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Correction to: "The intersection of two real forms in Hermitian symmetric spaces of compact type"

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Abstract. We correct the proof of Theorem 1.1 in our previous paper specially in the non-irreducible case.

In this paper we correct the proof of Theorem 1.1 in our previous paper [**4**], which was not complete. The proof works well if the Hermitian symmetric space is irreducible. On the other hand, the proof does not partly work well if the Hermitian symmetric space is not irreducible. We partly divide the proof into two cases where the Hermitian symmetric space is irreducible and non-irreducible and give a complete proof of antipodal property of the intersection of two real forms.

Theorem 1.1 ([**4**]). *Let M be a Hermitian symmetric space of compact type. If two real forms L_1 and L_2 in M intersect transversally, then $L_1 \cap L_2$ is an antipodal set of L_1 and L_2.*

We adopt the notational conventions in Section 2, [**4**]. We prepare the following lemma for the proof of Theorem 1.1.

Lemma. *Let M be a compact Riemannian symmetric space. If M is cubic (Definition 3.2 in [**4**]), then $s_o \operatorname{Exp} H = \operatorname{Exp} H$ for any vertex H of $\bar{S}$.*

Proof. If M is cubic, $2H \in \Gamma(A)$ for any vertex H of $\bar{S}$. Thus we have $\operatorname{Exp}(2H) = o$ and $s_o \operatorname{Exp} H = \operatorname{Exp}(-H) = \operatorname{Exp} H$.

Proof of Theorem 1.1. We partly divide the proof into two cases where M is irreducible and non-irreducible, however our argument starts with any Hermitian symmetric space M of compact type.

The holomorphic sectional curvature of M is positive, so $L_1 \cap L_2 \neq \emptyset$ by Theorem 1 in [**1**] or Lemma 3.1 in [**6**]. Because of the transitive action of the group of holomorphic isometries we may assume the origin o of M is contained in $L_1 \cap L_2$ without loss of

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generality. It is sufficient to prove that o and p are antipodal in L_1 and in L_2 for any point p in $L_1 \cap L_2 - \{o\}$.

Let A_i be a maximal torus of L_i $(i = 1, 2)$ which contains o and p. Let $\mathfrak{a}_i$ be the maximal abelian subspace corresponding to A_i $(i = 1, 2)$. We take a maximal torus A_i' of M containing A_i and denote the corresponding maximal abelian subspace by $\mathfrak{a}_i'$ $(i = 1, 2)$. Let

$$\mathrm{Exp}_o\, tH_2 \qquad (H_2 \in \mathfrak{a}_2,\ 0 \leq t \leq 1)$$

be a shortest geodesic in A_2 joining o to p. In particular, $p = \mathrm{Exp}_o\, H_2$. Since A_2 is geodesically convex in A_2' by Quast-Tanaka [**2**], $\mathrm{Exp}_o\, tH_2$ is also a shortest geodesic in A_2' joining o to p. Hence we can take a fundamental system Π_2 of the restricted root system with respect to $\mathfrak{a}_2'$ such that $H_2 \in \bar{S}_2$, where

$$S_2 = \{H \in \mathfrak{a}_2' \mid \langle \alpha, H \rangle > 0\ (\alpha \in \Pi_2),\ \langle \delta_i, H \rangle < \pi\ (\delta_i \in R_2^{\#})\}.$$

Since $\bar{S}_2$ is decomposed

$$\bar{S}_2 = \bigcup_{\Delta \subset \Pi_2^{\#}} S_2^{\Delta}$$

as a disjoint union, there exists $\Delta_2 \subset \Pi_2^{\#}$ such that $H_2 \in S_2^{\Delta_2}$. Lemma 3.6 in [**4**] implies $p \in \mathrm{Exp}_o\, S_2^{\Delta_2} \subset A_1' \cap A_2'$.

It is known that a Hermitian symmetric space of compact type is cubic. We express

$$\mathfrak{a}_2' = \{(x_1, \ldots, x_r)\}$$

with respect to a canonical coordinate of A_2' (see Definition 3.2 in [**4**]). Proposition 3.4 in [**4**] implies that there exists an involutive permutation λ of $\{1, \ldots, r\}$ satisfying

$$\mathfrak{a}_2 = \{(x_1, \ldots, x_r) \mid x_i = x_{\lambda(i)}\ (1 \leq i \leq r)\}. \qquad (*)$$

$S_2^{\Delta_2}$ is described as

$$\begin{aligned} S_2^{\Delta_2} = \{H \in \mathfrak{a}_2' \mid & \langle \alpha, H \rangle > 0\ (\alpha \in \Pi_2 \cap \Delta_2),\ \langle \beta, H \rangle = 0\ (\beta \in \Pi_2 - \Delta_2), \\ & \langle \delta_i, H \rangle < \pi\ (\delta_i \in R_2^{\#} \cap \Delta_2),\ \langle \delta_j, H \rangle = \pi\ (\delta_j \in R_2^{\#} - \Delta_2)\}. \end{aligned}$$

Since $H_2 \in S_2^{\Delta_2}$, we have

$$\Delta_2 = \{\alpha \in \Pi_2 \mid \langle \alpha, H_2 \rangle > 0\} \cup \{\delta_i \in R_2^{\#} \mid \langle \delta_i, H_2 \rangle < \pi\}.$$

From now on we divide the argument into two cases where M is irreducible and non-irreducible.

If M is irreducible, there exists $\Pi'_2 \subset \Pi_2$ such that

$$\mathfrak{a}_2 = \{H \in \mathfrak{a}'_2 \mid \langle \alpha, H \rangle = 0 \ (\alpha \in \Pi'_2)\}$$

by the expression $(*)$ of $\mathfrak{a}_2$. In fact, the restricted root system of irreducible Hermitian symmetric space of compact type is of type BC or type C and $\Pi_2^{\#} = \{x_1 - x_2, x_2 - x_3, \dots, x_{r-1} - x_r, x_r, 2x_1\}$ for type BC and $\Pi_2^{\#} = \{x_1 - x_2, x_2 - x_3, \dots, x_{r-1} - x_r, 2x_r, 2x_1\}$ for type C. In both cases x_i is a canonical coordinate and S_2 is described as

$$S_2 = \left\{ H \in \mathfrak{a}'_2 \;\middle|\; \frac{\pi}{2} > x_1(H) > \cdots > x_r(H) > 0 \right\}.$$

Hence by $(*)$ we have $\Pi'_2 \subset \Pi_2$ such that $H \in \mathfrak{a}'_2$ belongs to $\mathfrak{a}_2$ if and only if $\langle \alpha, H \rangle = 0$ for any $\alpha \in \Pi'_2$.

We show $\mathrm{Exp}_o\, S_2^{\Delta_2} \subset A_2$. Since Π_2 is a basis of $\mathfrak{a}'_2$, we can take a basis $\{u_\alpha \mid \alpha \in \Pi_2\}$ of $\mathfrak{a}'_2$ satisfying

$$\langle \alpha, u_\beta \rangle = \begin{cases} 1 & (\alpha = \beta), \\ 0 & (\alpha \neq \beta). \end{cases}$$

Let

$$H_2 = \sum_{\alpha \in \Pi_2} h_\alpha u_\alpha \qquad (h_\alpha \in \mathbb{R}).$$

Since $H_2 \in \bar{S}_2$, we have

$$0 \leq \langle \alpha, H_2 \rangle = h_\alpha$$

for any $\alpha \in \Pi_2$. Moreover, we have

$$\Pi_2 \cap \Delta_2 = \{\alpha \in \Pi_2 \mid h_\alpha > 0\},$$

and $\Pi'_2 \cap \Delta_2 = \emptyset$ because $H_2 \in \mathfrak{a}_2$. Let $H \in S_2^{\Delta_2}$, then $\langle \beta, H \rangle = 0$ for any $\beta \in \Pi_2 - \Delta_2$. Since $\Pi'_2 \subset \Pi_2 - \Delta_2$, we have $H \in \mathfrak{a}_2$. Hence we have $S_2^{\Delta_2} \subset \mathfrak{a}_2$, which implies

$$\mathrm{Exp}_o\, S_2^{\Delta_2} \subset A_2.$$

In order to see that we have a similar situation in $\mathfrak{a}'_1$, we show that there exists $\Delta_1 \subset \Pi_1$ such that

$$\mathrm{Exp}_o\, S_2^{\Delta_2} = \mathrm{Exp}_o\, S_1^{\Delta_1}$$

for a fundamental cell S_1 corresponding to a fundamental system of the restricted root system with respect to $\mathfrak{a}'_1$. Since $p \in A_1 \cap A_2$, there exists $H_1 \in \mathfrak{a}_1$ such that

$$\mathrm{Exp}_o\, tH_1 \qquad (0 \le t \le 1)$$

is a shortest geodesic in A_1 joining o to p. Since A_1 is geodesically convex in A_1', $\mathrm{Exp}_o\, tH_1$ is also a shortest geodesic in A_1' joining o to p. By the conjugacy of maximal tori, there exists $k_1 \in K$ such that $\mathrm{Ad}(k_1)\mathfrak{a}_1' = \mathfrak{a}_2'$, where K denotes the isotropy subgroup at o. Then

$$\mathrm{Exp}_o\, t\mathrm{Ad}(k_1)H_1 \qquad (0 \le t \le 1)$$

is a shortest geodesic in A_2' joining o to $k_1 p$. Hence there exists $k_2 \in N_K(\mathfrak{a}_2')$ such that

$$\mathrm{Ad}(k_2)\mathrm{Ad}(k_1)H_1 = \mathrm{Ad}(k_2k_1)H_1 \in \bar{S}_2.$$

So there exists $\Delta_2' \subset \Pi_2^{\#}$ which satisfies $\mathrm{Ad}(k_2k_1)H_1 \in S_2^{\Delta_2'}$. Put $k = k_2k_1 \in K$. Since we have

$$k^{-1}\,\mathrm{Exp}_o(\mathrm{Ad}(k)H_1) = \mathrm{Exp}_o\, H_1 = \mathrm{Exp}_o\, H_2,$$

there exists $w \in \bar{W}_{S_2}$ such that

$$wS_2^{\Delta_2'} = S_2^{\Delta_2},$$
$${}^{\forall}H \in S_2^{\Delta_2'},\ k^{-1}\,\mathrm{Exp}_o\, H = \mathrm{Exp}_o\, wH,$$
$$w\mathrm{Ad}(k)H_1 = H_2$$

by Takeuchi's result (Lemma 1.7 in [**3**] or see Lemma 2.1 in [**4**]). Since a Hermitian symmetric space of compact type is simply connected, we have $\bar{W}_{S_2} = \{1\}$ by Lemma 1.3 (2) in [**3**] and so $w = 1$. Or we know it by $\pi_1(M) \cong \bar{W}_{S_2}$ (Theorem 2.1 in [**3**]). Hence we rewrite the above as follows:

$$S_2^{\Delta_2'} = S_2^{\Delta_2},$$
$${}^{\forall}H \in S_2^{\Delta_2},\ k^{-1}\,\mathrm{Exp}_o\, H = \mathrm{Exp}_o\, H,$$
$$\mathrm{Ad}(k)H_1 = H_2.$$

Moreover, we have $k^{-1}\,\mathrm{Exp}_o\, S_2^{\Delta_2} = \mathrm{Exp}_o\, S_2^{\Delta_2}$. Since $\mathrm{Ad}(k)\mathfrak{a}_1' = \mathfrak{a}_2'$, the restricted root system R_1 with respect to $\mathfrak{a}_1'$ satisfies $R_1 = R_2 \circ \mathrm{Ad}(k)$ and there exist a fundamental root system Π_1 of R_1 and $\Delta_1 \subset \Pi_1^{\#}$ satisfying $\mathrm{Ad}(k)^{-1}S_2^{\Delta_2} = S_1^{\Delta_1} \ni H_1$. Thus

$$\mathrm{Exp}_o\, S_2^{\Delta_2} = k^{-1}\,\mathrm{Exp}_o\, S_2^{\Delta_2} = \mathrm{Exp}_o\, \mathrm{Ad}(k)^{-1}S_2^{\Delta_2} = \mathrm{Exp}_o\, S_1^{\Delta_1}.$$

Since $H_1 \in S_1^{\Delta_1} \cap \mathfrak{a}_1$, we conclude $S_1^{\Delta_1} \subset \mathfrak{a}_1$ and $\mathrm{Exp}_o\, S_1^{\Delta_1} \subset A_1$. Hence we obtain

$$\mathrm{Exp}_o\, S_1^{\Delta_1} = \mathrm{Exp}_o\, S_2^{\Delta_2} \subset A_1 \cap A_2.$$

Since $p \in \operatorname{Exp}_o S_2^{\Delta_2} \subset A_1 \cap A_2$, it contradicts the assumption that L_1 and L_2 intersect transversally if $\dim S_2^{\Delta_2} \geq 1$. Therefore $\dim S_2^{\Delta_2} = 0$ and $S_2^{\Delta_2}$ is a vertex of $\bar{S}_2$. By Lemma p is an antipodal point of o in M. Therefore o and p are antipodal both in L_1 and in L_2, which completes the proof of Theorem 1.1 in the case where M is irreducible.

Next we consider the case where M is not irreducible. In order to prove Theorem 1.1 in this case, we prepare the following special real forms. Let M_1 and M_2 be Hermitian symmetric spaces of compact type and $\tau : M_1 \to M_2$ be an anti-holomorphic isometric map. We denote

$$D_\tau(M_1) = \{(x, \tau(x)) \mid x \in M_1\} \subset M_1 \times M_2.$$

$D_\tau(M_1)$ is a real form in $M_1 \times M_2$ and we call it a *diagonal real form*. For more information on diagonal real forms see our sequent paper [**5**]. We use the following theorem in [**5**] in order to prove Theorem 1.1.

THEOREM 2.7 ([**5**]). *Let M be a Hermitian symmetric space of compact type and*

$$M = M_1 \times \cdots \times M_m$$

be a decomposition of M into irreducible factors. Then two real forms L_1 and L_2 in M are decomposed as

$$L_1 = L_{1,1} \times \cdots \times L_{1,n}, \qquad L_2 = L_{2,1} \times \cdots \times L_{2,n}$$

and for each a $(1 \leq a \leq n)$ the pair of $L_{1,a}$ and $L_{2,a}$ is one of the following.

(1) *Two real forms in M_i for some i $(1 \leq i \leq m)$.*
(2) *After renumbering irreducible factors of M if necessary,*

$$N_1 \times D_{\tau_2}(M_2) \times D_{\tau_4}(M_4) \times \cdots \times D_{\tau_{2s}}(M_{2s})$$

and

$$D_{\tau_1}(M_1) \times D_{\tau_3}(M_3) \times \cdots \times D_{\tau_{2s-1}}(M_{2s-1}) \times N_{2s+1},$$

where $\tau_i : M_i \to M_{i+1}$ $(1 \leq i \leq 2s)$ is an anti-holomorphic isometric map which determines $D_{\tau_i}(M_i)$ and $N_1 \subset M_1$ and $N_{2s+1} \subset M_{2s+1}$ are real forms. The intersection of these two real forms is

$$\{(x, \tau_1(x), \tau_2\tau_1(x), \ldots, \tau_{2s} \cdots \tau_1(x)) \mid x \in N_1 \cap (\tau_{2s} \cdots \tau_1)^{-1}(N_{2s+1})\}.$$

Here $(\tau_{2s} \cdots \tau_1)^{-1}(N_{2s+1})$ is a real form in M_1 and the intersection of the two real forms mentioned above is homothetic to the intersection of two real forms N_1 and $(\tau_{2s} \cdots \tau_1)^{-1}(N_{2s+1})$ in M_1.
(3) *After renumbering irreducible factors of M if necessary,*

$$N_1 \times D_{\tau_2}(M_2) \times D_{\tau_4}(M_4) \times \cdots \times D_{\tau_{2s-2}}(M_{2s-2}) \times N_{2s}$$

and

$$D_{\tau_1}(M_1) \times D_{\tau_3}(M_3) \times \cdots \times D_{\tau_{2s-3}}(M_{2s-3}) \times D_{\tau_{2s-1}}(M_{2s-1}),$$

where $\tau_i : M_i \to M_{i+1}$ $(1 \leq i \leq 2s-1)$ *is an anti-holomorphic isometric map which determines* $D_{\tau_i}(M_i)$ *and* $N_1 \subset M_1$ *and* $N_{2s} \subset M_{2s}$ *are real forms. The intersection of these two real forms is*

$$\{(x, \tau_1(x), \tau_2\tau_1(x), \ldots, \tau_{2s-1}\cdots\tau_1(x)) \mid x \in N_1 \cap (\tau_{2s-1}\cdots\tau_1)^{-1}(N_{2s})\}.$$

Here $(\tau_{2s-1}\cdots\tau_1)^{-1}(N_{2s})$ *is a real form in* M_1 *and the intersection of the two real forms mentioned above is homothetic to the intersection of two real forms* N_1 *and* $(\tau_{2s-1}\cdots\tau_1)^{-1}(N_{2s})$ *in* M_1.

(4) *After renumbering irreducible factors of* M *if necessary,*

$$D_{\tau_1}(M_1) \times D_{\tau_3}(M_3) \times \cdots \times D_{\tau_{2s-1}}(M_{2s-1})$$

and

$$D_{\tau_2}(M_2) \times D_{\tau_4}(M_4) \times \cdots \times D_{\tau_{2s}}(M_{2s}),$$

where $\tau_i : M_i \to M_{i+1}$ $(1 \leq i \leq 2s-1)$ *and* $\tau_{2s} : M_{2s} \to M_1$ *are anti-holomorphic isometric maps which determine* $D_{\tau_i}(M_i)$ $(1 \leq i \leq 2s)$. *The intersection of these two real forms is*

$$\{(x, \tau_1(x), \tau_2\tau_1(x), \ldots, \tau_{2s-1}\cdots\tau_1(x)) \mid (x, \tau_{2s}^{-1}(x)) \in D_{\tau_{2s-1}\cdots\tau_1}(M_1) \cap D_{\tau_{2s}^{-1}}(M_1)\}.$$

Here $D_{\tau_{2s-1}\cdots\tau_1}(M_1)$ *and* $D_{\tau_{2s}^{-1}}(M_1)$ *are diagonal real forms in* $M_1 \times M_{2s}$ *and the intersection of the two real forms mentioned above is homothetic to the intersection of these two diagonal real forms.*

In a case where a compact Riemannian symmetric space X is the product of compact Riemannian symmetric spaces X_1 and X_2, two points $p = (p_1, p_2)$ and $q = (q_1, q_2)$ in X are antipodal if and only if p_i and q_i are antipodal in X_i for $i = 1, 2$. Hence, to prove the intersection of two real forms is antipodal it suffices to consider the cases (1) to (4) in Theorem 2.7.

(1) is essentially the irreducible case and we have already proved the statement. In cases of (2) and (3) the intersection of two real forms is described by that of two real forms in an irreducible Hermitian symmetric space of compact type and its antipodal property follows from that in the case of (1).

For (4) we use a similar method to the irreducible case. We consider the intersection of two diagonal real forms $D_\sigma(M_1)$ and $D_\tau(M_1)$ in $M_1 \times M_2$ determined by anti-holomorphic isometric maps $\sigma, \tau : M_1 \to M_2$. We regard $L_1 = D_\sigma(M_1)$ and

$L_2 = D_\tau(M_1)$ and use the notation defined above in this proof. Since $D_\tau(M_1)$ is a real form in $M_1 \times M_2$, we can decompose $S_2, \Pi_2^\#$ and Δ_2 as

$$S_2 = S_{2,1} \times S_{2,2}, \quad \Pi_2^\# = \Pi_{2,1}^\# \cup \Pi_{2,2}^\#, \quad \Delta_2 = \Delta_{2,1} \cup \Delta_{2,2},$$

where $\Delta_{2,j} = \Delta_2 \cap \Pi_{2,j}^\#$ $(j = 1, 2)$. Since M_1 and M_2 are isomorphic, their fundamental systems are isomorphic and we obtain

$$S_2^{\Delta_2} \cap \mathfrak{a}_2 = \{(X, d\tau_o(X)) \mid X \in S_{2,1}^{\Delta_{2,1}}\}$$

and $\operatorname{Exp}_o(S_2^{\Delta_2} \cap \mathfrak{a}_2) \subset A_2$. In a way similar to that in the irreducible case we can take a canonical coordinate y_j of A_2' and we have

$$S_2 = \left\{ H \in \mathfrak{a}_2' \,\middle|\, \frac{\pi}{2} > y_1(H) > \cdots > y_n(H) > 0,\ \frac{\pi}{2} > y_{n+1}(H) > \cdots > y_{2n}(H) > 0 \right\}.$$

Therefore

$$S_2^{\Delta_2} \cap \mathfrak{a}_2 = \{H \in \mathfrak{a}_2' \mid H \in S_2^{\Delta_2},\ y_i(H) = y_{n+i}(H)\ (1 \le i \le n)\}.$$

Similarly if we take a suitable canonical coordinate x_i of A_1', we obtain

$$S_1^{\Delta_1} \cap \mathfrak{a}_1 = \{H \in \mathfrak{a}_1' \mid H \in S_1^{\Delta_1},\ x_i(H) = x_{n+i}(H)\ (1 \le i \le n)\}$$

and

$$\begin{aligned}
&\operatorname{Ad}(k)(S_1^{\Delta_1} \cap \mathfrak{a}_1) \\
&\quad = \operatorname{Ad}(k)\{H \in \mathfrak{a}_1' \mid H \in S_1^{\Delta_1}, x_i(H) = x_{n+i}(H)\ (1 \le i \le n)\} \\
&\quad = \{H \in \mathfrak{a}_2' \mid H \in S_2^{\Delta_2}, x_i(\operatorname{Ad}(k)^{-1}H) = x_{n+i}(\operatorname{Ad}(k)^{-1}H)\ (1 \le i \le n)\} \\
&\quad = S_2^{\Delta_2} \cap \mathfrak{a}_2.
\end{aligned}$$

Hence we have

$$\begin{aligned}
\operatorname{Exp}_o(S_2^{\Delta_2} \cap \mathfrak{a}_2) &= k^{-1} \operatorname{Exp}_o(S_2^{\Delta_2} \cap \mathfrak{a}_2) = k^{-1} \operatorname{Exp}_o(\operatorname{Ad}(k)(S_1^{\Delta_1} \cap \mathfrak{a}_1)) \\
&= \operatorname{Exp}_o(S_1^{\Delta_1} \cap \mathfrak{a}_1)
\end{aligned}$$

and

$$A_2 \supset \operatorname{Exp}_o(S_2^{\Delta_2} \cap \mathfrak{a}_2) = \operatorname{Exp}_o(S_1^{\Delta_1} \cap \mathfrak{a}_1) \subset A_1.$$

Since $p \in \operatorname{Exp}_o(S_2^{\Delta_2} \cap \mathfrak{a}_2) \subset A_1 \cap A_2$, it contradicts the assumption that L_1 and L_2 intersect transversally if $\dim S_{2,1}^{\Delta_{2,1}} = \dim(S_2^{\Delta_2} \cap \mathfrak{a}_2) \ge 1$. Therefore $\dim S_{2,1}^{\Delta_{2,1}} = 0$ and

$S_{2,1}^{\Delta_{2,1}}$ is a vertex of $\bar{S}_{2,1}$. By Lemma p is an antipodal point of o in M. Therefore o and p are antipodal both in L_1 and in L_2, which completes the proof of Theorem 1.1 in the case where M is not irreducible.

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References

[1] X. Cheng, The totally geodesic coisotropic submanifolds in Kähler manifolds, Geom. Dedicata, **90** (2002), 115–125.

[2] P. Quast and M. S. Tanaka, Convexity of reflective submanifolds in symmetric R-spaces, Tohoku Math. J., **64** (2012), 607–616.

[3] M. Takeuchi, On conjugate loci and cut loci of compact symmetric spaces I, Tsukuba J. Math., **2** (1978), 35–68.

[4] M. S. Tanaka and H. Tasaki, The intersection of two real forms in Hermitian symmetric spaces of compact type, J. Math. Soc. Japan, **64** (2012), 1297–1332.

[5] M. S. Tanaka and H. Tasaki, The intersection of two real forms in Hermitian symmetric spaces of compact type II, J. Math. Soc. Japan, **67** (2015), 275–291.

[6] H. Tasaki, The intersection of two real forms in the complex hyperquadric, Tohoku Math. J., **62** (2010), 375–382.

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