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Einstein-Hermitian connections on Hyper-Kähler quotients

Dedicated to Professor Tadashi Nagano on his 60th birthday

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1. Main result.

A *hyper-Kähler structure* on a Riemannian manifold (Y, g) is a set of three almost complex structures (I, J, K) which are parallel with respect to the Levi-Civita connection and satisfy the quaternion relations

$$IJ=-JI=K.$$

We have the associated Kähler forms $\omega_I, \omega_J, \omega_K$ defined by

$$\omega_I(v, w)=g(Iv, w), \qquad \omega_J(v, w)=g(Jv, w),$$
$$\omega_K(v, w)=g(Kv, w), \qquad \text{for } v, w\in TY$$

which are closed and parallel.

Let G be a compact Lie group acting on Y so as to preserve the metric g and the hyper-Kähler structure (I, J, K). Each element $\xi\in\mathfrak{g}$ of the Lie algebra of G defines a vector field ξ^* on Y which generates the action of ξ. The hyper-Kähler moment map defined below is the set of three moment maps.

DEFINITION 1.1. *A hyper-Kähler moment map* for the action of G on Y is a map $\mu=(\mu_I, \mu_J, \mu_K): Y\to\boldsymbol{R}^3\otimes\mathfrak{g}^*$ which satisfies

(a) $\qquad \mu_A(y\cdot g)=\mathrm{Ad}_g^*(\mu_A(y)), \qquad y\in Y,\ g\in G,\ A=I, J, K$

(b) $\qquad \langle\xi, d\mu_A(v)\rangle=\omega_A(\xi^*, v), \qquad v\in TY,\ \xi\in\mathfrak{g},\ A=I, J, K,$

where $\mathfrak{g}^*$ is the dual space of $\mathfrak{g}$, $\mathrm{Ad}^*: \mathfrak{g}^*\to\mathfrak{g}^*$ is the coadjoint map and $\langle\ ,\ \rangle$ denotes the dual pairing between $\mathfrak{g}$ and $\mathfrak{g}^*$.

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Let $Z=\{\zeta\in\mathfrak{g}^*|\mathrm{Ad}_g^*(\zeta)=\zeta$ for all $g\in G\}$. Taking a $\zeta\in\boldsymbol{R}^3\otimes Z$, we consider the quotient space $\mu^{-1}(\zeta)/G$. We assume that $\mu^{-1}(\zeta)$ is a submanifold and the action of G on $\mu^{-1}(\zeta)$ is free with Hausdorff quotient. Hence $\mu^{-1}(\zeta)/G$ is a smooth manifold. Let $i: \mu^{-1}(\zeta)\to Y$ be the inclusion and $\pi: \mu^{-1}(\zeta)\to\mu^{-1}(\zeta)/G$ the projection. Then by a result of Hitchin, Karlhede, Lindström and Roček [**6**].

FACT 1.2. *The quotient space $M=\mu^{-1}(\zeta)/G$ has a natural Riemannian metric g_M and a hyper-Kähler structure (I_M, J_M, K_M) such that*

$$\pi^*\omega_A^M = i^*\omega_A^Y \qquad A = I, J, K,$$

where ω_A^Y (resp. ω_A^M) is the Kähler form associated with the complex structure $A=I, J, K$ on Y (resp. M).

In the above situation, $\pi: \mu^{-1}(\zeta)\to M$ is a principal G-bundle and has a natural connection A where the horizontal space is the orthogonal complement of the tangent space of the orbit in $T_y\mu^{-1}(\zeta)$ $(y\in\mu^{-1}(\zeta))$.

Our main results is the following:

THEOREM 1.3. *The natural connection A satisfies the equation*

$$R_A(I_Mv, I_Mw) = R_A(J_Mv, J_Mw) = R_A(K_Mv, K_Mw) = R_A(v, w) \quad \text{for all } v, w \in TM.$$

In particular, A is an Einstein-Hermitian connection (with respect to the complex structure I_M) with zero Einstein constant. Namely the curvature form R_A is of type (1, 1) *and satisfies*

$$\mathrm{tr}_{\omega_I^M} R_A = \frac{1}{2}\sum_i R_A(e_i-\sqrt{-1}\,I_Me_i,\ e_i+\sqrt{-1}\,I_Me_i) = 0,$$

where $\{e_1, I_Me_1, \cdots, e_m, I_Me_m\}$ is a local orthonormal frame for the tangent bundle TM.

REMARKS. 1) When the base manifold is 4-dimensional, a connection A is anti-self-dual if and only if it is Einstein-Hermitian with zero Einstein constant.

2) Since everything is independent of the particular choice of the complex structure, A is also an Einstein-Hermitian connection with respect to J_M and K_M.

We shall give the both proofs of Fact 1.2 and Theorem 1.3 since they are closely related to each other.

PROOF. Let $y\in\mu^{-1}(\zeta)$. The tangent space of $\mu^{-1}(\zeta)$ at y decomposes as

$$T_y\mu^{-1}(\zeta) = T_y(G(y))\oplus H_y,$$

where H_y is the orthogonal complement of the tangent space $T_y(G(y))$ of the orbit through y.

We first show that complex structures I, J, K preserve H_y. Let $v \in T_y \mu^{-1}(\zeta)$, $\xi \in \mathfrak{g}$. The associated vector field ξ^* is orthogonal to Iv, since

$$g(Iv, \xi^*) = \omega_I(v, \xi^*) = -\langle \xi, d\mu_I(v) \rangle = 0.$$

Similarly ξ^* is orthogonal to Jv and Kv. These also imply that $I\xi^*$, $J\xi^*$, $K\xi^*$ are in the orthogonal complement $(T_y \mu^{-1}(\zeta))^{\perp}$ in $T_y Y$. If $v \in H_y$, then

$$\langle \xi, d\mu_I(Iv) \rangle = g(I\xi^*, Iv) = g(\xi^*, v) = 0,$$

$$\langle \xi, d\mu_J(Iv) \rangle = g(J\xi^*, Iv) = g(\xi^*, Kv) = 0,$$

$$\langle \xi, d\mu_K(Iv) \rangle = g(K\xi^*, Iv) = -g(\xi^*, Jv) = 0.$$

This shows $Iv \in \ker d\mu$. Since we have already seen that Iv is orthogonal to $T_y(G(y))$, Iv is in H_y. Similarly Jv, Kv are in H_y. The above argument also shows that the tangent space of Y at y has an orthogonal decomposition:

$$T_y Y = H_y \oplus T_y(G(y)) \oplus I T_y(G(y)) \oplus J T_y(G(y)) \oplus K T_y(G(y)). \tag{1.4}$$

The tangent space $T_{\pi(y)} M$ of the quotient space M at $\pi(y)$ is isomorphic to H_y via the map π_*. Hence I, J, K descend to almost complex structures on M, which we denote by I_M, J_M, K_M. They clearly satisfy the quaternion relations. We also have a Riemannian metric g_M induced from $g|H_y$. The projection $\pi: \mu^{-1}(\zeta) \to M = \mu^{-1}(\zeta)/G$ is a Riemannian submersion. Next we show that I_M, J_M, K_M are parallel with respect to the Levi-Civita connection ∇^M on M.

Let ∇^Y (resp., $\nabla^{\mu^{-1}(\zeta)}$) be the Levi-Civita connection of Y (resp., the submanifold $\mu^{-1}(\zeta) \subset Y$) and Π the second fundamental form of $\mu^{-1}(\zeta)$. We denote by $\tilde{v}$ the horizontal lift of $v \in T_m M$ to $\mu^{-1}(\zeta)$ and w^v the $T_y(G(y))$-component of a tangent vector $w \in T_y Y$. Then by O'Neill's formula for Riemannian submersions (see e.g., [1], p. 240), we have

$$\widetilde{\nabla_v^M w} = \nabla_{\tilde{v}}^{\mu^{-1}(\zeta)} \tilde{w} - \frac{1}{2}[\tilde{v}, \tilde{w}]^v = \nabla_{\tilde{v}}^Y \tilde{w} - \Pi(\tilde{v}, \tilde{w}) - \frac{1}{2}[\tilde{v}, \tilde{w}]^v,$$

where $[\tilde{v}, \tilde{w}]^v$ is the vertical component of $[\tilde{v}, \tilde{w}]$. Hence

$$\nabla_{\tilde{v}}^Y I\tilde{w} = (\widetilde{\nabla_v^M I_M w}) + \Pi(\tilde{v}, I\tilde{w}) + \frac{1}{2}[\tilde{v}, I\tilde{w}]^v.$$

On the other hand, since I is parallel with respect to ∇^Y, this equals to

$$I\nabla_{\tilde{v}}^Y \tilde{w} = (I_M \widetilde{\nabla_v^M} w) + I\Pi(\tilde{v}, \tilde{w}) + \frac{1}{2} I[\tilde{v}, \tilde{w}]^v.$$

Comparing H_y-component and $T_y G(y)$-component of the above equations, we have

(1.5) $$(I_M\widetilde{\nabla}_v^M w) = (\nabla_v^M \widetilde{I_M w}), \qquad \frac{1}{2}[\tilde{v}, I\tilde{w}]^v = (I\Pi(\tilde{v}, \tilde{w}))^v.$$

(Note that $I[\tilde{v}, \tilde{w}]^v \in (T_y\mu^{-1}(\zeta))^\perp$). The first equation shows that I_M is parallel with respect to ∇^M. The same holds for J_M and K_M, hence (M, g_M, I_M, J_M, K_M) is a hyper-Kähler manifold.

According to the decomposition (1.4), we can write $\Pi(\tilde{v}, \tilde{w})$ as

$$\Pi(\tilde{v}, \tilde{w}) = I\Pi_I + J\Pi_J + K\Pi_K,$$

where Π_I, Π_J and $\Pi_K \in T_y(G(y))$. Substituting this into the second equality in (1.5), we find

$$\widetilde{\nabla_v^M w} = \nabla_{\tilde{v}}^Y \tilde{w} - \frac{1}{2}([\tilde{v}, \tilde{w}]^v + I[I\tilde{v}, \tilde{w}]^v + J[J\tilde{v}, \tilde{w}]^v + K[K\tilde{v}, \tilde{w}]^v).$$

This implies

(1.6) $$g([I\tilde{v}, \tilde{w}]^v, \xi^*) = g(I[I\tilde{v}, \tilde{w}]^v, I\xi^*) = -2\langle \xi, D^2\mu_I(\tilde{v}, \tilde{w})\rangle,$$

where $D^2\mu_I$ is the hessian of μ_I. This formula is useful when we estimate the curvature of the natural connection.

Since the right hand side of the second equality in (1.5) is symmetric with respect to v and w, we have

(1.7) $$[\tilde{v}, I\tilde{w}]^v = [\tilde{w}, I\tilde{v}]^v = -[I\tilde{v}, \tilde{w}]^v.$$

The curvature R_A of the natural connection on the principal bundle $\mu^{-1}(\zeta) \to M$ is given by

$$R_A(v, w) = -\omega([\tilde{v}, \tilde{w}]^v),$$

where $\omega\colon T\mu^{-1}(\zeta) \to \mathfrak{g}$ is the connection form defined by

$$\omega | H_y \equiv 0, \qquad \omega(\xi^*) = \xi \qquad \text{for } \xi \in \mathfrak{g}.$$

Hence (1.7) implies

(1.8) $$R_A(I_M v, I_M w) = R_A(v, w).$$

Replacing I by J, K in (1.7), we have

(1.9) $$R_A(J_M v, J_M w) = R_A(K_M v, K_M w) = R_A(v, w).$$

Now we check that A is an Einstein-Hermitian connection. The equation (1.8) means that R_A is of type (1.1). The equation (1.9) implies

$$R_A(v, I_M v) + R_A(J_M v, K_M v) = 0.$$

A local orthonormal unitary frame for the tangent bundle TM is given by

$$\{v_i,\ I_M v_i,\ J_M v_i,\ K_M v_i\}_{i=1,\cdots,n} \qquad \left(n = \frac{1}{4}\dim_R M\right)$$

for some local vector fields $v_1, \cdots, v_n$. Then a direct calculation shows that A is an Einstein-Hermitian connection with zero Einstein constant.

REMARKS. 1) The above proof also shows that a Kähler quotient M has a natural Kähler structure and the curvature R_A of the natural connection A is of type (1, 1). (See [**6**] for the definition of Kähler quotients.)

2) After this work was done, Nitta pointed out that a similar result holds for quaternionic Kähler quotients [**12**] (see [**4**] for the definition of quaternionic Kähler quotients). He proved the result by adapting our proof to the case of quaternionic Kähler quotients, but it follows directly from our results by using Swann's "twistor like" fibration [**14**] as follows:

Swann defined a hyper-Kähler structure (the metric maybe indefinite) on the fundamental quaternionic line bundle $\mathcal{U}(M)$ (with the zero-section removed) of a quaternionic Kähler manifold M. If a Lie group G acts on M freely and preserving the quaternionic structure, then the action lifts to a free, isometric and triholomorphic action on $\mathcal{U}(M)$. And the (pseudo) hyper-Kähler quotient of $\mathcal{U}(M)$ is the fundamental quaternionic line bundle of the quaternionic quotient M' of M by G. The quaternionic quotient construction, as in the hyper-Kähler quotient case, induces naturally a principal bundle with a connection. Its pull-back to $\mathcal{U}(M')$ is come from the hyper-Kähler quotient construction of $\mathcal{U}(M')$. Our result (the signature of the metric is not essential in our proof) shows that this connection satisfies the equations (1.8), (1.9), which implies the connection on M' is a B_2-connection.

2. Examples.

We give a couple of examples of applications of Theorem 1.3.

The first example is ALE hyper-Kähler 4-manifolds constructed by Kronheimer [8]. Since the general case is hard to explain without long preparation of notation, we only treat the easiest case, the Eguchi-Hanson space, which is a hyper-Kähler structure on the holomorphic cotangent bundle of the projective line.

Let Y be a quaternion vector space

$$Y = \boldsymbol{H}^2 = \boldsymbol{C}^2 \times \boldsymbol{C}^2.$$

The Lie group G=U(1) acts on Y by

$$Y \ni (z_1, z_2, w_1, w_2) \longmapsto (e^{i\theta} z_1, e^{i\theta} z_2, e^{-i\theta} w_1, e^{-i\theta} w_2).$$

This action preserves both the hyper-Kähler structure and the metric. Let μ be the unique hyper-Kähler moment map which vanishes at the origin. The vector space $Z\subset\mathfrak{g}^*$ is identified with the set of pure imaginaries. If we take a non-zero $\zeta\in Z$, the hyper-Kähler quotient $M=\mu^{-1}(\zeta)/G$ is a (non-singular) complete hyper-Kähler 4-manifold. Moreover the metric is asymptotically locally Euclidean (ALE) which means it approximates the Euclidean metric on $\boldsymbol{R}^4/\Gamma$ for some finite subgroup $\Gamma\subset SO(4)$ (in this example $\Gamma=\{\pm 1\}$).

By Theorem 1.3 there exists a natural U(1)-bundle P and an anti-self-dual connection A on M. One can show that A has finite action. Moreover A has a very special property and is used to obtain the ADHM description for anti-self-dual connections on M.

These and other properties will be studied in [**9**].

In the second example, the hyper-Kähler quotient M is the moduli space of anti-self-dual connection, where the old hyper-Kähler manifold Y is the space of all connections on a principal bundle over an ALE hyper-Kähler 4-manifold and the Lie group G is the gauge group (cf. [**5**]). Although Y and G are infinite-dimensional, the proof of Theorem 1.3 works in this case since we can shows that $\mu^{-1}(0)$ is a submanifold and a slice for the action exists. We need to introduce the weighted Sobolev spaces in order to argue rigorously, but we omit the analytic details for the sake of brevity. See [**11**], [**3**] for detail.

Let (X, g) be an ALE Riemannian 4-manifold with a hyper-Kähler structure (I_X, J_X, K_X) and E a (complex) vector bundle over it of rank r. Let $\mathrm{Ad}E$ denote the adjoint bundle associated with E (i.e. the vector bundle of skew endomorphisms of E). We take a representation $R_\infty\colon \Gamma\to U(r)$ which will be identified with a flat connection on the ALE end which is diffeormophic to $(R, \infty)\times S^3/\Gamma$. We assume that E has a connection A_0 which is equal to A_0 on the ALE end.

Let Y be the space of connections A on E such that

$$|\nabla_{A_0}^{(l)}(A-A_0)| = O(t^{-3-l}),$$

where t is the Euclidean distance on the ALE end of X. The map $A\mapsto A-A_0$ gives an identification between Y and the vector space of $\mathrm{Ad}E$-valued 1-forms whose differentials decay in suitable orders. The L^2-inner product defines a Riemannian metric on Y. A hyper-Kähler structure (I_X, J_X, K_X) which are endomorphisms of the tangent bundle TX naturally induces endomorphisms of the cotangent bundle T^*X by

$$(I_X\alpha)(v) = -\alpha(I_X v), \text{ e.t.c.,} \qquad \text{for } \alpha\in T^*X, \qquad v\in TX.$$

These give endomorphisms (denoted by I_Y, J_Y and K_Y) of TY which are clearly parallel. Then (Y, g_Y, I_Y, J_Y, K_Y) is an infinite dimensional hyper-Kähler manifold.

Let G be the group of automorphisms of E converging to the identity at infinity. If we take a completion of G under the suitable weighted Sobolev norm, G becomes an infinite dimensional Lie group with the Lie algebra $\mathfrak{g}$. It acts on Y by pulling back the connection. This action preserves both g_Y and (I_Y, J_Y, K_Y). We define a map $\mu=(\mu_I, \mu_J, \mu_K): Y \to \boldsymbol{R}^3 \otimes \mathfrak{g}^*$ by

$$\langle \xi, \mu_I(A) \rangle = \int_X \mathrm{tr}(\xi R_A) \wedge \omega_I \qquad \text{for } \xi \in \mathfrak{g}, \text{ e.t.c.},$$

where R_A is the curvature of the connection A. It is easy to check that μ is a hyper-Kähler moment map.

Kähler forms ω_I, ω_J and ω_K are global parallel sections of Λ^+ and give a basis of Λ^+ at each point. Hence $\mu(A)$ is identified with $-R_A^+$, the self-dual part of the curvature form. Then the hyper-Kähler quotient $M=\mu^{-1}(0)/G$ is the (framed) moduli space of anti-self-dual connections. Note that the action is free since we consider the "reduced" gauge group (i.e., the group of automorphisms converging to the identity at infinity). This is the main difference from the case when the base manifold is compact. By Theorem 1.3 $\pi: \mu^{-1}(0) \to M$ is the principal G-bundle with a natural Einstein-Hermitian connection $\tilde{A}$.

Our connection $\tilde{A}$ relates to the universal connection. The gauge group G acts on the product $E \times \mu^{-1}(0)$ and gives a vector bundle

$$\boldsymbol{E} = E \times_G \mu^{-1}(0) \longrightarrow X \times M$$

by taking quotient. This bundle is called the universal bundle. For a point $x \in X$, the restriction $\boldsymbol{E} | \{x\} \times M$ is isomorphic to the bundle $\mu^{-1}(0) \times_G E_x$ associated with the principal bundle $\mu^{-1}(0) \to M$ by the action of G on E_x. Hence it has a connection induced from $\tilde{A}$. On the other hand, for $\pi(A) \in M$, if we fix $A \in \mu^{-1}(0)$, the restriction $\boldsymbol{E} | X \times \{\pi(A)\}$ is isomorphic to $E \to X$ via the map

$$\boldsymbol{E} | X \times \{\pi(A)\} \ni G(e, A) \longmapsto e \in E.$$

(Remark that this map depends on the choice of A). Pulling back the connection A by this map, we have a connection on $\boldsymbol{E} | X \times \{\pi(A)\}$ which depends only on the gauge equivalence class $\pi(A)$. Combining the above two connections, we have a natural connection $\boldsymbol{A}$ on $\boldsymbol{E}$ which we call the universal connection. If we introduce the product metric on $X \times M$, we have

PROPOSITION 2.1. *The universal connection* $\boldsymbol{A}$ *on the universal bundle* $\boldsymbol{E} = E \times_G \mu^{-1}(0)$ *over* $X \times M$ *is an Einstein-Hermitian connection with zero Einstein constant.*

The corresponding result is shown by Itoh [7] by a different method when the base manifold is a compact hyper-Kähler 4-manifold (i.e. a torus or a K3 surface). But we only have a $\mathrm{PU}(r)=\mathrm{U}(r)/\mathrm{U}(1)$-bundle as a universal bundle since we must take a quotient group $G/\mathrm{U}(1)$ to make the action free.

Proposition 2.1 has an interesting corollary. We can define a Fourier transform (or Nahm's transform) of anti-self-dual connections as in Mukai [**10**], Schenk [**13**], Braam and van Baal [**2**]:

Let F be a vector bundle with an anti-self-dual connection A over X. For each $m \in M$ defines a vector space $\hat{F}_m$ by

$$\hat{F}_m = L^2\text{-kernel of } D_{A_m}^- : \Gamma(S^- \otimes F \otimes \boldsymbol{E}|_{X \times \{m\}}) \longrightarrow \Gamma(S^+ \otimes F \otimes \boldsymbol{E}|_{X \times \{m\}}),$$

where $D_{A_m}^-$ is the Dirac operator (in the X-direction) twisted by the connection A_m induced by A and $\boldsymbol{A}|_{X \times \{m\}}$). As m varies in M, $\hat{F}$ forms a smooth vector bundle over M. Moreover it is a subbundle of a vector bundle (of infinite rank) whose fiber at m is $\Gamma(S^- \otimes F \otimes \boldsymbol{E}|_{X \times \{m\}})$ and inherits a natural connection as a subbundle. The similar calculation as in [**2**] shows this connection is also Einstein-Hermitian with zero constant. (Here we use Proposition 2.1.) If the moduli space M is also an ALE hyper-Kähler 4-manifold (may be different from X), we can define a similar transform from an anti-self-dual connection on M to one on X, reversing the roles of X and M. But in contrast with the torus case, the square of the transforms is not necessarily the identity. Note that in [**10**], [**13**], [**2**] X is a flat torus and M is its dual torus which is the moduli space of anti-self-dual connections on the trivial line bundle.

We have many other examples (e.g. the Taub-NUT space, the moduli space of monopoles, the moduli space of solutions of Nahm equations, e.t.c.) of spaces described as hyper-Kähler quotients. The above examples make an appeal to the importance of the natural connections, but we do not know their meaning in other examples yet.

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References

[1] A. L. Besse, Einstein Manifolds, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1987.

[2] P. J. Braam and P. van Baal, Nahm's transformation for instantons, Comm. Math. Phys., **122** (1989), 267-280.

[3] D. S. Freed and K. K. Uhlenbeck, Instantons and four-manifolds, MSRI Publ., **1**, Springer-Verlag, 1984.

[4] K. Galicki and H. B. Lawson, Quaternionic reduction and quaternionic orbifolds, Math. Ann., **282** (1988), 1-21.

[5] N. J. Hitchin, Metrics on moduli spaces, in Proceedings of the Lefschez Centennial Conference (Contemporary Math, 58, Part I), A. M. S., Providence, R. I., 1986.

[6] N. J. Hitchin, A. Karlhede, U. Lindström and M. Roček, Hyperkähler metrics and supersymmetry, Comm. Math. Phys., **108** (1987), 535-589.

[7] M. Itoh, Gauge fields and quaternion structure, Kodai Math. J., **11** (1988), 344-360.

[8] P. B. Kronheimer, The construction of ALE spaces as hyper-kähler quotients, J. Diff. Geom., **29** (1989), 665-683.

[9] P. B. Kronheimer and H. Nakajima, Yang-Mills instantons on ALE gravitational instantons, Math. Ann., **288** (1990), 263-307.

[10] S. Mukai, Duality between $D(X)$ and $D(\hat{X})$, with applications to Picard sheaves, Nagoya Math. J., **81** (1981), 153-175.

[11] H. Nakajima, Moduli spaces of anti-self-dual connections on *ALE* gravitational instantons, Invent. Math., **102** (1990), 267-303.

[12] T. Nitta, Yang-Mills connections on quaternionic Kähler quotients, Proc. Japan Acad., **66** (1990), 245-247.

[13] H. Schenk, On a generalised Fourier transform of instantons over flat tori, Comm. Math. Phys., **116** (1988), 177-183.

[14] A. F. Swann, Aspects symplectique de la géométrie quaternionique, C. R. Acad. Sci. Paris, **308** (1989), 225-228.

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