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Derived equivalence classification of symmetric algebras of domestic type

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Abstract. We give a complete derived equivalence classification of all symmetric algebras of domestic representation type over an algebraically closed field. This completes previous work by R. Bocian and the authors, where in this paper we solve the crucial problem of distinguishing standard and nonstandard algebras up to derived equivalence. Our main tool are generalized Reynolds ideals, introduced by B. Külshammer for symmetric algebras in positive characteristic, and recently shown by A. Zimmermann to be invariants under derived equivalences.

1. Introduction.

Throughout the paper K will denote a fixed algebraically closed field. By an algebra we mean a finite dimensional K-algebra. For an algebra A, we denote by $\operatorname{mod} A$ the category of finite dimensional right A-modules and by D the standard duality $\operatorname{Hom}_K(-,K)$ on $\operatorname{mod} A$. An algebra A is called *selfinjective* if $A \cong D(A)$ in $\operatorname{mod} A$, that is the projective A-modules are injective. Further, an algebra A is called *symmetric* if A and $D(A)$ are isomorphic as A-A-bimodules. Recall also that an algebra A is symmetric if and only if there exists an associative, symmetric, nondegenerate K-bilinear form $(-,-) : A \times A \to K$. The classical examples of selfinjective algebras (respectively, symmetric algebras) are provided by the finite dimensional Hopf algebras (respectively, the group algebras of finite groups). Moreover, for any algebra B, the trivial extension $\mathrm{T}(B) = B \ltimes D(B)$ of B by the B-B-bimodule $D(B)$ is a symmetric algebra, and B is a factor algebra of $\mathrm{T}(B)$. If A is a selfinjective algebra, then the left and the right socle of A concide, and we denote them by $\operatorname{soc}(A)$. Two selfinjective algebras A and Λ are said to be *socle equivalent* if the factor algebras $A/\operatorname{soc}(A)$ and $\Lambda/\operatorname{soc}(\Lambda)$ are isomorphic. For an algebra A, we denote by $D^b(\operatorname{mod} A)$ the derived category of bounded complexes from $\operatorname{mod} A$. Finally, two algebras A and Λ are said to be *derived equivalent* if the derived categories $D^b(\operatorname{mod} A)$ and $D^b(\operatorname{mod} \Lambda)$ are equivalent as triangulated categories.

Since Happel's work [**13**] interpreting tilting theory in terms of equivalences of derived categories, the machinery of derived categories has been of interest to representation theorists. In [**21**] J. Rickard proved his celebrated criterion: two algebras A and Λ are derived equivalent if and only if Λ is the endomorphism algebra of a tilting complex over A. Since a lot of interesting properties are preserved by derived equivalences, it is for many purposes important to classify classes of algebras up to derived equivalence, instead

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of Morita equivalence. For instance, for selfinjective algebras the representation type is an invariant of the derived category. Further, derived equivalent selfinjective algebras are stably equivalent [**22**], and hence have isomorphic stable Auslander-Reiten quivers. It has been also proved in [**23**] that the class of symmetric algebras is closed under derived equivalences. Finally, we note that derived equivalent algebras have the same number of pairwise nonisomorphic simple modules and isomorphic centers.

One central problem of modern representation theory is the determination of the derived equivalence classes of selfinjective algebras of tame representation type. Recall that for a tame algebra the indecomposable modules occur, in each dimension d, in a finite number of discrete and a finite number of one-parameter families. A distinguished class of tame algebras is formed by the representation-finite algebras for which there are only finitely many isomorphism classes of indecomposable modules. In [**22**] J. Rickard classified the Brauer tree algebras (for instance, representation-finite blocks of group algebras) up to derived equivalence in connection with Broué's conjecture [**12**]. The derived equivalence classification of all representation-finite selfinjective algebras has been established by H. Asashiba [**1**]. We refer also to [**16**] for the derived equivalence classification of algebras of the dihedral, semidihedral and quaternion type (for instance, representation-infinite tame blocks of group algebras), which are tame and symmetric.

In this paper, we are concerned with the problem of derived equivalence classification of all tame selfinjective algebras of domestic representation type. Recall that for algebras of domestic type there exists a common bound (independent of the fixed dimension) for the numbers of one-parameter families of indecomposable modules. The Morita equivalence classification of these algebras splits into two cases: the standard algebras, whose basic algebras admit simply connected Galois coverings, and the remaining nonstandard algebras. By general theory (see [**11**], [**17**], [**26**], [**27**]), the connected standard representation-finite (respectively, representation-infinite domestic) selfinjective algebras are Morita equivalent to the orbit algebras $\widehat{B}/G$ of the repetitive algebras $\widehat{B}$ of tilted algebras B of Dynkin (respectively, Euclidean) type with respect to actions of admissible infinite cyclic groups G of automorphisms of $\widehat{B}$. The nonstandard selfinjective algebras of domestic type are very exceptional and are Morita equivalent to socle and geometric deformations of the corresponding standard selfinjective algebras of domestic type (see [**10**], [**24**], [**27**], [**28**], [**30**]).

The aim of this paper is to give a complete derived equivalence classification of all connected representation-infinite symmetric algebras of domestic type. The Morita equivalence classification of these algebras has been established recently in [**8**], [**9**], [**10**], [**28**]. In Section 2 we define (by quivers and relations) the following families of representation-infinite domestic symmetric algebras:

$A(p,q)$, where $1 \leq p \leq q$,
$\Lambda(m)$, where $m \geq 2$,
$\Gamma(n)$, where $n \geq 1$,
$T(p,q)$, where $1 \leq p \leq q$,
$T(2,2,r)^*$, where $r \geq 2$,
$T(3,3,3)$, $T(2,4,4)$, and $T(2,3,6)$,
$\Omega(n)$, where $n \geq 1$ and $\operatorname{char} K = 2$.

The following theorem is the main result of the paper.

THEOREM 1.1. *The algebras $A(p,q)$, $\Lambda(m)$, $\Gamma(n)$, $T(p,q)$, $T(2,2,r)^*$, $T(3,3,3)$, $T(2,4,4)$, $T(2,3,6)$, and $\Omega(n)$ ($\operatorname{char} K = 2$) form a complete set of representatives of pairwise different derived equivalence classes of connected representation-infinite symmetric algebras of domestic type.*

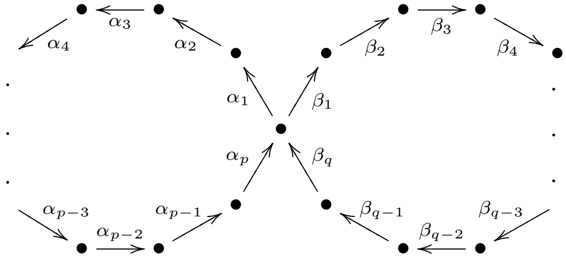
The derived equivalence classification of the standard (respectively, nonstandard) representation-infinite symmetric algebras of domestic type has been established in our joint papers with R. Bocian [**6**] (respectively, [**7**]). However, it remained open in these papers whether a standard and a nonstandard algebra can be derived equivalent, or not. (Recall that in general it is a notoriously difficult problem to distinguish algebras up to derived equivalence. The main problem is usually to find suitable derived invariants which are possible to compute.) In this paper we solve this problem, thereby completing the derived equivalence classification of symmetric algebras of domestic representation type. More precisely, we prove in Section 4 that the derived equivalence classes of the connected standard and nonstandard representation-infinite symmetric algebras of domestic type are disjoint. The crucial tool for proving this are the socalled generalized Reynolds ideals defined by B. Külshammer in [**19**] for symmetric algebras in positive characteristic. These sequences of ideals of the center of the algebra have recently been shown by A. Zimmermann to be invariant under derived equivalences [**32**]. This invariant is suitable for our purposes since the nonstandard symmetric algebras of domestic type occur only in characteristic 2. In the final Section 5 we present (for completeness) the derived equivalence classification of all representation-finite symmetric algebras from [**1**], and give an alternative proof of the important step in Asashiba's classification that the derived equivalence classes of the connected standard and nonstandard representation-finite symmetric algebras are disjoint. Our argument, using the above Reynolds ideals, considerably simplifies the original proof in [**1**].

For basic background on the representation theory applied here we refer to the books [**3**], [**5**], [**14**], [**18**] and to the survey articles [**27**], [**31**].

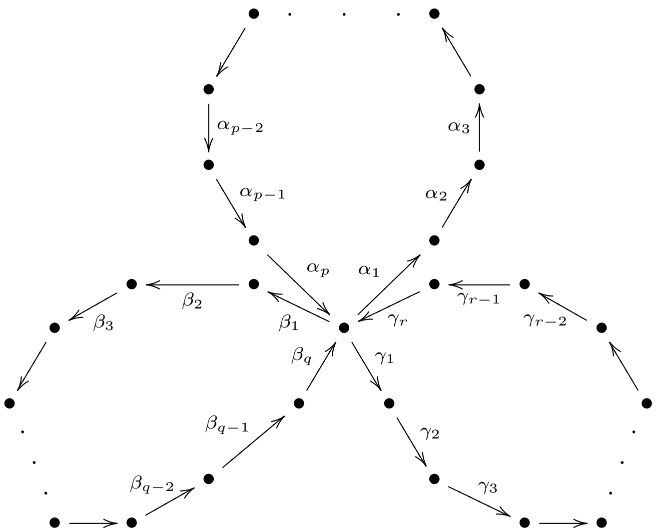
2. Derived normal forms of domestic symmetric algebras.

In order to define derived normal forms of the connected representation-infinite domestic symmetric algebras, consider the following families of quivers

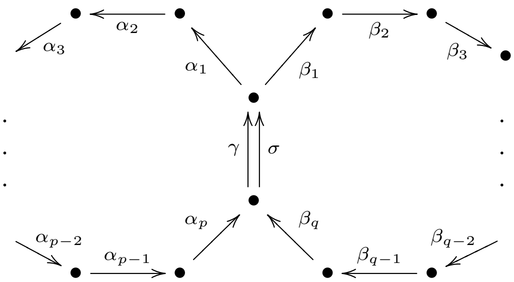
$\Delta(p,q)$:
$p, q \geq 1$



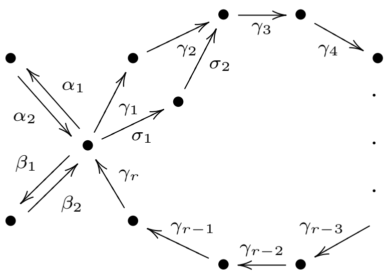
$\Delta(p,q,r)$:
$p,q,r \geq 1$



$\Sigma(p,q)$:
$p,q \geq 1$



$\Theta(r)$:
$r \geq 2$



The algebras $A(p,q)$. For $1 \leq p \leq q$, denote by $A(p,q)$ the algebra given by the quiver $\Delta(p,q)$ and the relations:

$$\alpha_1\alpha_2\dots\alpha_p\beta_1\beta_2\dots\beta_q = \beta_1\beta_2\dots\beta_q\alpha_1\alpha_2\dots\alpha_p,$$

$$\alpha_p\alpha_1 = 0, \quad \beta_q\beta_1 = 0,$$

$$\alpha_i\alpha_{i+1}\dots\alpha_p\beta_1\dots\beta_q\alpha_1\dots\alpha_{i-1}\alpha_i = 0, \ 2 \leq i \leq p-1,$$

$$\beta_j\beta_{j+1}\dots\beta_q\alpha_1\dots\alpha_p\beta_1\dots\beta_{j-1}\beta_j = 0, \ 2 \leq j \leq q-1.$$

We note that $A(p,q)$ is a standard one-parametric symmetric algebra of Euclidean type $\widetilde{\boldsymbol{A}}_{2(p+q)-3}$ (see [**6**, (5.3)(1)]).

THE ALGEBRAS $\Lambda(m)$. For $m \geq 2$, denote by $\Lambda(m)$ the algebra given by the quiver $\Delta(1,m)$ and the relations:

$$\alpha_1^2 = (\beta_1\beta_2\dots\beta_m)^2, \quad \alpha_1\beta_1 = 0, \quad \beta_m\alpha_1 = 0,$$
$$\beta_j\beta_{j+1}\dots\beta_m\beta_1\dots\beta_m\beta_1\dots\beta_{j-1}\beta_j = 0, \ 2 \leq j \leq m-1.$$

We note that $\Lambda(m)$ is a standard one-parametric symmetric algebra of Euclidean type $\widetilde{\boldsymbol{A}}_{2m-1}$ (see [**6**, (5.3)(2)]).

THE ALGEBRAS $\Gamma(n)$. For $n \geq 1$, denote by $\Gamma(n)$ the algebra given by the quiver $\Delta(2,2,n)$ and the relations:

$$\alpha_1\alpha_2 = (\gamma_1\gamma_2\dots\gamma_n)^2 = \beta_1\beta_2,$$
$$\alpha_2\gamma_1 = 0, \quad \beta_2\gamma_1 = 0, \quad \gamma_n\alpha_1 = 0,$$
$$\gamma_n\beta_1 = 0, \quad \alpha_2\beta_1 = 0, \quad \beta_2\alpha_1 = 0,$$
$$\gamma_j\gamma_{j+1}\dots\gamma_n\gamma_1\dots\gamma_n\gamma_1\dots\gamma_{j-1}\gamma_j = 0, \ 2 \leq j \leq n-1.$$

We note that $\Gamma(n)$ is a standard one-parametric symmetric algebra of Euclidean type $\widetilde{\boldsymbol{D}}_{2n+3}$ (see [**6**, (5.3)(3)]).

THE ALGEBRAS $T(p,q,r)$. For $2 \leq p \leq q \leq r$, denote by $T(p,q,r)$ the algebra given by the quiver $\Delta(p,q,r)$ and the relations:

$$\alpha_1\alpha_2\dots\alpha_p = \beta_1\beta_2\dots\beta_q = \gamma_1\gamma_2\dots\gamma_r,$$
$$\beta_q\alpha_1 = 0, \quad \gamma_r\alpha_1 = 0, \quad \alpha_p\beta_1 = 0,$$
$$\gamma_r\beta_1 = 0, \quad \alpha_p\gamma_1 = 0, \quad \beta_q\gamma_1 = 0,$$
$$\alpha_i\alpha_{i+1}\dots\alpha_p\alpha_1\dots\alpha_{i-1}\alpha_i = 0, \ 2 \leq i \leq p-1,$$
$$\beta_j\beta_{j+1}\dots\beta_q\beta_1\dots\beta_{j-1}\beta_j = 0, \ 2 \leq j \leq q-1,$$
$$\gamma_k\gamma_{k+1}\dots\gamma_r\gamma_1\dots\gamma_{k-1}\gamma_k = 0, \ 2 \leq k \leq r-1.$$

Observe that $T(p,q,r)$ is isomorphic to the trivial extension algebra $\mathrm{T}(H(p,q,r))$ of the path algebra $H(p,q,r)$ of the quiver $\Delta^*(p,q,r)$ obtained from $\Delta(p,q,r)$ by deleting the arrows α_1, β_1, γ_1. Further, $1/p+1/q+1/r > 1$ if and only if $(p,q,r) = (2,2,r)$, $(2,3,3)$, $(2,3,4)$, $(2,3,5)$, or equivalently $\Delta^*(p,q,r)$ is a Dynkin quiver of type $\boldsymbol{D}_{r+2}$, $\boldsymbol{E}_6$, $\boldsymbol{E}_7$, or $\boldsymbol{E}_8$, respectively. In this case, $T(p,q,r)$ is a standard representation-finite symmetric algebra (see [**17**]). Similarly, $1/p+1/q+1/r = 1$ if and only if $(p,q,r) = (3,3,3)$, $(2,4,4)$, $(2,3,6)$, or equivalently $\Delta^*(p,q,r)$ is an Euclidean quiver of type $\widetilde{\boldsymbol{E}}_6$, $\widetilde{\boldsymbol{E}}_7$, or $\widetilde{\boldsymbol{E}}_8$, respectively. In this case, $T(p,q,r)$ is a standard 2-parametric symmetric algebra (see [**2**]).

THE ALGEBRAS $T(p,q)$. For $1 \le p \le q$, denote by $T(p,q)$ the algebra given by the quiver $\Sigma(p,q)$ and the relations:

$$\alpha_1\alpha_2\dots\alpha_p\gamma = \beta_1\beta_2\dots\beta_q\sigma,$$
$$\gamma\alpha_1\alpha_2\dots\alpha_p = \sigma\beta_1\beta_2\dots\beta_q,$$
$$\alpha_p\sigma = 0, \quad \sigma\alpha_1 = 0, \quad \beta_q\gamma = 0, \quad \gamma\beta_1 = 0,$$
$$\alpha_i\alpha_{i+1}\dots\alpha_p\gamma\alpha_1\dots\alpha_{i-1}\alpha_i = 0, \ 2 \le i \le p-1,$$
$$\beta_j\beta_{j+1}\dots\beta_q\sigma\beta_1\dots\beta_{j-1}\beta_j = 0, \ 2 \le j \le q-1.$$

Then $T(p,q)$ is isomorphic to the trivial extension algebra $\mathrm{T}(H(p,q))$ of the path algebra $H(p,q)$ of the quiver $\Sigma^*(p,q)$ of Euclidean type $\widetilde{\boldsymbol{A}}_{p+q-1}$ obtained from $\Sigma(p,q)$ by deleting the arrows γ and σ. In particular, $T(p,q)$ is a standard 2-parametric symmetric algebra (see [**4**]).

THE ALGEBRAS $T(2,2,r)^*$. For $r \ge 2$, denote by $T(2,2,r)^*$ the algebra given by the quiver $\Theta(r)$ and the relations:

$$\alpha_1\alpha_2 = \beta_1\beta_2 = \gamma_1\gamma_2\dots\gamma_r, \quad \gamma_1\gamma_2 = \sigma_1\sigma_2,$$
$$\gamma_r\alpha_1 = 0, \quad \beta_2\alpha_1 = 0, \quad \gamma_r\beta_1 = 0, \quad \alpha_2\beta_1 = 0,$$
$$\alpha_2\gamma_1 = 0, \quad \alpha_2\sigma_1 = 0, \quad \beta_2\gamma_1 = 0, \quad \beta_2\sigma_1 = 0,$$
$$\alpha_2\alpha_1\alpha_2 = 0, \quad \beta_2\beta_1\beta_2 = 0,$$
$$\gamma_2\gamma_3\dots\gamma_r\sigma_1 = 0, \quad \sigma_2\gamma_3\dots\gamma_r\gamma_1 = 0,$$
$$\gamma_k\gamma_{k+1}\dots\gamma_r\gamma_1\gamma_2\dots\gamma_{k-1}\gamma_k = 0, \ 3 \le k \le r-1.$$

Then $T(2,2,r)^*$ is isomorphic to the trivial extension algebra $\mathrm{T}(H(2,2,r)^*)$ of the path algebra $H(2,2,r)^*$ of the quiver $\Theta^*(r)$ of Euclidean type $\widetilde{\boldsymbol{D}}_{r+2}$ obtained from $\Theta(r)$ by deleting the arrows α_1, β_1, γ_1 and σ_1. In particular, $T(2,2,r)^*$ is a standard 2-parametric symmetric algebra (see [**2**]).

THE ALGEBRAS $\Omega(n)$. For $n \ge 1$, denote by $\Omega(n)$ the algebra given by the quiver $\Delta(1,n)$ and the relations:

$$\alpha_1\beta_1\beta_2\dots\beta_n + \beta_1\beta_2\dots\beta_n\alpha_1 = 0,$$
$$\alpha_1^2 = \alpha_1\beta_1\beta_2\dots\beta_n, \quad \beta_n\beta_1 = 0,$$
$$\beta_j\beta_{j+1}\dots\beta_n\alpha_1\beta_1\beta_2\dots\beta_{j-1}\beta_j = 0, \ 2 \le j \le n-1.$$

Then $\Omega(n)$ is a nonstandard one-parametric selfinjective algebra. Furthermore, $\Omega(n)$ is a symmetric algebra if and only if $\operatorname{char} K = 2$. Moreover, in this symmetric case, $\Omega(n)$ is socle equivalent to the algebra $\Omega(n)' = A(1,n)$, called the standard form of $\Omega(n)$ (see [**10**]).

The following derived equivalence classifications of standard symmetric algebras of domestic type has been established in [**6**, Theorems 1 and 2].

Theorem 2.1. *The algebras $A(p,q)$, $\Lambda(m)$, $\Gamma(n)$, $T(p,q)$, $T(2,2,r)^*$, $T(3,3,3)$, $T(2,4,4)$, $T(2,3,6)$ form a complete set of representatives of pairwise different derived equivalence classes of connected, standard, representation-infinite symmetric algebras of domestic type.*

It has been proved in [**28**] (see also [**10**]) that the nonstandard representation-infinite symmetric algebras of domestic type occur only in characteristic 2. Furthermore, the following theorem, proved in [**7**, Theorem 1], gives the derived equivalence classification of these algebras.

Theorem 2.2. *The algebras $\Omega(n)$, for $n \geq 1$ and $\operatorname{char} K = 2$, form a complete set of representatives of pairwise different derived equivalence classes of connected, nonstandard, representation-infinite symmetric algebras of domestic type.*

3. Generalized Reynolds ideals.

In this section we briefly recall the definition of the sequence of generalized Reynolds ideals. For more details on this invariant we refer to [**19**], [**20**], [**15**], [**32**].

Let K be an algebraically closed field of characteristic $p > 0$. Let A be a finite dimensional symmetric K-algebra with associative, symmetric, nondegenerate K-bilinear form $(-,-) : A \times A \to K$. For a K-subspace M of A, denote by $M^\perp$ the orthogonal complement of M inside A with respect to the form $(-,-)$. Moreover, let $K(A)$ be the K-subspace of A generated by all commutators $[a,b] := ab - ba$, for any $a, b \in A$. Then for any $n \geq 0$ set

$$T_n(A) = \{x \in A \mid x^{p^n} \in K(A)\}.$$

Then, by [**19**], the orthogonal complements $T_n(A)^\perp, n \geq 0$, are ideals of the center $Z(A)$ of A, called generalized Reynolds ideals. They form a descending sequence

$$Z(A) = T_0(A)^\perp \supseteq T_1(A)^\perp \supseteq T_2(A)^\perp \supseteq T_3(A)^\perp \supseteq \dots$$

In fact, B. Külshammer proved in [**20**] that the equation $(\xi_n(z), x)^{p^n} = (z, x^{p^n})$ for any $x, z \in Z(A)$ defines a mapping $\xi_n : Z(A) \to Z(A)$ such that $\xi_n(A) = T_n(A)^\perp$.

Then we have the following theorem proved recently by A. Zimmermann [**32**].

Proposition 3.1. *Let A and B be derived equivalent symmetric algebras over an algebraically closed field of positive characteristic p. Then there is an isomorphism $\varphi : Z(A) \to Z(B)$ of the centers of A and B such that $\varphi(T_n(A)^\perp) = T_n(B)^\perp$ for all positive integers n.*

Proof. See [**32**, Theorem 1]. □

Hence the sequence of generalized Reynolds ideals gives a new derived invariant, for symmetric algebras over algebraically closed fields of positive characteristic.

In the next section we shall use this invariant for proving our main result. The algebras occurring in our context are all given by a quiver with relations (bound quiver) and a basis of the algebra is provided by the set of all pairwise distinct (modulo the ideal generated by the imposed relations) nonzero paths of the quiver.

The following simple observation will turn out to be useful.

PROPOSITION 3.2. *Let $A = KQ/I$ be a symmetric bound quiver algebra, and assume that a K-basis $\mathscr{B}$ of A is given by the pairwise distinct nonzero paths of the quiver Q (modulo the ideal I). Then the following statements hold:*

(1) *An associative nondegenerate symmetric K-bilinear form $(-,-)$ for A is given as follows*

$$(x,y) = \begin{cases} 1 & \text{if } xy \in \operatorname{soc}(A) \setminus \{0\} \\ 0 & \text{otherwise} \end{cases}$$

for $x, y \in \mathscr{B}$.

(2) *For any $n \geq 0$, the socle $\operatorname{soc}(A)$ is contained in the generalized Reynolds ideal $T_n(A)^{\perp}$.*

PROOF. (1) It is well-known (see [**31**, Section 2]) that an algebra A is symmetric if and only if there is a K-linear form $\psi : A \to K$ such that $\psi(ab) = \psi(ba)$ for all elements $a, b \in A$ and the kernel of ψ contains no nonzero left or right ideal of A. Moreover, for such a (symmetrizing) form $\psi : A \to K$, the K-bilinear form $(-,-) : A \times A \to K$ given by $(a,b) = \psi(ab)$ for all $a, b \in A$ is an associative, symmetric, nondegenerate form. For the symmetric algebra $A = KQ/I$ considered in the proposition, we may take the symmetrizing form $\varphi : A \to K$ which assigns 1 to any nonzero residue class of a path in Q in $A = KQ/I$ from the socle $\operatorname{soc}(A)$, and 0 to the residue classes of the remaining paths of Q. Then the bilinear form $(-,-)$ associated to φ satisfies the required statement (1).

(2) By [**15**] we have for any symmetric algebra A that

$$\bigcap_{n=0}^{\infty} T_n(A)^{\perp} = \operatorname{soc}(A) \cap Z(A).$$

But for the algebras described in the proposition we always have $\operatorname{soc}(A) \subseteq Z(A)$. □

4. Proof of the main result.

The aim of this section is to give the proof of Theorem 1.1, applying Theorems 2.1 and 2.2. We need some general facts.

For a selfinjective algebra A, we denote by Γ_A^s the *stable Auslander-Reiten quiver* of A, obtained from the Auslander-Reiten quiver Γ_A of A by removing the projective-injective vertices and the arrows attached to them. Recall that two selfinjective algebras A and B are called *stably equivalent* if their stable module categories $\underline{\operatorname{mod}}\, A$ and $\underline{\operatorname{mod}}\, B$ are equivalent.

PROPOSITION 4.1. *Let A and B be stably equivalent connected selfinjective algebras of Loewy length at least* 3. *Then the stable Auslander-Reiten quivers Γ_A^s and Γ_B^s are isomorphic.*

PROOF. See [**5**, Corollary X.1.9]. □

PROPOSITION 4.2. *Let A and Λ be derived equivalent selfinjective algebras. Then A and Λ are stably equivalent.*

PROOF. See [**22**, Corollary 2.2]. □

We know from Theorem 2.1 (respectively, Theorem 2.2) that the algebras $A(p,q)$, $\Lambda(m)$, $\Gamma(n)$, $T(p,q)$, $T(2,2,r)^*$, $T(3,3,3)$, $T(2,4,4)$, $T(2,3,6)$ (respectively, $\Omega(n)$, for $\operatorname{char} K = 2$) form a complete set of representatives of pairwise different derived equivalence classes of connected standard (respectively, nonstandard) representation-infinite symmetric algebras of domestic type. Moreover, these algebras are basic, connected and of Loewy length at least 3. It also follows from Propositions 4.1 and 4.2 that the stable Auslander-Reiten quivers of two derived equivalent connected selfinjective algebras of Loewy length at least 3 are isomorphic. In order to distinguish the derived equivalence classes of the standard and nonstandard representation-infinite symmetric algebras of domestic type, we need the shapes of the stable Auslander-Reiten quivers of algebras occuring in Theorems 2.1 and 2.2.

PROPOSITION 4.3. *The following statements hold*:

(1) $\Gamma^s_{A(p,q)}$ *consists of an Euclidean component of type* $\boldsymbol{Z}\widetilde{\boldsymbol{A}}_{2(p+q)-3}$ *and a* $\boldsymbol{P}_1(K)$-*family of stable tubes of tubular type* $(2p-1, 2q-1)$.
(2) $\Gamma^s_{\Lambda(n)}$ *consists of an Euclidean component of type* $\boldsymbol{Z}\widetilde{\boldsymbol{A}}_{2n-1}$ *and a* $\boldsymbol{P}_1(K)$-*family of stable tubes of tubular type* (n,n).
(3) $\Gamma^s_{\Gamma(n)}$ *consists of an Euclidean component of type* $\boldsymbol{Z}\widetilde{\boldsymbol{D}}_{2n+3}$ *and a* $\boldsymbol{P}_1(K)$-*family of stable tubes of tubular type* $(2,2,2n+1)$.

PROOF. See [**6**, Proposition 5.3]. □

PROPOSITION 4.4. *The following statements hold*:

(1) $\Gamma^s_{T(p,q)}$ *consists of two Euclidean components of type* $\boldsymbol{Z}\widetilde{\boldsymbol{A}}_{p+q-1}$ *and two* $\boldsymbol{P}_1(K)$-*families of stable tubes of tubular type* (p,q).
(2) $\Gamma^s_{T(2,2,r)^*}$ *consists of two Euclidean components of type* $\boldsymbol{Z}\widetilde{\boldsymbol{D}}_{r+1}$ *and two* $\boldsymbol{P}_1(K)$-*families of stable tubes of tubular type* $(2,2,r+2)$.
(3) $\Gamma^s_{T(3,3,3)}$ *consists of two Euclidean components of type* $\boldsymbol{Z}\widetilde{\boldsymbol{E}}_6$ *and two* $\boldsymbol{P}_1(K)$-*families of stable tubes of tubular type* $(2,3,3)$.
(4) $\Gamma^s_{T(2,4,4)}$ *consists of two Euclidean components of type* $\boldsymbol{Z}\widetilde{\boldsymbol{E}}_7$ *and two* $\boldsymbol{P}_1(K)$-*families of stable tubes of tubular type* $(2,3,4)$.
(5) $\Gamma^s_{T(2,3,6)}$ *consists of two Euclidean components of type* $\boldsymbol{Z}\widetilde{\boldsymbol{E}}_8$ *and two* $\boldsymbol{P}_1(K)$-*families of stable tubes of tubular type* $(2,3,5)$.

PROOF. We know that $T(p,q)$, $T(2,2,r)^*$, $T(3,3,3)$, $T(2,4,4)$, $T(2,3,6)$ are the trivial extension algebras of the hereditary algebras $H(p,q)$, $H(2,2,r)^*$, $H(3,3,3)$, $H(2,4,4)$, $H(2,3,6)$ of Euclidean types $\widetilde{\boldsymbol{A}}_{p+q-1}$, $\widetilde{\boldsymbol{D}}_{r+1}$, $\widetilde{\boldsymbol{E}}_6$, $\widetilde{\boldsymbol{E}}_7$, $\widetilde{\boldsymbol{E}}_8$, respectively. Then the required statements follow from the structure of the Auslander-Reiten quivers of the hereditary algebras of Euclidean types (see [**25**, (3.6)]) and the description of the Auslander-Reiten quivers of the trivial extensions of the hereditary algebras given in [**29**, Theorem 3.4], [**31**, Theorem 2.5.2] (see also [**4**], [**2**]). □

Proposition 4.5. *The stable Auslander-Reiten quiver* $\Gamma^s_{\Omega(n)}$ *of* $\Omega(n)$ *consists of an Euclidean component of type* $\boldsymbol{Z}\widetilde{\boldsymbol{A}}_{2n-1}$ *and a* $\boldsymbol{P}_1(K)$*-family of stable tubes of tubular type* $(2n-1)$.

Proof. See [**7**, Proposition 2.2]. □

As a direct consequence of the above three propositions we obtain the following fact.

Proposition 4.6. *Let A be an algebra of one of the forms $A(p,q)$, $\Lambda(n)$, $\Gamma(n)$, $T(p,q)$, $T(2,2,r)^*$, $T(3,3,3)$, $T(2,4,4)$, $T(2,3,6)$, and let B be an algebra of the form $\Omega(n)$. Assume that the stable Auslander-Reiten quivers of A and B are isomorphic. Then $A = A(1,n)$ and $B = \Omega(n)$ for some $n \geq 1$.*

Therefore the following proposition completes the proof of Theorem 1.1.

Proposition 4.7. *Let K be an algebraically closed field of characteristic 2. Then, for any $n \geq 1$, the symmetric algebras $\Omega(n)$ and $\Omega(n)' = A(1,n)$ are not derived equivalent.*

Proof. Let us denote by Ω either of the algebras $\Omega(n)$ or $\Omega(n)'$.

We shall compute the series of generalized Reynolds ideals for the symmetric algebras Ω,

$$Z(\Omega) \supseteq T_1(\Omega)^\perp \supseteq T_2(\Omega)^\perp \supseteq \dots$$

as described in Section 3.

We shall show that the ideals in these series have different dimensions for the algebras $\Omega(n)$ and $\Omega(n)' = A(1,n)$. Since the series of Reynolds ideals is a derived invariant (see Proposition 3.1), we can then distinguish these algebras up to derived equivalence.

The centers of these algebras have dimension $n+2$ as vector space over K. More precisely, it is straightforward to check that a K-basis is given as follows

$$Z(\Omega) = \big\langle 1,\ \beta_1 \dots \beta_n,\ s_1 := \alpha\beta_1 \dots \beta_n,\ s_j := \beta_j \dots \beta_n \alpha \beta_1 \dots \beta_{j-1}\ (2 \leq j \leq n) \big\rangle_K$$

where we abbreviate $\alpha = \alpha_1$. Note that the latter n elements $s_1, \dots, s_n$ form a basis of the socle of Ω.

Since we are dealing with characteristic 2, we have

$$T_1(\Omega) := \big\{ x \in \Omega \mid x^2 \in K(\Omega) \big\}.$$

Recall that $K(\Omega)$ is the subspace of the algebra Ω generated by all commutators $[x, y] = xy - yx$, where $x, y \in \Omega$. Now consider the first generalized Reynolds ideal

$$T_1(\Omega)^{\perp} := \{y \in Z(\Omega) \mid (x, y) = 0 \text{ for all } x \in T_1(\Omega)\},$$

where $(-,-)$ is the nondegenerate symmetric K-bilinear form for the symmetric algebra Ω, as defined in Proposition 3.2. Note that for such a basic symmetric algebra A, the socle is contained in any Reynolds ideal $T_m(A)^{\perp}$ (see Proposition 3.2(2)).

We consider the following sequence of ideals

$$\operatorname{soc}(\Omega) \subseteq T_1(\Omega)^{\perp} \subset Z(\Omega).$$

Here, the second inclusion is strict, since 1 is not contained in any Reynolds ideal of Ω. In fact, $\operatorname{soc}(\Omega) \subseteq T_1(\Omega)$, and $(1, s) = 1$ for every $s \in \operatorname{soc}(\Omega)$.

On the other hand, the socle of Ω has only codimension 2 in the center $Z(\Omega)$, leaving us with $\beta_1 \dots \beta_n$ as the crucial basis element to check.

But the element $\beta_1 \dots \beta_n$ is easily seen to be orthogonal to all basis elements in the ideal generated by the arrows of the quiver, except to α. In fact, $(\alpha, \beta_1 \dots \beta_n) = 1$ since $\alpha\beta_1 \dots \beta_n$ belongs to the socle of Ω.

Now we have to consider the algebras $\Omega(n)$ and $\Omega(n)'$ separately. Note that the distinction in the relations is that $\alpha^2 = \alpha\beta_1 \dots \beta_n$ is nonzero in $\Omega(n)$, whereas $\alpha^2 = 0$ in $\Omega(n)'$.

For $\Omega(n)$, the crucial fact to observe is that $\alpha \notin T_1(\Omega(n))$. In fact, $\alpha^2 = \alpha\beta_1 \dots \beta_n$ is nonzero, and it can be checked that it can not be written as a linear combination of commutators. But this implies that $\beta_1 \dots \beta_n \in T_1(\Omega(n))^{\perp}$. So we get the following series of ideals and their codimensions:

$$\operatorname{soc}(\Omega(n)) \underbrace{\subset}_{1} T_1(\Omega(n))^{\perp} = \langle \beta_1 \dots \beta_n, \operatorname{soc}(\Omega(n)) \rangle_K \underbrace{\subset}_{1} Z(\Omega(n)).$$

On the other hand, for $\Omega(n)'$, we have $\alpha \in T_1(\Omega(n)')$, since $\alpha^2 = 0$. Since $\beta_1 \dots \beta_n$ is not orthogonal to α, we conclude that $\beta_1 \dots \beta_n \notin T_1(\Omega(n)')^{\perp}$. Hence the corresponding series of ideals for $\Omega(n)'$ takes the form

$$\operatorname{soc}(\Omega(n)') \underbrace{=}_{0} T_1(\Omega(n)')^{\perp} \underbrace{\subset}_{2} Z(\Omega(n)').$$

Since the series of generalized Reynolds ideals, and in particular the codimensions occurring, is invariant under derived equivalences, we can now conclude that the nonstandard algebra $\Omega(n)$ and the standard algebra $\Omega(n)'$ are not derived equivalent. □

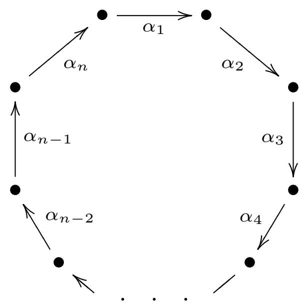
5. Derived normal forms of representation-finite symmetric algebras.

In [**1**, Theorem 2.2] H. Asashiba proved that the derived equivalence classes of connected representation-finite standard (respectively, nonstandard) selfinjective algebras are determined by the combinatorial data called the types, and the derived equivalence classes of the standard and nonstandard representation-finite selfinjective algebras are disjoint. Furthermore, by the results of C. Riedtmann [**24**, Proposition 5.7(b)] and J. Waschbüsch [**30**], the nonstandard representation-finite selfinjective algebras are symmetric, are given by some Brauer quivers, and occur only in characteristic 2 (see also [**28**, (3.6)–(3.8)]).

The aim of this section is to present the derived equivalence classification of all connected representation-finite symmetric algebras by quivers and relations.

For $m, n \geq 1$, denote by N_n^{mn} the algebra given by the quiver

$\Delta(n)$



and the relations:

$$\big(\alpha_i\alpha_{i+1}\dots\alpha_n\alpha_1\dots\alpha_{i-1}\big)^m\alpha_i = 0,\ 1 \leq i \leq n.$$

It is well-known that these algebras form a complete set of representatives of the Morita equivalence classes of the symmetric Nakayama algebras. In [**22**, Theorem 4.2] J. Rickard proved that they form a complete set of representatives of the derived equivalence classes of the Brauer tree algebras (which occur in the representation theory of representation-finite blocks of group algebras). Finally, we note also that the algebra N_n^n is the trivial extension algebra $\mathrm{T}(H(n))$ of the path algebra $H(n)$ of the quiver $\Delta(n)^*$ obtained from $\Delta(n)$ by deleting the arrow α_n.

For $m \geq 2$, denote by $D(m)'$ the algebra given by the quiver $\Delta(1, m)$ and the relations:

$$\alpha_1^2 = \beta_1\beta_2\dots\beta_m, \quad \beta_m\beta_1 = 0,$$
$$\beta_i\beta_{i+1}\dots\beta_m\alpha_1\beta_1\dots\beta_{i-1}\beta_i = 0,\ 2 \leq i \leq m-1.$$

Then $D(m)'$ is a standard representation-finite symmetric algebra of Dynkin type $\boldsymbol{D}_{3m}$ (see [**24**]).

In Section 2 we defined also the trivial extension algebras $T(2,2,r)$, $r \geq 2$, $T(2,3,3)$, $T(2,3,4)$, $T(2,3,5)$ of the hereditary algebras $H(2,2,r)$, $r \geq 2$, $H(2,3,3)$, $H(2,3,4)$, $H(2,3,5)$ of Dynkin types $\boldsymbol{D}_{r+2}$, $\boldsymbol{E}_6$, $\boldsymbol{E}_7$, $\boldsymbol{E}_8$, respectively. In [**17**, Theorems 3.1 and 3.7] D. Hughes and J. Waschbüsch proved that the trivial extension $\mathrm{T}(B)$ of a connected algebra B is representation-finite if and only if B is a tilted algebra of Dynkin type. Moreover, in [**22**, Theorem 3.1] J. Rickard proved that if A and B are derived equivalent algebras then their trivial extensions $\mathrm{T}(A)$ and $\mathrm{T}(B)$ are also derived equivalent. Therefore, the trivial extension algebras $N_n^n = T(H(n))$, $n \geq 1$, $T(2,2,r)$, $r \geq 2$, $T(2,3,3)$, $T(2,3,4)$, $T(2,3,5)$ form a complete set of representatives of pairwise different derived equivalence classes of the connected representation-finite trivial extension algebras $\mathrm{T}(B)$. We also note that by [**11**, Section 1] all connected representation-finite symmetric algebras of Dynkin types $\boldsymbol{E}_6$, $\boldsymbol{E}_7$, $\boldsymbol{E}_8$ are actually the trivial extension algebras of tilted algebras of Dynkin types $\boldsymbol{E}_6$, $\boldsymbol{E}_7$, $\boldsymbol{E}_8$.

In [**24**] C. Riedtmann proved that the Morita equivalence classes of the connected standard representation-finite symmetric algebras of Dynkin type $\boldsymbol{D}_n$, which are not trivial extension algebras, are given by some (looped) Brauer trees (see also [**28**, Theorem 3.11]). Then applying Rickard's constructions from [**22**, Section 4] one easily proves that the algebras $D(m)'$, $m \geq 2$, give a complete set of representatives of the derived equivalence classes of these symmetric algebras.

Applying the combinatorial descriptions of the stable Auslander-Reiten quivers of representation-finite selfinjective algebras (see [**1**, Section 2], [**11**, Section 1]) one easily shows that the stable Auslander-Reiten quivers of the standard symmetric algebras N_n^{mn}, $m, n \geq 1$, $D(m)'$, $m \geq 2$, $T(2,2,r)$, $r \geq 2$, $T(2,3,3)$, $T(2,3,4)$, $T(2,3,5)$ are pairwise nonisomorphic. Therefore, summing up the above discussion and invoking Propositions 4.1 and 4.2, we obtain the following result.

PROPOSITION 5.1. *The algebras K, N_n^{mn}, $m, n \geq 1$, $D(m)'$, $m \geq 2$, $T(2,2,r)$, $r \geq 2$, $T(2,3,3)$, $T(2,3,4)$, $T(2,3,5)$ form a complete set of representatives of pairwise different derived equivalence classes of connected, standard, representation-finite symmetric algebras.*

For $m \geq 2$, denote by $D(m)$ the algebra given by the quiver $\Delta(1,m)$ and the relations:

$$\alpha_1^2 = \beta_1\beta_2\ldots\beta_m, \quad \beta_m\beta_1 = \beta_m\alpha_1\beta_1,$$
$$\beta_i\beta_{i+1}\ldots\beta_m\alpha_1\beta_1\ldots\beta_{i-1}\beta_i = 0, \ 1 \leq i \leq m.$$

If $\operatorname{char} K \neq 2$, then $D(m)$ is a standard representation-finite symmetric algebra of Dynkin type $\boldsymbol{D}_{3m}$ and $D(m) \cong D(m)'$. On the other hand, if $\operatorname{char} K = 2$, then $D(m)$ is nonstandard (see [**24**, (5.7)] or [**30**]). Further, $D(m)$ and $D(m)'$ are socle equivalent, and $D(m)'$ is called the standard form of $D(m)$.

Applying Rickard's method from [**22**, Section 4], H. Asashiba proved in [**1**, Section 7] the following proposition.

PROPOSITION 5.2. *The algebras $D(m)$, $m \geq 2$, for* $\operatorname{char} K = 2$, *form a complete set of representatives of pairwise different derived equivalence classes of connected nonstandard representation-finite symmetric algebras.*

Applying again the combinatorial descriptions of the stable Auslander-Reiten quivers of representation-finite selfinjective algebras one easily deduces the following fact.

PROPOSITION 5.3. *Let A be an algebra of one of the forms K, N_n^{mn}, $D(m)'$, $T(2,2,r)$, $T(2,3,3)$, $T(2,3,4)$, $T(2,3,5)$, and B an algebra of the form $D(m)$. Assume* *that the stable Auslander-Reiten quivers of* A *and* B *are isomorphic. Then* $A = D(m)'$ *and* $B = D(m)$ *for some* $m \geq 2$.

In [**1**, Section 3] H. Asashiba proved that, if $\operatorname{char} K = 2$, then the algebras $D(m)$ and $D(m)'$ are not stably (hence not derived) equivalent. We will give an alternative, simplified, proof of the latter fact, applying Proposition 3.1.

Proposition 5.4. *Let* K *be an algebraically closed field of characteristic* 2. *Then, for any* $m \geq 2$, *the symmetric algebras* $D(m)$ *and* $D(m)'$ *are not derived equivalent.*

Proof. For a fixed $m \geq 2$, we denote by D either of the algebras $D(m)$ or $D(m)'$. Note that the socle $\operatorname{soc}(D)$ of the algebra D has as K-basis the following m elements

$$s_1 := \alpha\beta_1 \dots \beta_m = \alpha^3 = \beta_1 \dots \beta_m\alpha,$$
$$s_j := \beta_j \dots \beta_m\alpha\beta_1 \dots \beta_{j-1}, \quad \text{for } j = 2, \dots, m,$$

where we abbreviate $\alpha = \alpha_1$.

Then it is straightfoward to verify that the center of the algebra D is as K-vector space generated by the following basis

$$Z(D) = \langle 1, \beta_1 \dots \beta_m, s_1, s_2, \dots, s_m \rangle_K.$$

We shall study the series of ideals of the center

$$\operatorname{soc}(D) \subseteq T_1(D)^{\perp} \subseteq Z(D).$$

Since the socle only has codimension 2 in the center, and since the unit 1 can not be contained in the Reynolds ideal $T_1(D)^{\perp}$, the crucial question is whether $\beta_1 \dots \beta_m$ is contained in $T_1(D)^{\perp}$, or not.

Since we are dealing with characteristic 2, recall that

$$T_1(D) := \{x \in D \mid x^2 \in K(D)\},$$

where $K(D)$ is the subspace of D generated by all commutators.

First we consider the standard algebra $D(m)'$. We have the relation $\beta_m\beta_1 = 0$, so

$$\alpha^2 = \beta_1 \dots \beta_m = [\beta_1, \beta_2 \dots \beta_m] \in K(D(m)').$$

Thus, $\alpha \in T_1(D(m)')$. But by the definition of the nondegenerate symmetric bilinear form on $D(m)'$, given in Proposition 3.2, we have $(\alpha, \beta_1 \dots \beta_m) = 1$, because the product $\alpha\beta_1 \dots \beta_m = s_1$ is a nonzero socle element. Therefore, $\beta_1 \dots \beta_m \notin T_1(D(m)')^{\perp}$. The sequence of ideals of the center under consideration takes the form

$$\operatorname{soc}(D(m)') \underbrace{=}_{0} T_1(D(m)')^{\perp} \underbrace{\subset}_{2} Z(D(m)').$$

Secondly, we study the analogous sequence of ideals for the nonstandard algebra $D(m)$.

Claim. $\alpha^2 \notin K(D(m))$.

Proof of the Claim. This can be seen by working out an explicit basis for the commutator space $K(D(m))$. Let $\mathscr{B}$ be the monomial basis of $D(m)$ consisting of all pairwise distinct nonzero paths in the quiver with relations. Set $\bar{\mathscr{B}} := \mathscr{B} \setminus \{e_1, \dots, e_m, \alpha, \alpha^2, s_1, \dots, s_m\}$, where e_j is the trivial path at the vertex j. Then a K-basis of $K(D(m))$ is given as follows

$$K(D(m)) = \left\langle \bar{\mathscr{B}}, \alpha^2 - s_2, \dots, \alpha^2 - s_m, s_m - \alpha^3 \right\rangle_K.$$

Now it is readily checked that α^2 can indeed not be written as a linear combination of these basis elements.

From the claim we can conclude that $\alpha \notin T_1(D(m))$. But the element $\beta_1 \dots \beta_m$ is orthogonal to all basis elements in the ideal generated by the arrows of the quiver, except to α. In fact, the product of $\beta_1 \dots \beta_m$ with any path of length ≥ 1 becomes 0, except for α. Since $\alpha \notin T_1(D(m))$, we therefore get that

$$\beta_1 \dots \beta_m \in T_1(D(m))^\perp.$$

Hence the sequence of ideal under consideration takes the form

$$\operatorname{soc}(D(m)) \underbrace{\subset}_{1} T_1(D(m))^\perp = \langle \beta_1 \dots \beta_m, \operatorname{soc}(D(m)) \rangle_K \underbrace{\subset}_{1} Z(D(m)).$$

Comparing the codimensions of the ideals in these sequences for $D(m)'$ and $D(m)$, we can finally conclude, by invoking Proposition 3.1, that the algebras $D(m)'$ and $D(m)$ are not derived equivalent. □

Summing up, we obtain the following derived equivalence classification of the representation-finite symmetric algebras.

Theorem 5.5. *The algebras K, N_n^{mn}, $m, n \geq 1$, $D(m)$, $D(m)'$, $m \geq 2$, $T(2,2,r)$, $r \geq 2$, $T(2,3,3)$, $T(2,3,4)$, $T(2,3,5)$ form a complete set of representatives of pairwise different derived equivalence classes of connected representation-finite symmetric algebras.*

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