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Erratum to "Conformally flat 3-manifolds with constant scalar curvature"

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Abstract. In this note, we corrected the coefficients of formula (2.12) in Proposition 1 and the third formula in Proposition 2 in our paper [**1**]. We should remark that our theorems in [**1**] are not harmed.

In this note, we must mention that the coefficients of formula (2.12) in Proposition 1 and the third formula in Proposition 2 in our paper [**1**] should be corrected as follows:

$$\frac{1}{2}\Delta\sum_{i,j,k}R_{ij,k}^2=\sum_{i,j,k,l}R_{ij,kl}^2+\frac{3(n+2)}{n-2}\sum_{i,j,k}\lambda_iR_{ij,k}^2-\frac{(n+4)r}{(n-1)(n-2)}\sum_{i,j,k}R_{ij,k}^2. \tag{2.12}$$

$$\sum_{i,j,k,l}R_{ij,kl}^2=-24\sum_{i,j,k}\mu_iR_{ij,k}^2-\frac{3}{4}r\sum_{i,j,k}R_{ij,k}^2-\frac{9}{4}B\left(B-\frac{1}{6}r^2\right). \tag{2.21}$$

We should remark that their proofs do not need to be corrected except the statements of results of formulas.

For the proofs of Theorem 1 and Theorem 2 in the section 3 of our paper [**1**], except the coefficient of formula (3.7), we do not need to correct any thing. Hence, the assertions of Theorem 1 and Theorem 2 in [**1**] are true. Thus, Main Theorem 1 in [**1**] is also true. Although the proof of Theorem 3 in [**1**] needs to be corrected as follows (in fact, we use the same idea as before), we know that the assertion is true. Hence, the assertion in Main Theorem 2 in [**1**] holds.

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PROOF OF THEOREM 3. For the proof of Theorem 3 in [**1**], by the corrected formula (2.21), the formula (3.39) becomes

$$x = -\frac{1}{6}\left(B + r\frac{B_3^*}{B}\right) \pm \frac{1}{6}\sqrt{r^2\left(\frac{B_3^*}{B}\right)^2 + \frac{1}{2}rB_3^* + 10r^2B - \frac{121}{2}B^2}. \tag{3.39}$$

Since formula (3.42) and Lemma 3 in [**1**] hold, we obtain

$$0 \geq \left(-\frac{3}{B}x\left(\mu_l^{*2} - \frac{2}{3}B\right) + 3\mu_l^{*2} + \frac{1}{2}r\mu_l^* - B\right)\left(\mu_l^{*2} - \frac{1}{6}B\right), \tag{3.42}$$

and

$$x = -\frac{1}{6}\left(B + r\frac{B_3^*}{B}\right) - \frac{1}{6}\sqrt{r^2\left(\frac{B_3^*}{B}\right)^2 + \frac{1}{2}rB_3^* + 10r^2B - \frac{121}{2}B^2}. \tag{3.43}$$

Next we make necessary corrections of Lemma 4 as follows:

LEMMA 4. *If* $B_3^* \neq B^{3/2}/\sqrt{6}$, *then*

$$B < \frac{32}{121}\frac{r^2}{3}.$$

PROOF. By taking $l = 2$ in (3.42) and using the inequality $\mu_2^{*2} - B/6 < 0$ in (3.41), we have

$$0 \leq -\frac{3}{B}x\left(\mu_2^{*2} - \frac{2}{3}B\right) + 3\mu_2^{*2} + \frac{1}{2}r\mu_2^* - B. \tag{3.44}$$

First of all, we assert $B < 4r^2/27$. In fact, since B and r are constant by the assumption and $B < r^2/6$ as noted in Lemma 2, there exists a constant $a > 0$ such that $(1+a)B < r^2/6$. If $a \geq 1/8$, then $B < 4r^2/27$. In this case our assertion holds. If $a < 1/8$, then we have

$$x = -\frac{1}{6}\left(B + r\frac{B_3^*}{B}\right) - \frac{1}{6}\sqrt{\left((1-6a)B + r\frac{B_3^*}{B}\right)^2 + G}, \tag{3.45}$$

where G is defined by

$$G = -(1-6a)^2B^2 - 2(1-6a)rB_3^* + \frac{1}{2}rB_3^* + 10r^2B - \frac{121}{2}B^2.$$

According to $rB_3^* \leq -B^2$ in Lemma 2 and $(1+a)B < r^2/6$ we have $G > 0$. Therefore, we obtain $x \leq -aB$. By using this inequality, $\mu_2^{*2} - B/6 < 0$ in (3.41), $\mu_2^* < 0$ and (3.44), we can prove that it is impossible. Hence, we have

$$(1+6a)B < \frac{r^2}{6}.$$

If $6a \geq 1/8$, then our assertion is true. If $6a < 1/8$, then by applying the above procedure k times as in [**1**] so that $6^k a \geq 1/8$, then we have

$$(1 + 6^k a)B < \frac{r^2}{6}.$$

Thus we obtain $B < 4r^2/27$, that is, our assertion holds.

Next, by taking account of $B < 4r^2/27$, we will show $B < (32/121)(r^2/3)$. In fact, from (3.43), $rB_3^* \leq -B^2$ of (4) in Lemma 2 and $B < 4r^2/27$, we have $x < -(B/8)$. Hence, by substituting the above inequality into (3.44), from $0 < \mu_2^{*2} < B/6$ and $\mu_2^* < 0$, we infer

$$B < \frac{32}{121}\frac{r^2}{3}.$$

Thus the proof of Lemma 4 is completed. □

In view of the inequality $rB_3^* \leq -B^2$ and $r < 0$, we have $B_3^* > 0$. Thus, from Lemma 1 in [**1**], we have $0 < B_3^* = \sup B_3 < B^{3/2}/\sqrt{6}$. From Lemma 4, we have $r^2 > 11B$. Hence, we obtain, from $B_3^* < B^{3/2}/\sqrt{6}$ and $r^2 > 11B$,

$$\begin{aligned} &10r^2 B - \frac{121}{2}B^2 - r^2 B - \frac{2r^2 B_3^*}{\sqrt{B}} + \frac{rB_3^*}{2} \\ &\geq 9r^2 B - \frac{121}{2}B^2 - \frac{2r^2 B}{\sqrt{6}} - \frac{|r|B^{3/2}}{2\sqrt{6}} \\ &= \left(9 - \frac{2}{\sqrt{6}} - \frac{1}{4}\right)r^2 B + \left(\frac{|r|\sqrt{B}}{2} - \frac{B}{2\sqrt{6}}\right)^2 - \frac{121}{2}B^2 - \frac{B^2}{24} \\ &> 7 \times 11B^2 - 61B^2 > 0. \end{aligned}$$

Therefore, we have, from (3.43) and the above inequality,

$$\begin{aligned} x &< -\frac{1}{6}\left(B + r\frac{B_3^*}{B}\right) - \frac{1}{6}\sqrt{\left(r\sqrt{B} + r\frac{B_3^*}{B}\right)^2} \\ &= -\frac{1}{6}B + \frac{1}{6}r\sqrt{B}. \end{aligned}$$

Hence, by substituting the above inequality into (3.44), we infer, from $B > 0$, $r < 0$ and $0 < \mu_2^{*2} < B/6$,

$$
\begin{aligned}
0 &\leq -\frac{3}{B}\left(\mu_2^{*2} - \frac{2}{3}B\right)\left(-\frac{1}{6}B + \frac{1}{6}r\sqrt{B}\right) + 3\mu_2^{*2} + \frac{1}{2}r\mu_2^* - B \\
&= \left(\frac{7}{2} - \frac{r}{2\sqrt{B}}\right)\mu_2^{*2} + \frac{1}{2}r\mu_2^* - \frac{4}{3}B + \frac{1}{3}r\sqrt{B} \\
&< \left(\frac{7}{2} - \frac{r}{2\sqrt{B}}\right)\frac{B}{6} - \frac{1}{2}r\frac{\sqrt{B}}{\sqrt{6}} - \frac{4}{3}B + \frac{1}{3}r\sqrt{B} \\
&= -\frac{9}{12}B + \left(\frac{1}{3} - \frac{1}{12} - \frac{1}{2\sqrt{6}}\right)r\sqrt{B} < 0.
\end{aligned}
$$

But this is impossible. Thus the proof of Theorem 3 is completed. □

References

[1] Q.-M. Cheng, S. Ishikawa and K. Shiohama, Conformally flat 3-manifolds with constant scalar curvature, J. Math. Soc. Japan, **51** (1999), 209–226.

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