\cup L_7(L_5\cap L_7=\emptyset)$ are on the negative real axis. D_j's denote domains bounded by these curves. For example, D_7 is a domain bounded by curves l_3, L_6 and L_0.

There are four quarter-domains D_1, D_2, D_3 and D_4, two strip-domains D_5 and D_6, and a circle-domain around the origin (Fig. 1). $\xi=\xi(1,\ x)$ is considered as a mapping from the x-plane to the ξ-plane with perpendicular coordinates $(\mathrm{Re}\,\xi,\ \mathrm{Im}\,\xi)$. Some domain in the x-plane is conformally mapped by one-to-one onto some domain in the ξ-plane. We want to choose the whole ξ-plane with cuts as the second domain. We can map D_1 onto the quarter plane: $\mathrm{Re}\,\xi>0$, $\mathrm{Im}\,\xi>0$; D_2 onto the quarter plane: $\mathrm{Re}\,\xi>0$, $\mathrm{Im}\,\xi<0$; D_3 onto the quarter plane: $\mathrm{Re}\,\xi<0$, $\mathrm{Im}\,\xi>0$; D_4 onto the quarter plane: $\mathrm{Re}\,\xi<0$, $\mathrm{Im}\,\xi<0$; D_5 onto the strip domain: $\mathrm{Re}\,\xi<0$, $-\pi<\mathrm{Im}\,\xi<0$; and D_6 onto the strip domain: $\mathrm{Re}\,\xi<0$, $0<\mathrm{Im}\,\xi<\pi$. Thus the domain

$$D_2\cup L_1\cup D_1\cup l_1\cup D_3\cup L_2\cup D_5 \quad (\text{or } D_1\cup L_1\cup D_2\cup l_2\cup D_4\cup L_3\cup D_6)$$

is mapped conformally on-to-one onto a domain in the ξ-plane, which consists of a positive (or negative) imaginary axis, a right half plane: $\mathrm{Re}\,\xi>0$; and a quarter plane: $\mathrm{Re}\,\xi<0$, $\mathrm{Im}\,\xi>-\pi$ (or $\mathrm{Re}\,\xi<0$, $\mathrm{Im}\,\xi<\pi$), with a cut on the negative (or positive) imaginary axis. Therefore they are (incomplete) canonical regions. The domain $D_7\cup L_0\cup D_8$ is conformally mapped one-to-one onto a strip-domain on the right half plane: $|\mathrm{Im}\,\xi|<\pi$, $\mathrm{Re}\,\xi>0$.

If x varies beyond the negative real axis, we must check figures of level curves. As for x on the second sheet the integral ξ is modified such as $\xi(1,\ x)=\xi(1,\ 1')+\xi(1',\ x)$, where $1'$ is over 1 on the first sheet, and the path of integration of $\xi(1,\ 1')$ $(\xi(1',\ x))$ consists of a unit circle with positive direction around the origin (an appropriate bounded curve on the second sheet). Then $\xi(1,\ 1')=-2\pi i$, $\mathrm{Re}\,\xi(1',\ x)=\mathrm{Re}\,\xi(1,\ x)$. Thus the Stokes curves on the second sheet are projections of Stokes curves on the first. Generally the curves defined by $\mathrm{Re}\,\xi=0$ and $\mathrm{Im}\,\xi=-2n\pi$ for x on the $(n+1)$-th sheet correspond to respectively the curves defined by $\mathrm{Re}\,\xi=0$ and $\mathrm{Im}\,\xi=0$ for x on the first sheet.

The domain $D_4\cup L_3\cup D_6$ on the second sheet (denoted by $D_4'\cup L_3'\cup D_6'$) together with $L_4\cup D_5\cup L_2\cup D_3$ is conformally mapped one-to-one onto the left half plane of the ξ-plane, and then

$$D_2 \cup L_1 \cup D_1 \cup l_1 \cup D_3 \cup L_2 \cup D_5 \cup L_4 \cup D_6' \cup L_3' \cup D_4'$$

is conformally mapped one-to-one onto the whole ξ-plane with a cut on the negative imaginary axis. Thus this domain is a canonical region for the differential equation. Similarly,

$$D_3' \cup L_2' \cup D_5' \cup L_5 \cup D_6 \cup L_3 \cup D_4 \cup l_2 \cup D_2 \cup L_1 \cup D_1$$

is also a canonical region mapped onto the whole ξ-plane with a cut on the positive imaginary axis.

To find other canonical regions we need infinitely many sheets. As mentiond already, $D_7 \cup L_0 \cup D_8$ is mapped onto a strip, and infinitely many strips of this type fulfill the right half plane of the ξ-plane: $\mathrm{Re}\,\xi>0$. Thus we get two canonical regions each of which consists of infinitely many domains of the type $D_7 \cup L_0 \cup D_8$ together with domains mapped onto the left half plane which were got already. These canonical regions are mapped onto the whole ξ-plane with two cuts: $\mathrm{Im}\,\xi \geqq 2\pi$, $\mathrm{Im}\,\xi \leqq 0$, or $\mathrm{Im}\,\xi \geqq -2\pi$ on the imaginary axis.

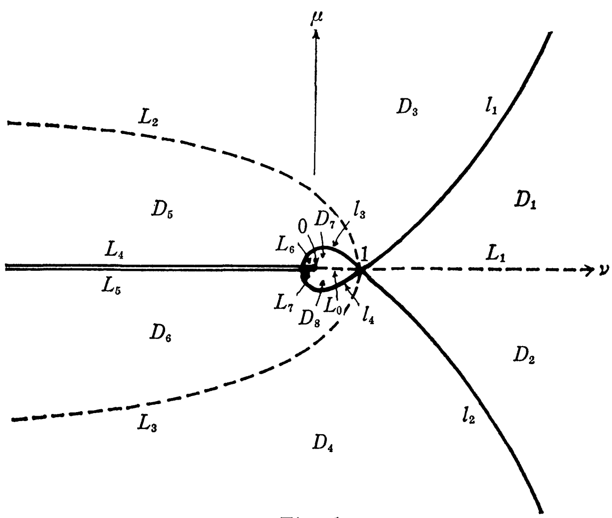

Fig. 1

§ 3. Canonical regions for the case $p(x)=x^{\nu}-1/x^2$.

Let $p(x)=x^{\nu}-1/x^{\mu}$, where ν is a positive integar. If μ is non-positive, $p(x)$ is a polynomial of x and the corresponding differential equation has turning points and an irregular singularity at $x=\infty$. If μ is positive, $p(x)$ is a rational function of x and the corresponding differential equation has turning points and singularities at the origin and at $x=\infty$. If μ is 1 or 2 the origin is the regular singularity. The case $\mu=1$ is treated in [4], and here we consider the case $\mu=2$. In this case the corresponding differential equation has $\nu+2$ turning poins, a regular singular point at the origin and an irregular singular point at $x=\infty$.

In order to construct canonical regions we consider hyperelliptic trajectories defined by an integral

$$\xi'=\int^{t}\left(\tau^{\nu}-\frac{1}{\tau^{2}}\right)^{1/2}d\tau,$$

which may not be represented by elementary functions. If τ is replaced by $\tau e^{2k\pi i/(\nu+2)}$ ($\tau>0$; $k=0, 1, \cdots, \nu+1$), the integral ξ' does not change its form. Thus Stokes curves defined by $\mathrm{Re}\,\xi'=$const. are direct lines $l_0^{(k)}$ ($k=0, 1\cdots, \nu+1$) proceeding from (secondary) turning points (i. e., zeros of the integrand (§ 4)) going to the origin (Fig. 2). Since the differential equation is of Airy type at each of turning points, three Stokes curves proceed from each of turning points, one of which is given by the direct line $l_0^{(k)}$, as above, and others are represented by $l_1^{(k)}$ and $\bar{l}_1^{(k)}$ in Fig. 2. At every turning point angle between any two Stokes curves proceeding from it is $2\pi/3$. Thus Stokes curves $l_0^{(k)}$, $l_1^{(k)}$ and $\bar{l}_1^{(k)}$ proceeding from the turning point $e^{2k\pi i/(\nu+2)}$ have respectively arguments of $2k\pi/(\nu+2)+\pi$, $2k\pi/(\nu+2)+\pi/3$ and $2k\pi/(\nu+2)-\pi/3$ at the turning point.

Stokes curves $l_1^{(k)}$, $\bar{l}_1^{(k)}$ (k=0, 1, $\cdots$, $\nu+1$) entering the point at infinity have $\nu+2$ arguments $\pm\pi/(\nu+2)$, $\pm 3\pi/(\nu+2)$, $\pm 5\pi/(\nu+2)$, $\cdots$ at the point at infinity. Anti-Stokes curves defined by $\mathrm{Im}\,\xi'=$const. and Stokes curves are perpendicular at any point except for turning points and the origin. Therefore we get anti-Stokes curves, that is, $\nu+2$ curves joining adjacent two turning points $L_1^{(k)}$ ($k=0$, $1, \cdots, \nu+1$) and $\nu+2$ direct lines $L_0^{(k)}$ ($k=0, 1, \cdots, \nu+1$) proceeding from turning points tending to the point at infinity (Fig. 2).

The function of t

$$\xi=\int_{1}^{t}\left(\tau^{\nu}-\frac{1}{\tau^{2}}\right)^{1/2}d\tau$$

can be regarded as a mapping from the t-plane to the ξ-plane. A domain containing $L_0^{(0)}$ in its interior and bounded by $l_1^{(0)}$ and $\bar{l}_1^{(0)}$ is a half plane domain, that is, it is mapped by ξ onto a right or left half plane of the ξ-plane. There are $\nu+2$ half plane domains like this, each of which contains $L_0^{(k)}$ ($k=0, 1, \cdots$, $\nu+1$) in its interior and bounded by $l_1^{(k)}$ and $\bar{l}_1^{(k)}$ ($k=0, 1, \cdots, \nu+1$). A domain containing $L_1^{(0)}$ in its interior and bounded by $l_0^{(0)}$, $l_0^{(1)}$, $l_1^{(0)}$ and $\bar{l}_1^{(1)}$ is a strip domain, that is, it is mapped by ξ onto a strip whose boundaries are images of $l_0^{(0)}\cup l_1^{(0)}$ and $l_0^{(1)}\cup\bar{l}_1^{(1)}$, and they are parallel to the imaginary axis. There are $\nu+2$ strip domains like this, each of which contains $L_1^{(k)}$ ($k=0, 1, \cdots, \nu+1$) in its interior and bounded by $l_0^{(k)}\cup l_1^{(k)}$ and $l_0^{(k+1)}\cup\bar{l}_1^{(k+1)}$ ($k=0, 1, \cdots, \nu+1$) ($l_0^{(\nu+2)}=l_0^{(0)}, \cdots$, $\bar{l}_1^{(\nu+2)}=\bar{l}_1^{(0)}$). A strip domain with adjacent two half plane domains constructs a canonical region, which is mapped by ξ onto the whole ξ-plane except for two slits. Those slits proceed from each of images of turning points and tend to

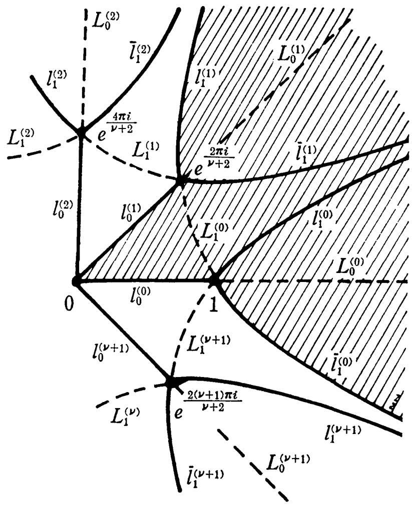

Fig. 2

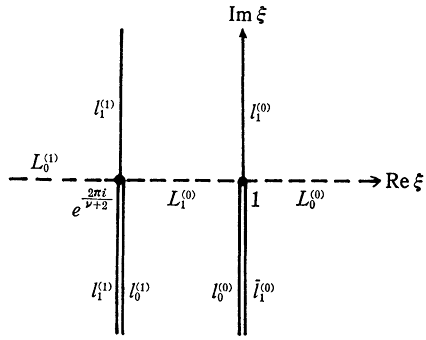

Fig. 3
Same letters are used for images and inverse images.

$\operatorname{Im}\xi=\infty$ or $-\infty$. We can construct $\nu+2$ canonical regions like this, one of which is shaded in Fig. 2 and whose image is shown in Fig. 3

§ 4. Asymptotic approximation 1.

The linear ordinary differential equation

$$\varepsilon\frac{dY}{dx}=\begin{bmatrix}0 & 1\\ x^{\nu}-\varepsilon/x^2 & 0\end{bmatrix}Y \quad (0<|x|\leqq x_0,\ 0<\varepsilon\leqq\varepsilon_0) \tag{4.1}$$

is transformed by

$$x=\varepsilon^{1/(\nu+2)}t,\quad Y=\operatorname{diag}[1,\ \varepsilon^{\nu/2(\nu+2)}]U$$

into the following type

$$\varepsilon^{1/2}\frac{dU}{dt}=\begin{bmatrix}0 & 1\\ p(t) & 0\end{bmatrix}U,\ p(t)=t^{\nu}-1/t^2 \quad (0<|t|<\infty). \tag{4.2}$$

The differential equation (4.1) has a turning point of order ν and a regular singular point, both of which are situated at the origin. The differential equation (4.2) is of the form given in § **1** if it is represented by vectors, and zeros of $p(t)$ are called secondary turning points of (4.1).

In order to get asymptotic approximations of solutions of the differential equation (4.2) we transform it by

$$U=\left(\begin{bmatrix}1 & -1\\ p^{1/2} & p^{1/2}\end{bmatrix}+\frac{p'}{4p^{3/2}}\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\varepsilon^{1/2}\right)V,$$

and we obtain the following differential equation, whose first two coefficients are diagonal

$$\varepsilon^{1/2}\frac{dV}{dt}=\left(\begin{bmatrix}\sqrt{p} & 0\\ 0 & -\sqrt{p}\end{bmatrix}-\frac{p'}{4p}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\varepsilon^{1/2}+\frac{7p'^2-4pp''}{32p^{5/2}}\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix}\varepsilon\right)V.$$

Formal solutions of the differential equation can be shown to be asymptotic expansions of the true solutions in every canonical region given in § **3**. This procedure is similar to [4] and we omit anaylytic theory. Thus we get the asymptotic approximation (i. e., first approximation) in a canonical region as $\varepsilon\to 0$

$$\text{(4.3)}\qquad Y\sim\begin{bmatrix}1 & 0\\ 0 & \varepsilon^{\nu/2(\nu+2)}\end{bmatrix}p^{1/4\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\exp\left(\frac{1}{\sqrt{\varepsilon}}\int_1^t\sqrt{p}\,dt\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\right).$$

§ 5. Asymptotic approximation 2.

In the region

$$K\varepsilon^{1/(\nu+2)}\leqq|x|\leqq x_0\quad (K=\text{const.}),$$

we rewrite the differential equation (4. 1) in a form

$$\text{(5.1)}\qquad x^{\nu/2+2}(x^{-(\nu+2)}\varepsilon)\frac{dZ}{dx}=\left(\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ -1 & -\frac{\nu}{2}x^{(\nu+2)/2}\end{bmatrix}(x^{-(\nu+2)}\varepsilon)\right)Z.$$

where

$$Y=\text{diag}[1,\ x^{\nu/2}]Z.$$

Furthermere we transform this differential equation by

$$Z=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & -b/2\\ a/2 & 0\end{bmatrix}(x^{-(\nu+2)}\varepsilon)+\begin{bmatrix}0 & *\\ * & 0\end{bmatrix}(x^{-(\nu+2)}\varepsilon)^2+\cdots\right)W$$

into

$$x^{\nu/2+2}(x^{-(\nu+2)}\varepsilon)\frac{dW}{dx}=\left(\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}+\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}(x^{-(\nu+2)}\varepsilon)\right.$$
$$\left.+\begin{bmatrix}* & 0\\ 0 & *\end{bmatrix}(x^{-(\nu+2)}\varepsilon)^2+\cdots\right)W,$$

where $a=\frac{1}{2}\left(-1-\frac{\nu}{2}x^{(\nu+2)/2}\right)$, $b=\frac{1}{2}\left(1-\frac{\nu}{2}x^{(\nu+2)/2}\right)$ and asterisks designate some values. Since the last differential equation has diagonal coefficients we can get formal solutions which are asymptotic expansions of the true solutions in sector domain with central angle $4\pi/(\nu+2)$. We choose a sector domain such that arguments of its boundaries correspond to ones of any one of canonical regions, for instance, the sector domain $K\varepsilon^{1/(\nu+2)}\leqq|x|\leqq x_0$, $-\pi/(\nu+2)<\arg x<3\pi/(\nu+2)$ corresponds to the canonical region with boundaries $\bar{l}_1^{(0)}$, $l_0^{(0)}$, $l_0^{(1)}$ and $l_1^{(1)}$. In such a sector we get an asymptotic approximation of the solution of (5.1). Thus we get

$$Y\sim x^{\nu/4\left[\begin{smallmatrix}-1 & 0\\ 0 & 1\end{smallmatrix}\right]}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\exp\left\{\left(\frac{1}{\varepsilon}\frac{2}{\nu+2}x^{(\nu+2)/2}+\frac{1}{\nu+2}x^{-(\nu+2)/2}\right)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\right\}. \tag{5.2}$$

We remark that such a sector intersects a canonical region when the parameter is sufficiently small.

§ 6. Matching matrix.

The 2-by-2 matching matrix M connecting two solutions (4.3) and (5.2) is given by

$$G_0F_0E_0M=G_iF_iE_i, \tag{6.1}$$

where

$$G_0=\mathrm{diag}[x^{-\nu/4},\ x^{\nu/4}],\quad G_i=\mathrm{diag}[p^{-1/4},\ \varepsilon^{\nu/2(\nu+2)}p^{1/4}],$$

$$F_0=F_i=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix},\quad E_0=\exp(\alpha\,\mathrm{diag}[1,\ -1]),$$

$$E_i=\exp(\beta\,\mathrm{diag}[1,\ -1]),\quad \alpha=\frac{1}{\varepsilon}\frac{2}{\nu+2}x^{(\nu+2)/2}+\frac{1}{\nu+2}x^{-(\nu+2)/2},$$

$$\beta=\frac{1}{\sqrt{\varepsilon}}\int_1^t\sqrt{p(t)}\,dt,\quad M=(m_{ij}).$$

The equation (6.1) is written in

$$E_0ME_i^{-1}=F_0^{-1}G_0^{-1}G_iF_i, \tag{6.2}$$

whose right hand side is asymptotically represented:

$$F_0^{-1}G_0^{-1}G_iF_i\sim\frac{1}{2}\begin{bmatrix}x^{\nu/4}p^{-1/4}+\rho x^{-\nu/4}p^{1/4} & x^{\nu/4}p^{-1/4}-\rho x^{-\nu/4}p^{1/4}\\ -x^{\nu/4}p^{1/4}+\rho x^{-\nu/4}p^{1/4} & x^{\nu/4}p^{-1/4}+\rho x^{-\nu/4}p^{1/4}\end{bmatrix},$$

$$\rho=\varepsilon^{\nu/2(\nu+2)}.$$

As we remarked in § 5 an outer sector and a canonical region intersect each other for small ε. Then let $x=\eta\rho$, $t=\eta\rho^{-1}(|\eta|=1)$. We get

$$x^{\nu/4}p^{-1/4} \quad \text{and} \quad x^{-\nu/4}p^{1/4}\sim\rho^{\nu/2} \quad \text{as} \quad \rho\to 0.$$

Therefore the right hand side of (6.2) has an asymptotic approximation:

$$F_0^{-1}G_0^{-1}G_\imath F_\imath \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rho^{\nu/2} \quad (\rho\to 0).$$

Next we calculate the left hand side of (6.2) which is given by

$$E_0 M E_\imath^{-1} = \begin{bmatrix} m_{11}e^{\alpha-\beta} & m_{12}e^{\alpha+\beta} \\ m_{21}e^{-\alpha-\beta} & m_{22}e^{-\alpha+\beta} \end{bmatrix}.$$

By replacing x, t by η, ρ, we get

$$\beta=\alpha+O(\eta^{-(6+3\nu)/2}\rho^{1+\nu/2})-c\rho^{-\nu-2} \quad (\rho\to 0),$$

where c is a constant. Therefore hold the following asymptotic relations

$$\alpha-\beta=c\rho^{-\nu-2}+O(\rho^{1+\nu/2}), \quad \alpha+\beta=2\alpha-c\rho^{-\nu-2}+O(\rho^{1+\nu/2}) \quad (\rho\to 0).$$

Since $\alpha=O(\eta^{1+\nu/2}\rho^{-3\nu/2-3})$, $\mathrm{Re}\,\alpha$ tends to infinity if we take η such that $|\eta|=1$ and $\rho\to 0$, and $\mathrm{Re}\,\alpha$ tends to minus infinity if $\eta=e^{2\pi i/(\nu+2)}$ and $\rho\to 0$ (Fig. 3). Thus $\alpha+\beta$ takes arbitrarily large and small values. Therefore we get m_{12}, $m_{21}\sim 0$ after equating both hand sides, and the matching matrix M has an asymptotic approximation such as

$$M\sim\varepsilon^{\nu/4(\nu+2)}\exp\left(\frac{c}{\sqrt{\varepsilon}}\ \mathrm{diag}[-1, 1]\right) \quad (\varepsilon\to 0).$$

Since each of other canonical regions has two directions such that $\mathrm{Re}\,\alpha\to\pm\infty$ as $\varepsilon\to 0$, a matching matrix corresponding to each canonical region can be calculated in the similar way and has a similar form.

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