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κ-Ohio completeness

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Abstract. Generalizing the Ohio completeness property, we introduce the notion of κ-Ohio completeness. Although many results from a previous paper by the authors may easily be adapted for this new property, there are also some interesting differences. We provide several examples to illustrate this. We also have a consistency result; depending on the value of the cardinal $\mathfrak{d}$, the countable union of open and ω_1-Ohio complete subspaces may or may not be ω_1-Ohio complete.

1. Introduction.

All spaces under consideration are Tychonoff. For all undefined notions we refer to [**6**]. A topological space X is *Ohio complete* if for every compactification γX of X there is a G_δ-subset S of γX such that $X \subseteq S$ and for every $y \in S \setminus X$, there is a G_δ-subset of γX which contains y and misses X. Ohio completeness was introduced by Arhangel'skiĭ in [**1**] where it turned out to be a useful concept for the study of properties of remainders in compactifications.

Let κ be an infinite cardinal number. It is quite natural to generalize Ohio completeness by saying that a space X is *κ-Ohio complete* if for every compactification γX of X there is a G_κ-subset S of γX such that $X \subseteq S$ and for every $y \in S \setminus X$, there is a G_κ-subset of γX which contains y and misses X. Here a subspace of a space X is called a G_κ-subset if it is the intersection of at most κ-many open subsets of X.

Observe that any space is κ-Ohio complete, for some large enough κ. Also, if either the Čech-number or the compact covering number of a space does not exceed κ, then this space is κ-Ohio complete.

Ohio complete spaces were studied in [**2**] and [**3**]. In this paper we will focus our attention on unions of κ-Ohio complete subspaces. Since all the positive results proved in [**3**] can be easily generalized for κ-Ohio completeness, our main purpose will be to construct counterexamples for the κ-Ohio complete case. This will be done in the main section. We will construct a non κ-Ohio complete space which is the union of a locally countable family of closed and κ-Ohio complete

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subspaces. Next we will show that, if κ is a regular cardinal, the union of κ-many open and κ-Ohio complete subspaces need not be a κ-Ohio complete space.

The last section is devoted to positive results about open sums. We shall prove that the union of λ-many open and κ-Ohio complete subspaces is $kcov(\kappa^\lambda)$-Ohio complete. This result implies several interesting consistency results. In particular, if $\mathfrak{d} = \omega_1$, then the union of countably many open and ω_1-Ohio complete subspaces is again ω_1-Ohio complete. This statement may fail if $\mathfrak{d} > \omega_1$. Here the cardinal $\mathfrak{d}$ is the compact covering number of the space ω^ω of irrationals (see [**5**] for more information).

Throughout the paper we will use the terminology introduced in [**3**]. Of course the terminology there was only introduced for Ohio completeness, but the κ-Ohio complete generalization is straightforward.

2. Examples.

In [**3**] it was proved that the union of a locally finite family of closed and Ohio complete subspaces is again Ohio complete and the same holds for κ-Ohio completeness. In contrast, the union of a locally countable family of closed and Ohio complete subspaces need not be Ohio complete by [**3**, Example 5.3]. The obvious generalization of this example shows that the union of a locally-κ family of closed and Ohio complete subspaces need not be κ-Ohio complete. The following example is much better, since it shows that even a locally countable union of closed and Ohio complete subspaces may fail to be κ-Ohio complete.

Example 2.1. Fix an infinite cardinal number κ and let $\tau = \kappa^+$. The space τ carries the discrete topology. Let $\gamma\tau = \tau \cup \{\infty\}$ be its one-point compactification. Let $\mathscr{A}$ be a family which is maximal with respect the following property:

(1) $\mathscr{A} \subseteq [\tau]^\omega$,
(2) $\forall A, B \in \mathscr{A}\ (A \neq B \to |A \cap B| < \omega)$.

Recall that the well-known space $Y = \psi(\tau, \mathscr{A})$ is defined as follows (see for example [**5**, p.153]). The underlying set of Y is $\tau \cup \mathscr{A}$, the points of τ are isolated and a basic neighborhood of $A \in \mathscr{A}$ has the form $\{A\} \cup (A \setminus F)$, where F is finite. It is clear from the definition that Y is covered by countable open sets.

We claim that the subspace $\mathscr{A}$ is not a G_κ-subset of Y. To prove this it suffices to show that every closed subset of Y which misses $\mathscr{A}$ is finite. If this were not the case, then we could find a countably infinite closed subset C of Y which misses $\mathscr{A}$. Since the family $\mathscr{A}$ was chosen maximal, there must exist some $A \in \mathscr{A}$ such that $A \cap C$ is countably infinite. But then, every neighborhood of A intersects C, a contradiction.

We let $Z = Y \times \gamma\tau$ and X be the subspace of Z given by:

$$X = (Y \times \tau) \cup (\mathscr{A} \times \{\infty\}).$$

We use [**3**, Lemma 5.1] (the proof can be easily adapted for κ-Ohio completeness) to show that X is not κ-Ohio complete. Since $\mathscr{A}$ is not a G_κ-subset of Y, the space X is not a G_κ-subset of Z. To conclude that X is not κ-Ohio complete, observe that $Z \setminus X$ contains no non-empty G_κ-subset of Z.

It remains to verify that X is the union of a locally countable family of closed and Ohio complete subspaces of X. Let π be the projection of X onto the first coordinate. We let the closed cover $\mathscr{C}$ of X be given by:

$$\mathscr{C} = \{\pi^{-1}(y) : y \in Y\}.$$

Note that the fibers of π are homeomorphic to either $\gamma\tau$ or τ, which are both Ohio complete since they are locally compact. Furthermore, since Y is locally countable in itself and π is continuous, it follows that $\mathscr{C}$ is locally countable in X.

We now turn towards open sum theorems. In [**3**, Example 5.7] it was shown that the countable union of open and Ohio complete subspaces need not be Ohio complete. We can then ask whether the union of κ-many open and κ-Ohio complete subspaces is κ-Ohio complete. From the next theorem it will follow that, at least for κ regular, this is not the case. Moreover it will follow that, under the assumpion $\kappa < \mathfrak{d}$, the countable union of open and Ohio complete subspaces need not even be κ-Ohio complete. In the final section of this paper we will show that on the other hand the countable union of open and Ohio complete subspaces is always $\mathfrak{d}$-Ohio complete.

Fix an infinite regular cardinal κ and consider the space 2^κ. We call a set a $G_{<\kappa}$-subset if it is a G_λ-subset for some $\lambda < \kappa$. We denote by $(2^\kappa)_{<\kappa}$ its $G_{<\kappa}$-modification. So the topology on $(2^\kappa)_{<\kappa}$ is generated by the collection of all $G_{<\kappa}$-subsets of 2^κ. Now consider the following subset of (2^κ):

$$E_{<\kappa} = \{x \in 2^\kappa : |\{\alpha < \kappa : x_\alpha \neq 0\}| < \kappa\}.$$

Note that this set is dense in the space $(2^\kappa)_{<\kappa}$. Recall that the Baire number of a space with no isolated points, also called the Novák number, is the minimal cardinality of a family of closed nowhere dense subsets whose union is the whole space. In [**8**, Lemma 1.3(b)] it is proved that the Baire number of the space $(2^\kappa)_{<\kappa}$ is always greater than or equal to κ^+. This result implies the following.

LEMMA 2.2. *For regular κ, the set $E_{<\kappa}$ is not a G_κ-subset of $(2^\kappa)_{<\kappa}$.*

PROOF. Note that since κ is regular, the set $E_{<\kappa}$ is equal to the union of sets of the form $2^\alpha \times \{0\}^{\kappa\setminus\alpha}$, for $\alpha < \kappa$. These sets are all closed and nowhere dense in $(2^\kappa)_{<\kappa}$.

So if $E_{<\kappa}$ were a G_κ-subset, then its complement would be the union of κ-many closed sets which are all nowhere dense since $E_{<\kappa}$ is dense in $(2^\kappa)_{<\kappa}$. So by the previous observation, we would have that the Baire number of the space $(2^\kappa)_{<\kappa}$ were less than or equal to κ, contradicting [**8**, Lemma 1.3(b)]. □

The example constructed in the following theorem is very similar to [**3**, Example 5.7], see also [**4**, Example 2.4].

THEOREM 2.3. *Let κ and λ be infinite cardinal numbers, with κ regular. There exists a space X with the following properties*:

(1) *If X is λ-Ohio complete, then $E_{<\kappa}$ is a G_λ-set in $(2^\kappa)_{<\kappa}$,*
(2) *X is the union of κ-many open and κ-Ohio complete subspaces.*

PROOF. We set $E = E_{<\kappa}$. For every $e \in E$, we fix a collection $A(e)$ of one-to-one functions from κ into $D = 2^\kappa \setminus E$, which is maximal with respect to the following conditions:

(i) $\forall f \in A(e)\ \forall \alpha < \kappa\ (f(\alpha) \restriction \alpha = e \restriction \alpha)$,
(ii) $\forall f, g \in A(e)\ (f \neq g \to |\mathrm{ran}(f) \cap \mathrm{ran}(g)| < \kappa)$.

Fix a discrete space Y of cardinality λ^+ and let $\omega Y = Y \cup \{\infty\}$ be its one-point compactification. For every $\alpha \in \kappa$, with E_α we denote the subspace $2^\alpha \times \{0\}^{\kappa\setminus\alpha}$. Put $A_\alpha = \bigcup_{e\in E_\alpha} A(e)$ and $A = \bigcup_{\alpha\in\kappa} A_\alpha$ and let $Z = A \cup (D \times \omega Y)$. If $f \in A$ and $\alpha < \kappa$, we let

$$U(f, \alpha) = \{f\} \cup \bigcup\{f(\beta) \times \omega Y : \alpha < \beta < \kappa\}.$$

The collection $\mathscr{B}$, which serves as a base for a topology on Z, is given by

$$\{U(f, \alpha) : f \in A,\ \alpha < \kappa\} \cup \{R \times U : R \subseteq D,\ U \text{ is an open subset of } \omega Y\}.$$

We leave it to the reader to verify that topologized in this way, the space Z is Hausdorff and zero-dimensional and hence Tychonoff.

We let X be the subspace of Z given by $A \cup (D \times Y)$. In the following claim we will prove assertion (1). The proof is almost identical to the argument used in [**4**, Example 2.4].

CLAIM 1. *If X is λ-Ohio complete, then E is a G_λ-set in $(2^\kappa)_{<\kappa}$.*

PROOF OF CLAIM. Striving for a contradiction, assume that X is λ-Ohio complete but E is not a G_λ-subset of $(2^\kappa)_{<\kappa}$.

Since for every $d \in D$, the subspace $\{d\} \times \omega Y$ of Z is homeomorphic to ωY, the set $Z \setminus X$ contains no non-empty G_λ-subsets of Z. Then, by [**3**, lemma 5.1] X must be a G_λ-subset of Z. This implies that A is a G_λ-subset of the subspace $A \cup (D \times \{\infty\})$ of Z. Hence $D = \bigcup_{\alpha<\lambda} G_\alpha$, where each $G_\alpha \times \{\infty\}$ is closed in $A \cup (D \times \{\infty\})$.

By assumption D is not the union of λ-many closed subsets of $(2^\kappa)_{<\kappa}$, so for some $\alpha < \lambda$, $E \cap Cl_{<\kappa}(G_\alpha) \neq \emptyset$ (where the closure is taken in $(2^\kappa)_{<\kappa}$). So we may fix $e \in E$, such that for every $\beta < \kappa$, there is some $g \in G_\alpha$ such that $g \restriction \beta = e \restriction \beta$. But this means that we may find an injective function $f\colon \kappa \to G_\alpha$ such that for every $\beta \in \kappa$, $f(\beta) \restriction \beta = e \restriction \beta$.

Since the collection $A(e)$ was maximal, it follows that for some $f' \in A(e)$, we have that $|\mathrm{ran}(f) \cap \mathrm{ran}(f')| = \kappa$. But this means that f' is in the closure of the set $G_\alpha \times \{\infty\}$ (closure in $A \cup (D \times \{\infty\})$), which is a contradiction. $\lhd$

We will now prove assertion (2), that is that X is the union of κ-many open and κ-Ohio complete subspaces. For each $\alpha < \kappa$, we let $X_\alpha = A_\alpha \cup (D \times Y)$. It is not hard to verify that X_α is an open subspace of X and of course $X = \bigcup\{X_\alpha : \alpha < \kappa\}$. It remains to prove that each X_α is κ-Ohio complete.

CLAIM 2. *For each $\alpha < \kappa$, the space X_α is κ-Ohio complete.*

PROOF OF CLAIM. Fix $\alpha < \kappa$. Note that both A_α and $D \times Y$ are discrete subspaces of X_α. Since a discrete space is (κ-)Ohio complete, we have that X_α is the union of two (κ-)Ohio complete subspaces. The space $D \times Y$ is clearly an open subspace of X_α, and the set A_α is a G_κ-subset of X_α. Indeed, one verifies easily that $A_\alpha = \bigcap_{\alpha \le \beta < \kappa} \bigcup_{f \in A_\alpha} (U(f, \beta) \cap X_\alpha)$.

So it follows that X_α is the union of two G_κ-subsets which are both κ-Ohio complete. By Corollary 4.2 in [**3**] it follows that X_α is κ-Ohio complete. $\lhd$

This completes the proof of the theorem. $\square$

The previous theorem may be applied to obtain several interesting examples.

EXAMPLE 2.4. Assume $\lambda < \mathfrak{d}$. Then the union of countably many open and Ohio complete subspaces need not be λ-Ohio complete.

PROOF. Consider the space X constructed in the previous theorem with $\kappa = \omega$. If X were λ-Ohio complete, then $E_{<\omega}$ would be a G_λ-subset of $(2^\omega)_{<\omega}$.

However in this case $E_{<\omega}$ is homeomorphic to the space of rationals in $(2^\omega)_{<\omega}$, which is just the Cantor set. Of course, since $\lambda < \mathfrak{d}$, the set of rationals is not a G_λ-subset of 2^ω. □

EXAMPLE 2.5. Assume κ regular. Then the union of κ-many open and κ-Ohio complete subspaces need not be κ-Ohio complete.

PROOF. It suffices to apply Theorem 2.3 with $\lambda = \kappa$, and then use Lemma 2.2. □

3. Positive results.

Having obtained several counterexamples, we now provide some positive results on open sum theorems for κ-Ohio completeness. In particular, the results of this section will allow us to prove that, under the assumption $\mathfrak{d} = \omega_1$, the countable open union of ω_1-Ohio complete subspaces is ω_1-Ohio complete.

Recall that the covering number of a space X, denoted by $kcov(X)$, is the minimal cardinality of a collection $\mathscr{K}$ of compact subsets of X which covers X. In the next lemma we will refer to $kcov(\kappa^\lambda)$. In this case the space κ always carries the discrete topology. Of course $\kappa \leq kcov(\kappa^\lambda) \leq \kappa^\lambda$.

LEMMA 3.1. *Let X be a space. Then the union of λ-many G_κ-subsets of X is a $G_{kcov(\kappa^\lambda)}$-subset of X.*

PROOF. Let $\mathscr{G}$ be a family of G_κ-subsets of X, with $|\mathscr{G}| = \lambda$. For every $G \in \mathscr{G}$, we fix a sequence $(G_\alpha)_{\alpha\in\kappa}$ of open subsets of X such that $G = \bigcap_{\alpha\in\kappa} G_\alpha$. Since $|\mathscr{G}| = \lambda$, the space $\kappa^{\mathscr{G}}$ is homeomorphic to κ^λ, and then we may write $\kappa^{\mathscr{G}} = \bigcup_{\tau\in kcov(\kappa^\lambda)} K_\tau$, where each K_τ is a compact subset of $\kappa^{\mathscr{G}}$.

For every $\tau \in kcov(\kappa^\lambda)$, we let $f_\tau : \mathscr{G} \to [\kappa]^{<\omega}$ be the function defined as $f_\tau(G) = p_G(K_\tau)$, where p_G denotes the projection of $\kappa^{\mathscr{G}}$ onto the G-th factor, and we let $\mathscr{F} = \{f_\tau : \tau \in kcov(\kappa^\lambda)\}$.

For every $f \in \mathscr{F}$, we let $W_f = \bigcup_{G\in\mathscr{G}} \bigcap_{\alpha\in f(G)} G_\alpha$ and $W = \bigcap_{f\in\mathscr{F}} W_f$. It is clear that W_f is an open subset of X containing $\bigcup\mathscr{G}$, and then W is a $G_{kcov(\kappa^\lambda)}$-subset of X containing $\bigcup\mathscr{G}$. We shall now prove that actually $W = \bigcup\mathscr{G}$.

To this end, suppose that $x \notin \bigcup\mathscr{G}$. Then, for every $G \in \mathscr{G}$ we may fix an index $\alpha_G \in \kappa$ such that $x \notin G_{\alpha_G}$. Since the point $y = (\alpha_G)_{G\in\mathscr{G}} \in \kappa^{\mathscr{G}}$, there exists some $\tau \in kcov(\kappa^\lambda)$ such that $y \in K_\tau$. By construction, $x \notin W_{f_\tau}$ so that $x \notin W$. □

We say that a subspace X of a space Z is κ-*Ohio embedded* in Z, if there is a G_κ-subset S of Z such that $X \subseteq S$ and for every $y \in S \setminus X$, there is a G_κ-subset of Z which contains y and misses X. Such a G_κ-subset S will be called a κ-*good* G_κ-*subset* with respect to X. For more information about κ-Ohio embedded spaces and κ-good G_κ-subsets see [**3**].

THEOREM 3.2. *Let X be a space. Suppose that $\mathscr{U}$ is a cover of X consisting of G_κ-subsets, with $|\mathscr{U}| = \lambda$. If $X \subseteq Z$, and every element of $\mathscr{U}$ is κ-Ohio embedded in Z, then X is $kcov(\kappa^\lambda)$-Ohio embedded in Z.*

PROOF. Let $\mathscr{U} = \{U_\alpha : \alpha \in \lambda\}$. For every $\alpha \in \lambda$, we may fix a G_κ-subset S_α of Z which is κ-good with respect to U_α. Note that since U_α is a G_κ-subset of X, we may assume without loss of generality that $S_\alpha \cap X = U_\alpha$. Then, by Lemma 3.1, the set $S = \bigcup_{\alpha\in\lambda} S_\alpha$ is a $G_{kcov(\kappa^\lambda)}$-subset of Z. We claim that S is $kcov(\kappa^\lambda)$-good with respect to X. First of all, note that $X \subseteq S$, since $U_\alpha \subseteq S_\alpha$, for $\alpha \in \lambda$. So it remains to show that every point in $S \setminus X$ can be separated from X by a $G_{kcov(\kappa^\lambda)}$-subset of Z. Actually we will prove more: such a point can be separated from X by a G_κ-subset of Z.

So, fix an arbitrary point $z \in S \setminus X$. Then $z \in S_\alpha \setminus U_\alpha$, for some $\alpha \in \lambda$. Then, by construction, there is a G_κ-subset T of Z such that $z \in T$ and $T \cap U_\alpha = \emptyset$. But then, since $S_\alpha \cap X = U_\alpha$, the set $S_\alpha \cap T$ is a G_κ-subset of Z which contains z and misses X. □

Since $\mathfrak{d} = kcov(\omega^\omega)$, the following corollary shows that the union of countably many open and Ohio complete subspaces is $\mathfrak{d}$-Ohio complete. So Example 2.4 is best possible.

COROLLARY 3.3. *Let X be a space. Let $\mathscr{U}$ be a cover of X consisting of G_κ-subsets, with $|\mathscr{U}| = \lambda$. Suppose that every element of $\mathscr{U}$ is contained in a κ-Ohio complete subspace of X. Then X is $kcov(\kappa^\lambda)$-Ohio complete.*

PROOF. Fix an arbitrary compactification γX of X. Since every element of $\mathscr{U}$ is contained in a κ-Ohio complete subspace of X, it follows from [**3**, Proposition 2.1] and [**3**, Corollary 2.5] that every element of $\mathscr{U}$ is κ-Ohio embedded in γX. So by the previous theorem, X is $kcov(\kappa^\lambda)$-Ohio embedded in γX. Since γX was an arbitrary compactification of X, this shows that X is $kcov(\kappa^\lambda)$-Ohio complete. □

Our main interest is in open sum theorems. In particular we have the following.

COROLLARY 3.4. *Assume $\mathfrak{d} = \omega_1$. Let $\mathscr{U}$ be a countable open cover of X such that every element of $\mathscr{U}$ is contained in an ω_1-Ohio complete subspace of X, then X is ω_1-Ohio complete.*

PROOF. It suffices to observe that $kcov(\omega_1^\omega) = \mathfrak{d}$ (see for example [**9**, Proposition 3.6]), and then apply Corollary 3.3. □

Let us denote the least cardinal λ for which the union of λ-many open and κ-Ohio complete subspace fails to be κ-Ohio complete with the symbol $\mathscr{O}(\kappa)$. Then the following theorem holds.

Theorem 3.5. *Let κ be an infinite cardinal number.*

(1) *If κ is regular then $\mathscr{O}(\kappa) \leq \kappa$.*
(2) *If λ is a cardinal number such that $kcov(\kappa^\lambda) = \kappa$, then $\mathscr{O}(\kappa) > \lambda$.*
(3) *Assuming the* **GCH**, *then $\mathscr{O}(\kappa) \geq \mathrm{cf}(\kappa)$.*
(4) *Assuming the* **GCH** *and κ regular then $\mathscr{O}(\kappa) = \kappa$.*
(5) *If $\kappa < \mathfrak{d}$, then $\mathscr{O}(\kappa) = \omega$.*
(6) $\mathscr{O}(\omega_1) = \begin{cases} \omega_1, & \text{if } \mathfrak{d} = \omega_1, \\ \omega, & \text{if } \mathfrak{d} > \omega_1. \end{cases}$

Proof. Assertion (1) follows from Example 2.5. Corollary 3.3 implies (2). For assertion (3), observe that **GCH** implies that, if $\lambda < \mathrm{cf}(\kappa)$, then $\kappa^\lambda = \kappa$ (see [**7**, Theorem 5.15]) and then $kcov(\kappa^\lambda) = \kappa$. This implies $\mathscr{O}(\kappa) \geq \mathrm{cf}(\kappa)$. Assertions (1) and (3) imply (4). Assertion (5) follows from Example 2.4. Finally, (6) follows from Corollary 3.4 and Example 2.4 ($\lambda = \omega_1$). □

Question 3.6. If κ is a singular cardinal, is it still true that $\mathscr{O}(\kappa) \leq \kappa$?

Finally, we consider locally countable and point-countable open sum theorems. By [**3**, Corollary 4.5], a point-finite and hence also locally finite union of open and ω_1-Ohio complete subspaces is again ω_1-Ohio complete. However as we have seen, if $\mathfrak{d} > \omega_1$, then even a countable union of open and ω_1-Ohio complete subspaces may fail to be ω_1-Ohio complete.

Now, if $\mathfrak{d} = \omega_1$, then the countable open sum theorem is true for ω_1-Ohio completeness, but we do not know the answer to the following.

Question 3.7. Assume $\mathfrak{d} = \omega_1$. Does the point-countable or locally countable open sum theorem for ω_1-Ohio completeness hold?

Remark 3.8. Let us point out that the notion of Ohio completeness could be generalized in even a more general way. Given infinite cardinal numbers κ and λ, we say that a space X is (κ, λ)-*Ohio complete* if for every compactification γX of X there is a G_κ-subset S of γX such that $X \subseteq S$ and for every $y \in S \setminus X$, there is a G_λ-subset of γX which contains y and misses X.

So this notion is a further elaboration of the Ohio completeness property. Of course, many of the results in [**3**] may be rephrased in terms of this notion, with the two (possibly distinct) variables κ and λ. The interested reader may verify that in certain results the first of these two variables plays a more important role than the second and in other results it is the other way around. In particular, in the second example of this paper (Theorem 2.3) we added a one-point compactification of a space Y in the second coordinate, where the size of Y may be arbitrarily large. So if κ is regular, then for any cardinal λ, the union of κ-many open and κ-Ohio complete spaces may fail to be (κ, λ)-Ohio complete.

References

[1] A. V. Arhangel'skiĭ, Remainders in compactifications and generalized metrizability properties, Topology Appl., **150** (2005), 79–90.

[2] D. Basile and J. van Mill, Ohio completeness and products, Topology Appl., **155** (2008), 180–189.

[3] D. Basile and J. van Mill, and G. J. Riddderbos, Sum theorems for Ohio completeness, Colloq. Math., **113** (2008), 91–104.

[4] J. Chaber, Metacompactness and the class MOBI, Fund. Math., **91** (1976), 211–217.

[5] E. K. van Douwen, The integers and topology, Handbook of set-theoretic topology, North-Holland, Amsterdam, 1984, pp. 111–167.

[6] R. Engelking, General topology, second ed., Sigma Series in Pure Mathematics, **6**, Heldermann Verlag, Berlin, 1989, Translated from the Polish by the author.

[7] T. Jech, Set theory, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 2003, The third millennium edition, revised and expanded.

[8] A. Landver, Baire numbers, uncountable Cohen sets and perfect-set forcing, J. Symbolic Logic, **57** (1992), no. 3, 1086–1107.

[9] O. Okunev and A. Tamariz-Mascarúa, On the Čech number of $C_p(X)$, Topology Appl., **137** (2004), 237–249, IV Iberoamerican Conference on Topology and its Applications.

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