J. Math. Soc. Japan
Vol. 42, No. 1, 1990

On a p-adic interpolating power series of the generalized Euler numbers

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(Received July 11, 1988)
(Revised Nov. 30, 1988)

§1. Introduction.

Let $u\neq 1$ be an algebraic number. The n-th Euler number $H^{n}(u)$ belonging to u is defined by

$$\frac{1-u}{e^{t}-u}=\sum_{n=0}^{\infty}\frac{H^{n}(u)}{n!}t^{n}.$$

Let p be a prime number and χ a primitive Dirichlet character. Shiratani-Yamamoto ([**6**]) constructed a p-adic interpolating function $G_{p}(s, u)$ of the Euler numbers $H^{n}(u)$, and as its applications to the p-adic L-functions $L_{p}(s, \chi)$, they derived an explicit formula for $L_{p}'(0, \chi)$ including Ferrero-Greenberg's formula ([**2**]), and gave an explanation of Diamond's formula ([**1**]).

Let f_{χ} be the conductor of χ. As analogous to the generalized Bernoulli numbers, Tsumura ([**10**]) defined the n-th generalized Euler number $H_{\chi}^{n}(u)$ for χ belonging to u by

$$\sum_{a=0}^{f_{\chi}-1}\frac{(1-u^{f_{\chi}})\chi(a)e^{at}u^{f_{\chi}-a-1}}{e^{f_{\chi}t}-u^{f_{\chi}}}=\sum_{n=0}^{\infty}\frac{H_{\chi}^{n}(u)}{n!}t^{n}, \tag{1.1}$$

and he constructed a p-adic interpolating function $l_{p}(s, u, \chi)$, which is an extension of $G_{p}(s, \chi)$. Further, by considering the expansion of $l_{p}(s, u, \chi)$ at $s=1$, he obtained some congruences for the generalized Euler numbers.

Sinnott ([**7**]) showed how to calculate the μ-invariants of the Γ-transforms of rational functions, and gave a new proof of the well-known theorem of Ferrero-Washington that Iwasawa's μ-invariants are zero for the basic $\boldsymbol{Z}_{p}$-extensions of all abelian number fields. By similar technique, an analytic property of the interpolating power series of $L_{p}(s, \chi)$ was investigated in [**8**], and a new proof of the Friedman's result in [**3**] was given in [**9**].

In the present paper, by similar methods used in [**7**], [**8**] and [**9**] we shall investigate an interpolating power series of the generalized Euler numbers. In §2, we shall reconstruct the function $l_{p}(s, u, \chi)$ by constructing an interpolating power series $F_{\chi,u}(T)$, and calculate the μ-invariant of $F_{\chi,u}(T)$. (The power series $F_{\chi,u}(T)$ becomes equal to the power series $e_u(T, \theta)$ in [**10**] if slight modifications are made.) In § 3, we shall investigate an analytic property of the series $F_{\chi,u}(T)$. In § 4, fixing a finite set S of prime numbers distinct from p, we shall investigate the p-adic valuation of $H^n_{\chi\omega^{-n}\phi\varphi}(u)$, where ω is the Teichmüller character for p and ϕ (resp. φ) is a character of the second kind for S (resp. p).

In the rest of this paper, we denote the set of positive integers by $\boldsymbol{N}$ as usual, and put $\bar{\boldsymbol{N}}=\boldsymbol{N}\cup\{0\}$. We fix a prime number p and denote by $\boldsymbol{Q}_p$, $\boldsymbol{Z}_p$ and $\boldsymbol{C}_p$, the p-adic rational number field, the ring of integers of $\boldsymbol{Q}_p$ and the completion of the algebraic closure of $\boldsymbol{Q}_p$, respectively. We write $|\ |$ for the p-adic valuation of $\boldsymbol{C}_p$ normalized by $|p|=1/p$. Let $\bar{\boldsymbol{Q}}$ denote the algebraic closure of the rational number field $\boldsymbol{Q}$ in the complex number field $\boldsymbol{C}$. We fix an embedding of $\bar{\boldsymbol{Q}}$ into $\boldsymbol{C}_p$ and regard $\bar{\boldsymbol{Q}}$ also as a field contained in $\boldsymbol{C}_p$. For any Dirichlet characters $\chi^{(1)}, \cdots, \chi^{(m)}$, and any $u\in\bar{\boldsymbol{Q}}$, we write $\boldsymbol{Q}_p(\chi^{(1)}, \cdots, \chi^{(m)}, u)$ for the field generated by u and the values of $\chi^{(1)}, \cdots, \chi^{(m)}$ over $\boldsymbol{Q}_p$, and $O_{\chi^{(1)},\dots,\chi^{(m)},u}$ for the ring of integers of $\boldsymbol{Q}_p(\chi^{(1)}, \cdots, \chi^{(m)}, u)$. In general, if R is a ring, we write $R^\times$ for the multiplicative group of units of R, $R[T]$ for the ring of polynomials in an indeterminate T with coefficients in R, and $R[[T-1]]$ for the ring of formal power series in an indeterminate $T-1$ with coefficients in R. If F is a field, we write $F(T)$ for the field of rational functions with coefficients in F. Denoting the Teichmüller character for p by ω, we put $\langle x\rangle=x/\omega(x)$ for any $x\in\boldsymbol{Z}_p^\times$. A Dirichlet character always means a primitive one.

§ 2. An interpolating power series and the μ-invariant.

Let χ be a Dirichlet character with conductor f_χ and $u\neq1$ an element of $\bar{\boldsymbol{Q}}$. As in the introduction, we define the n-th generalized Euler number $H^n_\chi(u)$ by (1.1). In what follows, we assume

$$(2.1) \qquad |1-u^{f_\chi p^N}|\geqq 1 \qquad \text{for all } N\in\bar{\boldsymbol{N}}.$$

If $g\in\boldsymbol{N}$ is any multiple of f_χ, we put

$$(2.2) \qquad R_{\chi,u}(T)=\sum_{a=0}^{g-1}\frac{\chi(a)u^{g-a}T^a}{T^g-u^g}\in \boldsymbol{Q}_p(\chi, u)(T),$$

which is independent of the choice of g.

PROPOSITION 2.1. *$R_{\chi,u}(T)$ lies in $O_{\chi,u}[[T-1]]$.*

PROOF. Put $g=f_\chi$. If $|u|\leqq1$, then (2.1) implies $|1-u^g|=1$, and so $T^g-u^g=(1-u^g)+\sum_{a=0}^{g-1}\binom{g}{a}(T-1)^a\in(O_{\chi,u}[[T-1]])^\times$. Hence, $R_{\chi,u}(T)$ lies in $O_{\chi,u}[[T-1]]$. If $|u|>1$, then $|(1/u^g)-1|=1$, and so, $(T/u)^g-1\in(O_{\chi,u}[[T-1]])^\times$. Hence, $R_{\chi,u}(T)=\sum_{a=0}^{g-1}\chi(a)u^{-a}T^a/((T/u)^g-1)\in O_{\chi,u}[[T-1]]$.

Now, we recall some of the basic properties of the p-adic Γ-transform.

If O is the ring of integers of a finite extension k of $\boldsymbol{Q}_p$, we denote by Λ_O the ring of O-valued measures on $\boldsymbol{Z}_p$. If $\alpha\in\Lambda_O$, we put

$$\hat{\alpha}(T)=\sum_{n=0}^{\infty}\Big(\int_{\boldsymbol{Z}_p}\binom{x}{n}d\alpha(x)\Big)(T-1)^n\in O[[T-1]].$$

The map $\alpha\rightarrow\hat{\alpha}(T)$ gives an isomorphism of Λ_O with $O[[T-1]]$. If $a\in\boldsymbol{Z}_p^{\times}$, we write $\alpha\circ a$ for the measure defined by $\alpha\circ a(X)=\alpha(aX)$ for any compact and open subset X of $\boldsymbol{Z}_p$. Then, $\widehat{(\alpha\circ a)}(T)=\hat{\alpha}(T^{a^{-1}})$, where we put $T^x=\sum_{n=0}^{\infty}\binom{x}{n}(T-1)^n$ for $x\in\boldsymbol{Z}_p$.

We define the Γ-transform Γ_α: $\boldsymbol{Z}_p\rightarrow O$ of $\alpha\in\Lambda_O$ by

$$\Gamma_\alpha(s)=\int_{\boldsymbol{Z}_p^{\times}}\langle x\rangle^s d\alpha(x).$$

We fix a topological generator u_0 of the multiplicative group $1+2p\boldsymbol{Z}_p$. If $x\in\boldsymbol{Z}_p^{\times}$, let $l(x)\in\boldsymbol{Z}_p$ be such that $\langle x\rangle=u_0^{l(x)}$. We put

$$F_\alpha(T)=\sum_{n=0}^{\infty}\Big(\int_{\boldsymbol{Z}_p^{\times}}\binom{l(x)}{n}d\alpha(x)\Big)(T-1)^n\in O[[T-1]].$$

Then,

$$\Gamma_\alpha(s)=F_\alpha(u_0^s).$$

For any $\alpha\in\Lambda_O$, we denote by $\tilde{\alpha}$ the measure on $\boldsymbol{Z}_p$ obtained by restricting α to $\boldsymbol{Z}_p^{\times}$ and extending by 0. For any $g(T)\in O[[T-1]]$, we put $\tilde{g}(T)=g(T)-(1/p)\sum_{\zeta^p=1}g(\zeta T)\in O[[T-1]]$. Then, we have

$$(\hat{\tilde{\alpha}})(T)=(\tilde{\hat{\alpha}})(T).$$

We also note that $F_\alpha(T)=F_{\tilde{\alpha}}(T)$.

For any $g(T)\in O[[T-1]]$ and any Dirichlet character ν with conductor p^n, we put

$$g_\nu(T)=1/\tau(\nu^{-1},\zeta_{p^n})\sum_{a=1}^{p^n}\nu^{-1}(a)g(\zeta_{p^n}^a T),$$

where ζ_{p^n} is a primitive p^n-th root of unity and $\tau(\nu^{-1},\zeta_{p^n})=\sum_{a=1}^{p^n}\nu^{-1}(a)\zeta_{p^n}^a$. It is easy to see that $g_\nu(T)$ is independent of the choice of ζ_{p^n}. Let O_ν' denote the ring of integers of the field $k(\zeta_{p^n})$, which contains the values of ν. Then, $g_\nu(T)\in O_\nu'[[T-1]]$. For any $\alpha\in O$ and any $n\in\bar{\boldsymbol{N}}$, we have

$$F_\alpha(u_0^n)=(T\cdot d/dT)^n((\hat{\alpha}))_{\omega^{-n}}(T)|_{T=1}=(d/dz)^n((\hat{\alpha}))_{\omega^{-n}}(e^z)|_{z=0}. \tag{2.3}$$

For any $\alpha\in\Lambda_O$ and any Dirichlet character φ of the second kind for p, we denote by α_φ the measure in $\Lambda_{O_\varphi'}$ satisfying $(\widehat{\alpha_\varphi})(T)=(\hat{\alpha})_\varphi(T)\in O_\nu'[[T-1]]$. Then, we have

$$F_{\alpha_\varphi}(T)=F_\alpha(\varphi(u_0)T). \tag{2.4}$$

In what follows, we put $\Lambda_{\chi,u}=\Lambda_{o_{\chi,u}}$. Let $\alpha_{\chi,u}\in\Lambda_{\chi,u}$ be the measure satisfying $\widehat{\alpha_{\chi,u}}(T)=R_{\chi,u}(T)$ and put

$$F_{\chi,u}(T)=F_{\alpha_{\chi,u}}(T)\,(=F_{\widetilde{\alpha_{\chi,u}}}(T)).$$

LEMMA 2.2. (1) *If $g\in\boldsymbol{N}$ is a common multiple of f_χ and p, then*

$$\widetilde{R_{\chi,u}}(T)=\sum_{\substack{a=0\\(a,p)=1}}^{g-1}\frac{\chi(a)u^{g-a}T^a}{T^g-u^g}=R_{\chi,u}(T)-\chi(p)R_{\chi,u^p}(T^p).$$

(2) *If ν is a Dirichlet character with conductor a power of p, then*

$$(\widetilde{R_{\chi,u}})_\nu(T)=\widetilde{R_{\chi\nu,u}}(T).$$

PROOF. If $g\in\boldsymbol{N}$ is divisible by f_χ and p, then

$$\sum_{\zeta^p=1}R_{\chi,u}(\zeta T)=\sum_{\zeta^p=1}\sum_{a=0}^{g-1}\frac{\chi(a)u^{g-a}\zeta^aT^a}{T^g-u^g}=p\sum_{\substack{a=0\\p\mid a}}^{g-1}\frac{\chi(a)u^{g-a}T^a}{T^g-u^g}.$$

Considering the special case $g=f_\chi p$, we see that $\sum_{\zeta^p=1}R_{\chi,u}(\zeta T)=p\chi(p)R_{\chi,u^p}(T^p)$. Hence, we obtain the assertion of (1).

As for (2), the assertion is obvious if $\nu=1$. Suppose that $\nu\neq1$ and $f_\nu=p^n$, and put $g=f_\chi p^n$. Then, a direct calculation shows

$$\begin{aligned}(\widetilde{R_{\chi,u}})_\nu(T)&=1/\tau(\nu^{-1},\zeta_{p^n})\sum_{a=1}^{p^n}\sum_{\substack{b=0\\(b,p)=1}}^{g-1}\frac{\nu^{-1}(a)\chi(b)u^{g-b}\zeta_{p^n}^{ab}T^b}{T^g-u^g}\\&=\sum_{\substack{b=0\\(b,p)=1}}^{g-1}\frac{\chi(b)\nu(b)u^{g-b}T^b}{T^g-u^g}=R_{\chi\nu,u}(T)-(1/p)\sum_{\zeta^p=1}R(\zeta T).\end{aligned}$$

Hence, we deduce our assertion.

In what follows, we put $\chi_n=\chi\omega^{-n}$ for each $n\in\bar{\boldsymbol{N}}$.

PROPOSITION 2.3. *For each $n\in\bar{\boldsymbol{N}}$, let f_n denote the conductor of χ_n. Then, we have*

$$F_{\chi,u}(u_0^n)=\frac{u}{1-u^{f_n}}H_{\chi_n}^n(u)-\frac{\chi_n(p)p^nu^p}{1-u^{pf_n}}H_{\chi_n}^n(u^p).$$

PROOF. From (2.3) and the definition of $F_{\chi,u}(T)$, we have

$$F_{\chi,u}(u_0^n)=(d/dz)^n\widehat{(\widetilde{\alpha_{\chi,u}})_{\omega^{-n}}}(e^z)|_{z=0}.$$

Since $\widehat{(\widetilde{\alpha_{\chi,u}})}(T)=\widetilde{(\widehat{\alpha_{\chi,u}})}(T)=\widetilde{R_{\chi,u}}(T)$, Lemma 2.2 shows that

$$\widehat{(\widetilde{\alpha_{\chi,u}})_{\omega^{-n}}}(T)=R_{\chi_n,u}(T)-\chi_n(p)R_{\chi_n,u^p}(T^p).$$

Hence, the assertion follows from the definition of the generalized Euler numbers (1.1).

PROPOSITION 2.4. *Let φ be a Dirichlet character of the second kind for p.*

Then,

$$F_{\chi\varphi,u}(T)=F_{\chi,u}(\varphi(u_0)T).$$

PROOF. Lemma 2.2 shows that $\widetilde{\alpha_{\chi\varphi,u}}=(\widetilde{\alpha_{\chi,u}})_\varphi$. Hence, the assertion follows from (2.4).

REMARK. From Proposition 2.3 and Theorem 1 of [**10**], we see that the function $l_p(u, s, \chi)$ in [**10**] is equal to $F_{\chi,u}(u_0^{-s})$.

Let π be a prime of $O_{\chi,u}$. For any $x\in \boldsymbol{C}_p-\{0\}$, let $\mathrm{ord}_\pi(x)\in\boldsymbol{Q}$ be such that $|x|=|\pi|^{\mathrm{ord}_\pi(x)}$. For any power series $f(T)=\sum_{n=0}^\infty a_n(T-1)^n\neq0$ with $a_n\in \boldsymbol{C}_p$ and $|a_n|\leqq1$ for all $n\in\bar{\boldsymbol{N}}$, we define its μ-invariant by $\mu(f(T))=\inf\{\mathrm{ord}_\pi(a_n)\,|\,n\in\bar{\boldsymbol{N}},\ a_n\neq0\}$. We put further $\mathrm{ord}_\pi(0)=\mu(0)=\infty$. Note that if $f(T)$ is a polynomial such that $f(T)=\sum_{n=0}^m b_nT^n$, then $\mu(f(T))=\min\{\mathrm{ord}_\pi(b_n)\,|\,0\leqq n\leqq m\}$. Put $\mu_{\chi,u}=\mu(F_{\chi,u}(T))$. Then, we have the following

THEOREM 2.5. *Suppose that u satisfies the condition* (2.1). *If $|u|>1$, then $\mu_{\chi,u}=-\mathrm{ord}_\pi(u)$. If $|u|<1$, then $\mu_{\chi,u}=\mathrm{ord}_\pi(u)$. If $|u|=1$ and if χ is even, then $\mu_{\chi,u}=\mathrm{ord}_\pi(1+u)$. Otherwise, $\mu_{\chi,u}=0$.*

PROOF. Theorem 1 of [7] states that $\mu_{\chi,u}=\mu(\widetilde{R_{\chi,u}}(T)+\widetilde{R_{\chi,u}}(T^{-1}))$. Put $g=f_\chi p$. Then, a direct calculation shows

$$\begin{aligned}(2.5)\quad \widetilde{R_{\chi,u}}(T)+\widetilde{R_{\chi,u}}(T^{-1}) &= -\sum_{\substack{a=0\\(a,g)=1}}^{g-1}\frac{\chi(a)u^a(u^{2(g-a)}-\chi(-1))(T^a+T^{-a})}{(T^g-u^g)(T^{-g}-u^g)}\\ &= \sum_{\substack{a=0\\(a,g)=1}}^{g-1}\frac{\chi(a)u^a(u^{2(g-a)}-\chi(-1))(T^{g+a}+T^{g-a})}{(T^g-u^g)(T^gu^g-1)}.\end{aligned}$$

If $|u|>1$, then

$$\mu_{\chi,u}=\mu\Big(\sum_{\substack{a=0\\(a,g)=1}}^{g-1}\frac{\chi(a)u^{a-2g}(u^{2(g-a)}-\chi(-1))(T^{g+a}+T^{g-a})}{((T/u)^g-1)(T^g-(1/u)^g)}\Big).$$

Since $\mu(((T/u)^g-1)(T^g-(1/u)^g))=0$, we have

$$\mu_{\chi,u}=\min\{\mathrm{ord}_\pi(u^{a-2g}(u^{2(g-a)}-\chi(-1)))\,|\,1\leqq a<g,\ (a,g)=1\}=-\mathrm{ord}_\pi(u).$$

Next, suppose that $|u|\leqq1$. Then $\mu((T^g-u^g)(T^gu^g-1))=0$, hence we have

$$\mu_{\chi,u}=\min\{\mathrm{ord}_\pi(u^a(u^{2(g-a)}-\chi(-1)))\,|\,1\leqq a<g,\ (a,g)=1\}.$$

If $|u|<1$, then $\mu_{\chi,u}=\mathrm{ord}_\pi(u)$. If $|u|=1$ and if χ is even, then

$$\mu_{\chi,u}=\min\{\mathrm{ord}_\pi(u^{2a}-1)\,|\,1\leqq a<g,\ (a,g)=1\}=\mathrm{ord}_\pi(u^2-1).$$

Moreover, (2.1) implies $|u-1|=1$ if $|u|=1$. Hence, $\mu_{\chi,u}=\mathrm{ord}_\pi(1+u)$. If $|u|=1$ and if χ is odd, then we have

$$\mu_{\chi,u}=\min\{\mathrm{ord}_\pi(u^{2a}+1)\,|\,1\leqq a<g,\ (a,g)=1\}.$$

If $2|g$, then $4|f_\chi$ or $p=2$, and so, $4|f_\chi p^2$. Then, (2.1) implies $|1-u^4|=1$ and we deduce $|1+u^2|=1$. Thus, we obtain $\mu_{\chi,u}=0$. If $2\nmid g$, then both u^2+1 and u^4+1 belong to the set $\{u^{2a}+1 \mid 1\leqq a<g, (a, g)=1\}$. If $|u^2+1|=1$, then we immediately have $\mu_{\chi,u}=0$. If $|u^2+1|<1$, then $|u+1|=|(u^2+1)+(u-1)|=1$, hence, $|u^4+1|=|u^2(u+1)(u-1)+(u^2+1)|=1$. Thus, we obtain $\mu_{\chi,u}=0$.

§ 3. Analytic property of the function $F_{\chi,u}(z)$.

Let $F_{\chi,u}(T)\in O_{\chi,u}[[T-1]]$ be as in the previous section. We put $D=\{z\in \boldsymbol{C}_p \mid |z-1|<1\}$. Then, $F_{\chi,u}(z)$ is an analytic function on D. Here and throughout this section, "analytic" means "Krasner analytic" ([4]). By Theorem 2.5, $F_{\chi,u}(T)=0$ holds if and only if $u=0$ or both $\chi(-1)=1$ and $u=-1$ hold. In this section, by a method in [8], we prove the following

THEOREM 3.1. *If $u\neq 0$ and if either $\chi(-1)=-1$ or $u\neq -1$ holds, then $F_{\chi,u}(z)$ has no analytic continuation to any quasi-connected subset of $\boldsymbol{C}_p$ properly containing D.*

PROOF. Let $R_{\chi,u}(T)$ and $\alpha_{\chi,u}$ be as in the previous section, and put $\widetilde{R_{\chi,u}}^+(T)=\widetilde{R_{\chi,u}}(T)+\widetilde{R_{\chi,u}}(T^{-1})$ and $\widetilde{\alpha_{\chi,u}}^+=\widetilde{\alpha_{\chi,u}}+\widetilde{\alpha_{\chi,u}}\circ(-1)$. Then, $\widetilde{\alpha_{\chi,u}}^+$ is an even measure supported on $\boldsymbol{Z}_p^\times$ and we have $\widehat{(\alpha_{\chi,u})}^+(T)=\widetilde{R_{\chi,u}}^+(T)$ and $(1/2)\Gamma_{\widetilde{\alpha_{\chi,u}}^+}(s)=\Gamma_{\alpha_{\chi,u}}(s)=F_{\chi,u}(u_0^s)$. Now, we apply Theorem 1 or 2 of [8].

Case 1. $|u|=1$. Put $F=O_{\chi,u}/\pi O_{\chi,u}$. We first consider the case $\mu_{\chi,u}=0$. Suppose that the assertion of Theorem 3.1 does not hold. Then, it follows from the first paragraph of Section 3 of [8] that $F_{\chi,u}(T) \bmod \pi \in F(T)$, and Theorem 1 of [8] shows that $T^n\widetilde{R_{\chi,u}}^+(T) \bmod \pi \in F[T]$ for a sufficiently large $n\in \boldsymbol{N}$. Putting $g=f_\chi p$, it follows from (2.5) that $(T^g-u^g)(T^g u^g-1) \bmod \pi$ divides $\sum_{a=0, (a,g)=1}^{g-1} \chi(a)u^a(u^{2(g-a)}-\chi(-1))(T^{g+a}-T^{g-a}) \bmod \pi$ in $F[T]$. Considering the degrees of these polynomials, we deduce $|u^{2a}-\chi(-1)|<1$ for any a with $1\leqq a<g$ and $(a, g)=1$. This contradicts to the fact that $|u^{2a}-\chi(-1)|=1$ for some a with $1\leqq a<g$ and $(a, g)=1$, as is known from the proof of Theorem 2.5. Hence, Theorem 3.1 must hold in the case $\mu_{\chi,u}=0$.

Next, if $\mu_{\chi,u}>0$, then Theorem 2.5 shows that $\chi(-1)=1$ and that $\mu_{\chi,u}=\mathrm{ord}_\pi(1+u)$. Applying the above argument to $(1+u)^{-1}F_{\chi,u}(T)$ and $(1+u)^{-1}\widetilde{R_{\chi,u}}^+(T)$ instead of $F_{\chi,u}(T)$ and $\widetilde{R_{\chi,u}}^+(T)$, we obtain the assertion of Theorem 3.1.

Case 2. $|u|>1$. In this case, putting $g=f_\chi p$, we have

$$\widetilde{R_{\chi,u}}(T)=\sum_{\substack{a=0\\(a,g)=1}}^{g-1}\frac{\chi(a)u^{-a}T^a}{(T/u)^g-1}=-\sum_{\substack{a=0\\(a,g)=1}}^{g-1}\chi(a)u^{-a}T^a\sum_{m=0}^{\infty}(u^{-1}T)^{gm}$$
$$=-\sum_{\substack{m=0\\(m,g)=1}}^{\infty}\chi(m)u^{-m}T^m,$$

and so,

$$(3.1)\qquad \widetilde{R_{\chi,u}}^{+}(T)=-\sum_{\substack{m=0\\(m,g)=1}}^{\infty}\chi(m)u^{-m}(T^{m}+T^{-m}).$$

In order to apply Theorem 2 of [8], we first prove the following

LEMMA 3.2. *Let O be the ring of integers of a finite extension of $\boldsymbol{Q}_p$, and let $\{a_m\}$ and $\{b_m\}$ be sequences of O such that $\lim_{m\to\infty}a_m=\lim_{m\to\infty}b_m=0$. If both of the elements $\sum_{m=0}^{\infty}a_m(T^m+T^{-m})$ and $\sum_{m=0}^{\infty}b_m(T^m+T^{-m})$ in $O[[T-1]]$ are equal, then we have $a_m=b_m$ for all $m\in\bar{\boldsymbol{N}}$.*

PROOF. Put $f(T)=\sum_{m=0}^{\infty}a_m(T^m+T^{-m})$. It is sufficient to show that if $f(T)=0$, then we have $a_m=0$ for all $m\in\bar{\boldsymbol{N}}$.

If $c\in\boldsymbol{Z}_p$, let δ_c denote the Dirac measure of mass 1 supported at c. Then, $\widehat{\delta_c}(T)=T^c$. Let $\alpha_f\in\Lambda_O$ be the measure satisfying $\widehat{\alpha_f}(T)=f(T)$. Then, for any $n\in\boldsymbol{N}$ and any integer l, we have $\alpha_f(l+p^n\boldsymbol{Z}_p)=\sum_{\substack{m=0\\m\equiv\pm l(\mathrm{mod}\ p^n)}}^{\infty}a_m$. Now, assume that $a_{m_0}\neq 0$ for some $m_0\in\bar{\boldsymbol{N}}$. Since $\lim_{m\to\infty}a_m=0$, there is an integer $m_1\in\boldsymbol{N}$ such that $|a_m|<|a_{m_0}|$ for any $m>m_1$. Choose an integer $n\in\boldsymbol{N}$ with $p^n-m_0>m_1$. Then, we have $\alpha_f(m_0+p^n\boldsymbol{Z}_p)\neq 0$. In fact, if $m_0\neq 0$, then $|\alpha_f(m_0+p^n\boldsymbol{Z}_p)|=|a_{m_0}+\sum_{m=1}^{\infty}(a_{m_0+mp^n}+a_{-m_0+mp^n})|=|a_{m_0}|\neq 0$, and if $m_0=0$, then $|\alpha_f(m_0+p^n\boldsymbol{Z}_p)|=|2(a_0+\sum_{m=1}^{\infty}a_{mp^n})|=|2a_0|\neq 0$. Hence, we have $\alpha_f\neq 0$ if $f(T)\neq 0$. Thus, we conclude that if $f(T)=0$, then $a_m=0$ for all $m\in\bar{\boldsymbol{N}}$. This completes the proof.

Let V be the torsion subgroup of $\boldsymbol{Z}_p^{\times}$. Lemma 1 of [8] states that there is a rational number $r\geqq 1$ such that

$$Vu_0^{\boldsymbol{Z}}\cap\boldsymbol{Q}^{\times}=\{1,-1\}r^{\boldsymbol{Z}}.$$

Now, we continue to prove Theorem 3.1 in the case $|u|>1$. From (3.1), Lemma 3.2 and Theorem 2 of [8], it is sufficient to show that there exists an integer $n\in\boldsymbol{N}$ prime to g which does not belong to $r^{\boldsymbol{Z}}$. Indeed, for any $k\in\boldsymbol{N}$, the integer $kg+1$ is prime to g, but not all of the integers of this form belong to $r^{\boldsymbol{Z}}$.

Case 3. $|u|<1$. From Lemma 2.2, a direct calculation shows that $\widetilde{R_{\chi,u^{-1}}}(T)=-\chi(-1)\widetilde{R_{\chi,u}}(T^{-1})$, namely $\widetilde{\alpha_{\chi,u^{-1}}}=-\chi(-1)\widetilde{\alpha_{\chi,u}}$. Hence, the case $|u|<1$ is reduced to Case 2.

§4. p-adic valuation of the generalized Euler numbers.

Let $S=\{l_1,\cdots,l_t\}$ be a finite set of prime numbers distinct from p. Let χ be a Dirichlet character with conductor f_χ and we assume, in this section, that $u\in\bar{\boldsymbol{Q}}$ satisfies

$$|1-u^{f\chi l_1^{N_1}\cdots l_t^{N_t}p^N}|\geqq 1 \tag{4.1}$$

for all $N_1, \cdots, N_t, N\in\bar{N}$.

Let Ψ be the set of Dirichlet characters of the second kind for S, namely Dirichlet characters of the form $\prod_{i=1}^t\psi_i$, where ψ_i is a Dirichlet character of the second kind for l_i. We denote by Φ the set of Dirichlet characters of the second kind for p.

In this section, we prove the following theorem, which is an analogous result on Euler numbers to that on Bernoulli numbers described in the last paragraph of Section 3 of [3] or in Theorem 4.1 (1)(2) of [9].

THEOREM 4.1. *If the condition* (4.1) *is satisfied, then, for almost all* $\psi\in\Psi$, *we have*

$$\operatorname{ord}_\pi\Big(\frac{u}{1-u^{f_{\chi_n\psi\varphi}}}H^n_{\chi_n\psi\varphi}(u)\Big)=\mu_{\chi,u} \tag{4.2}$$

for all $\varphi\in\Phi$ *and all* $n\in\bar{N}$. *(Here and throughout this section, "almost all" means "all but finitely many".) In particular, if* $|u|\neq 1$, *then,* (4.2) *holds for all* $\psi\in\Psi$, *all* $\varphi\in\Phi$ *and all* $n\in\bar{N}$, *except for the case that* $n=0$, $\chi\psi\varphi=1$ *and* $|u|>1$, *in which case, we have* $\operatorname{ord}_\pi((u/(1-u))H^0(u))=0$.

PROOF. Fix $\psi\in\Psi$, $\varphi\in\Phi$ and $n\in\bar{N}$ arbitrarily, and denote the conductor of $\chi_n\psi\varphi$ simply by f.

In the case $|u|\neq 1$ and $n\geqq 1$, we see from Lemma 1 of [10] and Theorem 2.5 that, for any sufficiently large $N\in N$,

$$\begin{aligned}\frac{u}{1-u^f}H^n_{\chi_n\psi\varphi}(u)&\equiv\sum_{a=1}^{fp^N-1}\chi_n\psi\varphi(a)a^n\frac{u^{fp^N-a}}{1-u^{fp^N}}\pmod{\pi^{\mu_{\chi,u}+1}O_{\chi\psi\varphi,u}}\\&\equiv\sum_{\substack{a=1\\(a,p)=1}}^{fp^N-1}\chi_n\psi\varphi(a)a^n\frac{u^{fp^N-a}}{1-u^{fp^N}}\pmod{\pi^{\mu_{\chi,u}+1}O_{\chi\psi\varphi,u}}.\end{aligned}$$

(Note that, though the case $p=2$ is excluded in [10], Lemma 1 of it is valid also in this case.) Taking account of Theorem 2.5 again, we see

$$\Big|\sum_{\substack{a=1\\(a,p)=1}}^{fp^N-1}\chi_n\psi\varphi(a)a^n\frac{u^{fp^N-a}}{1-u^{fp^N}}\Big|=|\pi|^{\mu_{\chi,u}}.$$

Hence, we obtain (4.2).

Next, we consider the case $|u|\neq 1$ and $n=0$. Since $H^0_{\chi\psi\varphi}(u)=\sum_{a=0}^{f-1}\chi\psi\varphi(a)u^{f-a-1}$, it is easy to deduce the required assertion.

In the case $|u|=1$, we use a method in [9]. For that purpose, we first introduce notations and propositions without assuming $|u|=1$.

We put $Z_S=\prod_{l\in S}Z_l$ and $l_S=\prod_{l\in S}l$. For each $m\in N$, let μ_m denote the group of m-th root of unity in $\bar{Q}$ and put $\mu_S=\bigcup_{n=0}^\infty\mu_{l_S^n}$. Put $k=Q_p(\chi, u)$ and $k_S=k(\mu_S)$.

For any k_S-valued measure ν on $\boldsymbol{Z}_S$, its Fourier transform $\hat{\nu}: \mu_S \to k_S$ is defined by $\hat{\nu}(\zeta)=\int_{\boldsymbol{Z}_S} \zeta^x d\nu(x)$. If there exists $R(T) \in k_S(T)$ such that $\hat{\nu}(\zeta)=R(\zeta)$ holds for almost all $\zeta \in \mu_S$, we call ν a rational function measure and $R(T)$ the associated rational function of ν. Any rational function in $k_S(T)$ can occur as the associated rational function of a certain k_S-valued rational function measure on $\boldsymbol{Z}_S$ ([**9**], § 2).

Let $\nu_{\chi,u}$ be a rational function measure on $\boldsymbol{Z}_S$ whose associated rational function is $R_{\chi,u}(T)$. We assume that, in the case $|u| \neq 1$, $R_{\chi,u}(\zeta)=\hat{\nu}_{\chi,u}(\zeta)$ holds for all $\zeta \in \mu_S$. For any $\psi \in \Psi$, regarding ψ as a character of $\boldsymbol{Z}_S^\times$, we put

$$\Gamma_{\nu_{\chi,u}}(\psi) = \int_{\boldsymbol{Z}_S^\times} \psi(x) d\nu_{\chi,u}(x).$$

For any $S' \subset S$, we put

$$\Psi_{S'} = \{\psi \in \Psi \mid \ l \mid f_{\chi\psi} \text{ if } l \in S' \text{ and } l \nmid f_{\chi\psi} \text{ if } l \in S-S'\}.$$

PROPOSITION 4.2. *For almost all $\psi \in \Psi_S$, we have*

$$\Gamma_{\nu_{\chi,u}}(\psi) = \frac{u}{1-u^{f_{\chi\psi}}} H^0_{\chi\psi}(u). \tag{4.3}$$

If $|u| \neq 1$, then (4.3) *holds for all $\psi \in \Psi_S$.*

PROOF. For any $\psi \in \Psi$, choose $n_{1,\psi}, \cdots, n_{t,\psi} \in \boldsymbol{N}$ such that the conductor f_ψ of ψ divides $F_\psi = \prod_{i=1}^t l_i^{n_{i,\psi}}$. As in the proof of Proposition 2.2 of [**9**], we have

$$\Gamma_{\nu_{\chi,u}}(\psi) = \sum_{\zeta \in \mu_{F_\psi}} a_\psi(\zeta) \hat{\nu}_{\chi,u}(\zeta),$$

where $a_\psi(\zeta) = 1/F_\psi \sum_{\substack{x=0 \\ (x,l_S)=1}}^{F_\psi - 1} \psi(x) \zeta^{-x}$. If ζ is a F_ψ-th root of unity whose order is not divisible by f_ψ, then $a_\psi(\zeta)=0$. Since $\nu_{\chi,u}$ is a rational function measure with the associated rational function $R_{\chi,u}(T)$, there is an integer $L \in \boldsymbol{N}$, divisible only by the primes in S, such that $\hat{\nu}_{\chi,u}(\zeta) = R_{\chi,u}(\zeta)$ for all $\zeta \in \mu_S - \mu_L$. Now, there are only finitely many ψ with $f_\chi \mid L$. If $f_\chi \nmid L$, then we have $\zeta^L \neq 1$ for any F_ψ-th root of unity ζ whose order is divisible by f_ψ. Hence,

$$\Gamma_{\nu_{\chi,u}}(\psi) = \sum_{\zeta \in \mu_{F_\psi}} a_\psi(\zeta) R_{\chi,u}(\zeta T)|_{T=1}. \tag{4.4}$$

Put $g = f_\chi F_\psi$. Then, a direct calculation shows

$$\sum_{\zeta \in \mu_{F_\psi}} a_\psi(\zeta) R_{\chi,u}(\zeta T) = \sum_{\substack{a=1 \\ (a,l_S)=1}}^{g-1} \frac{\chi\psi(a) u^{g-a} T^a}{T^g - u^g}. \tag{4.5}$$

In the case $\psi \in \Psi_S$, we have

$$\sum_{\zeta \in \mu_{F_\psi}} a_\psi(\zeta) R_{\chi,u}(\zeta T) = R_{\chi\psi,u}(T). \tag{4.6}$$

Since $R_{\chi\phi,u}(1)=(u/(1-u^{f_{\chi\phi}}))H^{0}_{\chi\phi}(u)$, we see that (4.3) holds for any $\phi\in\Psi_S$ with $f_\phi \nmid L$.

If $|u|\neq 1$, (4.4) holds for all $\phi\in\Psi$, and so, if $\phi\in\Psi_S$, we see from (4.6) that (4.3) holds. This completes the proof.

PROPOSITION 4.3. *For almost all $\phi\in\Psi$ we have*

$$\text{(4.7)} \qquad \mathrm{ord}_\pi(\Gamma_{\nu_{\chi,u}}(\phi)) = \mu_{\chi,u}.$$

If $|u|\neq 1$, then, (4.7) holds for all $\phi\in\Psi$.

PROOF. Let $\nu^*_{\chi,u}$ be the measure on $\boldsymbol{Z}_S$ obtained by restricting $\nu_{\chi,u}$ to $\boldsymbol{Z}_S^\times$ and extending by 0. Let $\phi(x)$ be the characteristic function on $\boldsymbol{Z}_S^\times$. Then,

$$\phi(x) = \sum_{\zeta\in\mu_{l_S}} a_\zeta \zeta^x,$$

where $a_\zeta=l_S^{-1}\sum_{\substack{a=1\\(a,l_S)=1}}^{l_S}\zeta^{-a}$. Put

$$R^*_{\chi,u}(T) = \sum_{\zeta\in\mu_{l_S}} a_\zeta R_{\chi,u}(\zeta T).$$

Then, $\nu^*_{\chi,u}$ is a rational function measure whose associated rational function is $R^*_{\chi,u}(T)$. Let $\nu^*_{\chi,u}\circ(-1)$ be the measure on $\boldsymbol{Z}_S$ defined by $\nu^*_{\chi,u}\circ(-1)(X)=\nu^*_{\chi,u}(-X)$ for any compact and open subset X of $\boldsymbol{Z}_S$. Then, $\nu^*_{\chi,u}\circ(-1)$ is also a rational function measure whose associated rational function is $R^*_{\chi,u}(T^{-1})$. Put $g=f_\chi l_S$. Then, a direct calculation shows

$$R^*_{\chi,u}(T) = \sum_{\substack{a=0\\(a,l_S)=1}}^{g-1} \frac{\chi(a)u^{g-a}T^a}{T^g-u^g}$$

and in the same way as we have shown Proposition 2.1, we see that $R^*_{\chi,u}(T)\in O_{\chi,u}[[T-1]]$. From the remark after Theorem 3.1 of [**9**], we have, for almost all $\phi\in\Psi$,

$$\mathrm{ord}_\pi(\Gamma_{\nu_{\chi,u}}(\phi)) = \mu(R^*_{\chi,u}(T)+R^*_{\chi,u}(T^{-1})).$$

Further, in the same way in the proof of Theorem 2.5, we see

$$\mu(R^*_{\chi,u}(T)+R^*_{\chi,u}(T^{-1})) = \mu_{\chi,u}.$$

Hence, (4.7) holds for almost all $\phi\in\Psi$.

In the case $|u|\neq 1$, (4.4) holds for all $\phi\in\Psi$. Hence, we see from (4.5) and Theorem 2.5 that (4.7) holds for all $\phi\in\Psi$.

PROPOSITION 4.4. *Suppose that $\mu_{\chi,u}\neq\infty$. Then, for almost all $\phi\in\Psi_S$,*

$$\text{(4.8)} \qquad F_{\chi\phi,u}(T)/\pi^{\mu_{\chi,u}} \in (O_{\chi,\phi,u}[[T-1]])^\times.$$

If $|u|\neq 1$, then (4.8) holds for all $\phi\in\Psi_S$.

PROOF. If p divides f_χ, then p divides $f_{\chi\phi}$, and Proposition 2.3 shows

$F_{\chi\psi,u}(1)=(u/(1-u^{f_{\chi\psi}}))H^{0}_{\chi\psi}(u)$. Hence, by Propositions 4.2 and 4.3, we have

$$\text{(4.9)} \qquad \mathrm{ord}_{\pi}(F_{\chi\psi,u}(1))=\mu_{\chi,u}$$

for almost all $\psi\in\Psi_S$. Now, Theorem 2.5 shows $\mu_{\chi,u}=\mu_{\chi\psi,u}$. Hence, (4.8) holds for all $\psi\in\Psi_S$ satisfying (4.9).

Unless p divides f_χ, then p divides $f_{\chi\varphi}$ for any $\varphi\in\Phi-\{1\}$. For any $\psi\in\Psi$, we have $\psi\in\Psi_S$ if and only if $l_S|f_{\chi\varphi\psi}$. Hence, for almost all $\psi\in\Psi_S$, we have

$$\text{(4.10)} \qquad \mathrm{ord}_{\pi}(F_{\chi\varphi\psi,u}(1))=\mu_{\chi\varphi,u}\,.$$

From Proposition 2.4 and Theorem 2.5, we see that (4.10) is equivalent to (4.8). Therefore, (4.8) holds for almost all $\psi\in\Psi_S$.

Finally, if $|u|\neq 1$, we see from Propositions 4.2 and 4.3 that, for all $\psi\in\Psi_S$, (4.9) or (4.10) holds. Hence, (4.8) holds for all $\psi\in\Psi_S$. This completes the proof.

PROOF OF THEOREM 4.1 IN THE CASE $|u|=1$. Again, we fix $\psi\in\Psi$, $\varphi\in\Phi$ and $n\in\bar{\boldsymbol{N}}$, and write f for the conductor of $\chi_n\psi\varphi$.

We first assume that $\psi\in\Psi_S$. Suppose that $n\geqq 1$ in the first place.

Case 1. $\mu_{\chi,u}=0$. Let $\mathfrak{p}$ be the prime ideal of $O_{\chi,\psi,\varphi,u}$. From Lemma 1 of [**10**], we have, for any sufficiently large $N\in\boldsymbol{N}$,

$$\begin{aligned}\frac{u}{1-u^f}H^{n}_{\chi_n\psi\varphi}(u)&\equiv\sum_{a=0}^{fp^N-1}\chi_n\psi\varphi(a)a^n\frac{u^{fp^N-a}}{1-u^{fp^N}} \pmod{\mathfrak{p}}\\&\equiv\sum_{\substack{a=0\\(a,p)=1}}^{fp^N-1}\chi\psi(a)\frac{u^{fp^N-a}}{1-u^{fp^N}} \pmod{\mathfrak{p}}.\end{aligned}$$

Hence, we have $(u/(1-u^f))H^{n}_{\chi_n\psi\varphi}(u)\equiv\widetilde{R_{\chi\psi,u}}(1)\equiv F_{\chi\psi,u}(1) \pmod{\mathfrak{p}}$. Therefore, if ψ satisfies (4.8), we have $|(u/(1-u^f))H^{n}_{\chi_n\psi\varphi}(u)|=|F_{\chi\psi,u}(1)|=1$ and (4.2) holds.

Case 2. $\mu_{\chi,u}>0$. In this case, Theorem 2.5 states that χ is even and that $\mu_{\chi,u}=\mathrm{ord}_{\pi}(1+u)$, and (4.1) implies $2\nmid fp$. Choose $N_0\in\boldsymbol{N}$ arbitrarily. Then, Lemma 1 of [**10**] shows that, for any sufficiently large $N\in\boldsymbol{N}$, we have

$$\begin{aligned}\frac{u}{1-u^f}H^{n}_{\chi_n\psi\varphi}(u)&\equiv\sum_{a=1}^{fp^N-1}\chi_n\psi\varphi(a)a^n\frac{u^{fp^N-a}}{1-u^{fp^N}} \pmod{\mathfrak{p}^{N_0}}\\&\equiv\sum_{a=1}^{(fp^N-1)/2}\chi_n\psi\varphi(a)\frac{a^n u^{fp^N-a}+\omega^{-n}(-1)(fp^N-a)^n u^a}{1-u^{fp^N}} \pmod{\mathfrak{p}^{N_0}}\\&\equiv\sum_{a=1}^{(fp^N-1)/2}\chi_n\psi\varphi(a)a^n\frac{u^a(u^{fp^N-2a}+1)}{1-u^{fp^N}} \pmod{\mathfrak{p}^{N_0}}.\end{aligned}$$

Since $2\nmid fp^N$, we have $|u^{fp^N-2a}+1|\leqq|u+1|$. If $u=-1$, then $H^{n}_{\chi_n\psi\varphi}(u)=0$, and (4.2) holds. If $u\neq-1$, choose the above N_0 such that $\mathfrak{p}^{N_0}\subset(1+u)\mathfrak{p}$. Then,

$$\frac{u}{(1-u^f)(1+u)}H^n_{\chi_n\psi\varphi}(u)\equiv\sum_{a=1}^{(fp^N-1)/2}\chi_n\psi\varphi(a)a^n\frac{u^a(u^{fp^N-2a}+1)}{(1-u^{fp^N})(1+u)}\pmod{\mathfrak{p}}$$
$$\equiv\sum_{\substack{a=1\\(a,p)=1}}^{(fp^N-1)/2}\chi\psi(a)\frac{u^a(u^{fp^N-2a}+1)}{(1-u^{fp^N})(1+u)}\pmod{\mathfrak{p}}$$
$$\equiv\frac{1}{1+u}\sum_{\substack{a=1\\(a,p)=1}}^{fp^N-1}\chi\psi(a)\frac{u^{fp^N-a}}{1-u^{fp^N}}\pmod{\mathfrak{p}}.$$

Thus, we have shown $(u/(1-u^f)(1+u))H^n_{\chi_n\psi\varphi}(u)\equiv(1/(1+u))F_{\chi\psi,u}(1)\pmod{\mathfrak{p}}$. Therefore, if ψ satisfies (4.8), (4.2) holds.

Next, we consider the case $n=0$. If $p|f_{\chi_\varphi}$, then Propositions 2.3 and 2.4 show that $F_{\chi\psi,u}(\varphi(u_0))=(u/(1-u^f))H^0_{\chi\psi\varphi}(u)$. In the case $\mu_{\chi,u}=\infty$, we have $H^0_{\chi\psi\varphi}(u)=0$, and (4.2) holds. In the case $\mu_{\chi,u}\neq\infty$, if ψ satisfies (4.8), then (4.2) holds. If $p\nmid f_{\chi_\varphi}$, then Theorem 2.5 and Propositions 4.2 and 4.3 show that $\mathrm{ord}_\pi((u/(1-u^f))H^0_{\chi\psi'\varphi}(u))=\mu_{\chi_\varphi,u}=\mu_{\chi,u}$ holds for almost all $\psi'\in\Psi_S$. Further, we note that such φ is unique in Φ if it exists.

Thus, we conclude that for almost all $\psi\in\Psi_S$, (4.2) holds for all $\varphi\in\Phi$ and all $n\in\bar{N}$.

Finally, let S' be a proper subset of S. If S' is empty, then $\Psi_{S'}$ is empty or a set consisting of only one element. If $\Psi_{S'}$ is an infinite set, then each element ψ of $\Psi_{S'}$ is expressed as $\psi=\psi_{S'}\psi'_{S'}$, where $\psi_{S'}$ (resp. $\psi'_{S'}$) is a Dirichlet character of the second kind for S' (resp. $S-S'$), and it is easy to see that $\psi'_{S'}$ depends only on the set S'. Hence, putting $\Psi'_{S'}=\{\psi\psi'^{-1}_{S'}|\psi\in\Psi_{S'}\}$ and applying the above argument to S', $\Psi'_{S'}$ and $\chi\psi'_{S'}$ instead of S, Ψ_S and χ, we deduce that, for almost all $\psi\in\Psi_{S'}$, (4.2) holds for all $\varphi\in\Phi$ and all $n\in\bar{N}$.

Thus, we complete the proof of Theorem 4.1.

References

[1] J. Diamond, On the values of p-adic L-functions at positive integers, Acta Arith., **35** (1979), 223-237.

[2] B. Ferrero and R. Greenberg, On the behavior of p-adic L-functions at $s=0$, Invent. Math., **50** (1978), 91-102.

[3] E. Friedman, Ideal class groups in basic $\boldsymbol{Z}_{p_1}\times\cdots\times\boldsymbol{Z}_{p_s}$-extensions of abelian number fields, Invent. Math., **65** (1982), 425-440.

[4] M. Krasner, Rapport sur le prolongement analytiques dans les corps values complets par la methode des élements analytiques quasi-connexes, Bull. Soc. Math. France, Mémoire, **39-40** (1974), 131-254.

[5] Y. Morita, On the radius of convergence of the p-adic L-function, Nagoya Math. J., **75** (1979), 177-193.

[6] K. Shiratani and S. Yamamoto, On a p-adic interpolation function for the Euler numbers and its derivatives, Mem. Fac. Sci. Kyushu Univ., **39** (1985), 113-125.

[7] W. Sinnott, On the μ-invariant of the Γ-transform of a rational function, Invent. Math., **75** (1984), 273-282.

[8] W. Sinnott, On the power series attached to p-adic L-functions, J. Reine Angew. Math., **382** (1987), 22-34.

[9] W. Sinnott, Γ-transforms of rational function measures on $\boldsymbol{Z}_S$, Invent. Math., **89** (1987), 139-157.

[10] H. Tsumura, On a p-adic interpolation of the generalized Euler numbers and its applications, Tokyo J. Math., **10** (1987), 281-293.

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