J. Math. Soc. Japan
Vol. 41, No. 4, 1989

Ergodicity for an infinite particle system in R^d of jump type with hard core interaction

By Hideki TANEMURA

(Received Nov. 30, 1987)
(Revised Aug. 24, 1988)

§ 0. Introduction.

In this paper, we consider a system of infinitely many hard balls with the same diameter r moving discontinuously in R^d. We denote the configuration space of hard balls by $\mathcal{X}$:

$$\mathcal{X}=\{\xi=\{x_i\}: |x_i-x_j|\geqq r, i\neq j\}, \tag{1}$$

the position of a ball being represented by its center.

The ball of the system moves by random jump under the hard core condition. The system is completely specified by the measure $c(x, dy, \xi)$ which gives the rate of the movement of the ball at the position x to the position y when the entire configuration is ξ. We shall consider the case where $c(x, dy, \xi)$ is given by

$$c(x, dy, \xi)=\exp\{-\sum_{z\in\xi\setminus\{x\}}\Phi(|y-z|)\}p(|x-y|)dy,$$

where $p(\cdot)$ is a non-negative function on $[0, \infty)$ such that $\int_{R^d}p(|x|)dx=1$ and $p(\cdot)>0$ on $[0, 2h)$ for some $h>0$ and Φ is a measurable function on $[0, \infty)$ satisfying the following properties:

(Φ.1) $\quad\Phi(\cdot)\geqq -C\quad$ for some constant $C\geqq 0$;

(Φ.2) $\quad\Phi(a)=\infty\quad$ if and only if $a\in[0, r)$;

(Φ.3) $\quad\Phi(\cdot)=0\quad$ on $[\hat{r}, \infty)$ for some constant $\hat{r}\geqq r$.

Φ is regarded as a hard pair potential which is rotation invariant, stable and of finite range.

In the previous paper [8] we studied the case where $r=\hat{r}$.

We construct the Markov process ξ_t which describes our system. This process has the Gibbs state μ associated with the potential Φ as a reversible measure.

The purpose of this paper is to show the ergodicity of the stationary Markov process in the case where the density of balls is sufficiently small. It is important to develop a topological argument on the configuration space to prove the ergodicity. If the configuration space were connected, any configuration would be attained from any other configuration by moving balls continuously. However, due to the hard core potential, our configuration space is not connected: it has more than one connected component. We prove that every pair of different connected components, Γ_1 and Γ_2 say, are jointed to each other with a chain of connected components $\Gamma_1=\Lambda_1, \Lambda_2, \cdots, \Lambda_k=\Gamma_2$ in which Λ_i and Λ_{i+1} are in "h-communication" (the precise definition is given in § 2) for all i with $0\leqq i\leqq k-1$. This means that any configuration is attained from any other configuration by means of a finite number of jumps of magnitude equal or less than h. This argument constitutes the most crucial part for our proof of ergodicity.

In § 1, we construct the Markov process describing our model by using Liggett's theorem [5] and show that a Gibbs state is a reversible measure for the process. In § 2, using a lemma about the topological property of the configuration space, we prove the ergodicity of the process. The proof of the lemma is given in § 3. In § 4, we study the central limit theorem of the tagged particle of our process. Kipnis-Varadhan [3] proved the central limit theorem for a tagged particle of simple exclusion process on a lattice. Using the ergodicity of the process and the same technique as employed in [3], we can discuss the central limit theorem, except for the non-degeneracy problem of the covariance matrix that remains open.

§ 1. Construction of a Markov process.

Let $\mathcal{M}$ be the set of all countable subsets ξ of $\boldsymbol{R}^d$ satisfying $\#(\xi\cap K)<\infty$ for any compact subset $K\subset\boldsymbol{R}^d$. We regard $\xi\in\mathcal{M}$ as a non-negative integer valued Radon measure on $\boldsymbol{R}^d$: $\xi(\cdot)=\sum\delta_{x_i}(\cdot)$ and accordingly equip $\mathcal{M}$ with the vague topology. The space $\mathcal{X}$ defined by (1) is then a compact subset of $\mathcal{M}$.

For any $\xi\in\mathcal{M}$ and $y\in\boldsymbol{R}^d$ we denote $\xi\cup\{y\}$ by $\xi\cdot y$. Also we denote $\xi\setminus\{z\}$ by $\xi\setminus z$ if $z\in\xi$. For any Borel subset K of $\boldsymbol{R}^d$ we denote by ξ_K the restriction of ξ to K; ξ_K is a Radon measure on K; however, we regard it as an element of $\mathcal{M}$ in a natural way.

Let $C(\mathcal{X})$ be the space of all real valued continuous functions on $\mathcal{X}$ with supremum norm $\|\ \|_\infty$. We denote by $C_0(\mathcal{X})$ the set of functions of $C(\mathcal{X})$ each of which depends only on the configurations in some compact set K:

$$C_0(\mathcal{X})=\{f\in C(\mathcal{X}):\ f(\xi)=f(\xi_K) \text{ for some compact set } K\}.$$

It is easily seen that $C_0(\mathcal{X})$ is dense in $C(\mathcal{X})$. We define σ-fields $\mathcal{B}(\mathcal{X})$ and $\mathcal{B}_K(\mathcal{X})$ by

$$\mathcal{B}(\mathcal{X})=\sigma(N_A: A\in\mathcal{B}(\boldsymbol{R}^d)),$$

and

$$\mathcal{B}_K(\mathcal{X})=\sigma(N_A: A\in\mathcal{B}(\boldsymbol{R}^d),\ A\subset K),$$

where $N_A(\xi)$ is the number of particles of ξ in A. The σ-field $\mathcal{B}(\mathcal{X})$ coincides with the topological Borel field on $\mathcal{X}$.

Before defining a linear operator on $C(\mathcal{X})$ which generates a Markov process, we define the function $\chi(\underline{x}|\xi)$ for $\underline{x}=\{x_1, x_2, \cdots, x_n\}$ and $\xi\in\mathcal{M}$ by

$$\chi(\underline{x}|\xi)=\exp\Big\{-\frac{1}{2}\sum_{\substack{y,z\in\underline{x}\\ y\neq z}}\Phi(|y-z|)-\sum_{y\in\underline{x},z\in\xi}\Phi(|y-z|)\Big\},$$

where Φ is a given measurable function on $[0, \infty)$ satisfying $(\Phi.1)\sim(\Phi.3)$. Let $p(\cdot)$ be a non-negative function on $[0, \infty)$ satisfying

$$\int_{\boldsymbol{R}^d}dx\,p(|x|)=1, \tag{1.1}$$

$$\int_{\boldsymbol{R}^d}dx|x|^2p(|x|)<\infty, \tag{1.2}$$

$$p(\cdot)>0 \quad \text{on } [0, 2h) \text{ for some } h>0. \tag{1.3}$$

Now, we define a linear operator on $C_0(\mathcal{X})$ by

$$Lf(\xi)=\sum_{x\in\xi}\int_{\boldsymbol{R}^d}\{f(\xi^{x,y})-f(\xi)\}\chi(y|\xi\diagdown x)p(|x-y|)dy,$$

where

$$\xi^{x,y}=\begin{cases}(\xi\diagdown x)\cdot y, & \text{if } x\in\xi,\ y\notin\xi,\\ \xi, & \text{otherwise.}\end{cases}$$

Since L is dissipative and $C_0(\mathcal{X})$ is dense in $C(\mathcal{X})$, L has the smallest closed extension $\bar{L}$. Define bounded operators $L_{j,k}$ on $C(\mathcal{X})$ for $j=(j_1, \cdots, j_d)$, $k=(k_1, \cdots, k_d)\in\boldsymbol{Z}^d$ by

$$L_{j,k}f(\xi)=\begin{cases}p_{j,k}^{-1}\sum_{x\in\xi\cap I_j}\int_{I_k}\{f(\xi^{x,y})-f(\xi)\}\chi(y|\xi\diagdown x)p(|x-y|)dy, & \text{if } p_{j,k}>0,\\ 0, & \text{otherwise,}\end{cases}$$

where

$$p_{j,k}=\int_{(-1,1]^d}dx\,p(|j-k-x|),$$

and

$$I_j=\prod_{m=1}^{d}\Big(j_m-\frac{1}{2}, j_m+\frac{1}{2}\Big].$$

Then, $L_{j,k}$ satisfies the following conditions:

$$Lf(\xi)=\sum_{j,k\in\boldsymbol{Z}^d}p_{j,k}L_{j,k}f(\xi), \qquad \text{for } f\in C_0(\mathcal{X}); \tag{1.4}$$

(1.5) there is a constant M_1 such that for all $j, k\in \boldsymbol{Z}^d$

$$\|L_{j,k}f\|_\infty \leqq M_1\|f\|_\infty, \qquad \text{for } f\in C(\mathscr{X});$$

(1.6) $$L_{j,k}L_{j',k'} = L_{j',k'}L_{j,k},$$

for all $j, k, j', k'\in \boldsymbol{Z}^d$ with $d(I_j\cup I_k, I_{j'}\cup I_{k'})\geqq \hat{r}$, where $d(A_1, A_2)=\inf\{|x-y|: x\in A_1, y\in A_2\}$, for $A_1, A_2\subset \boldsymbol{R}^d$.

From (1.5) and (1.6) it is easily seen that there is a constant M_2 such that for all $j, k\in \boldsymbol{Z}^d$

$$\sum_{j',k'\in Z^d} p_{j',k'}\left(\sup_{f\in C(\mathscr{X})} \frac{\|L_{j,k}L_{j',k'}f-L_{j',k'}L_{j,k}f\|_\infty}{\|L_{j,k}f\|_\infty+\|L_{j',k'}f\|_\infty}\right)\leqq M_2.$$

Therefore, a slight modification of Liggett's theorem [5] implies that $(\bar{L}, \mathscr{D}(\bar{L}))$ generates a unique strongly continuous Markov semigroup T_t on $C(\mathscr{X})$.

Since T_t is a Feller semigroup, for each initial distribution μ there exists a Markov process (ξ_t, P_μ), with semigroup T_t, which is right continuous and has left limits.

For any compact subset $K\subset \boldsymbol{R}^d$, we denote by $\mathscr{M}(K)$ and $\mathscr{M}(K, n)$ the set of all finite subsets of K and the set of all subsets of K having n points respectively.

An alternative description of $\mathscr{M}(K, n)$ is given by

(1.7) $$\mathscr{M}(K, n) = \begin{cases} \{\emptyset\}, & \text{if } n=0, \\ (K^n)'/S_n, & \text{if } n\geqq 1, \end{cases}$$

where $(K^n)'=\{(x,\cdots,x_n)\in K^n: x_i\neq x_j \text{ if } i\neq j\}$ and S_n is the symmetric group of degree n. By means of the factorization (1.7) we introduce a measure $\lambda_{K,z}$ on $\mathscr{M}(K)=\cup_{n=0}^\infty \mathscr{M}(K, n)$ (direct sum) such that

$$\lambda_{K,z}(\emptyset)=1,$$

and

$$\lambda_{K,z}(A)=\frac{z^n}{n!}\int_{\tilde{A}} dx_1dx_2\cdots dx_n \qquad \text{for a Borel set } A \text{ in } \mathscr{M}(K, n),\ n\geqq 1,$$

where $z\geqq 0$ and $\tilde{A}$ is a preimage of A by the factor mapping in the factorization (1.7). The integral of a measurable function f on $\mathscr{M}(K)$ with respect to this measure is denoted by $\int f(\underline{x})d^z\underline{x}$.

Now, we are going to define a Gibbs state. We will see that this Gibbs state is a reversible measure for our process ξ_t.

DEFINITION 1.1 ([2]). A probability measure μ on $\mathscr{X}$ is called a (grand canonical) Gibbs state with activity $z\geqq 0$, if for any compact subset K of $\boldsymbol{R}^d$, the restriction of μ on $\mathscr{B}_K(\mathscr{X})$ is absolutely continuous with respect to $d^z\underline{x}$ and the density is given by

$$\sigma_K(\underline{x}) = \int_{\eta(K)=0} \mu(d\eta)\chi(\underline{x} \mid \eta).$$

The activity z is the parameter which controls the density of particles. Denote by $\mathcal{G}(z)$ the set of all Gibbs states with activity $z \geqq 0$. This set $\mathcal{G}(z)$ is convex and compact with respect to the topology of weak convergence, so any element of $\mathcal{G}(z)$ is represented by the extremal points of $\mathcal{G}(z)$. We denote by $\mathrm{ex}\,\mathcal{G}(z)$ the set of all extremal points of $\mathcal{G}(z)$.

REMARK 1.1 ([4]). There exists a positive constant z_0 such that for any $z \in (0, z_0)$ $\mathcal{G}(z)$ consists of one element, i.e., $\#\mathcal{G}(z)=1$.

REMARK 1.2 ([7]). Let $\mathcal{G}(z)$ consist of one element μ. Then the following limit exists,

$$\rho(z) = \lim_{K \uparrow \boldsymbol{R}^d} \frac{1}{|K|} \int_{\mathcal{X}} \xi(K)\mu(d\xi)$$

when K tends to $\boldsymbol{R}^d$ in the sense of Van Hove. We call $\rho(z)$ the particle density of μ. Note that $\rho(z) \to 0$ as $z \to 0$. Also the following fact is known. For any $\varepsilon > 0$,

$$\mu\left(\left|\frac{\xi(K)}{|K|} - \rho(z)\right| \geqq \varepsilon\right) \longrightarrow 0 \qquad \text{as } K \uparrow \boldsymbol{R}^d .$$

LEMMA 1.2. *If μ is a Gibbs state, then μ is a reversible measure for ξ_t, i.e.*

$$\langle T_t f, g\rangle_\mu = \langle f, T_t g\rangle_\mu \qquad \textit{for any } f, g \in C(\mathcal{X}),\ t \geqq 0,$$

where $\langle \cdot, \cdot \rangle_\mu$ is the L^2 inner product with respect to μ.

PROOF. Let $j, k \in \boldsymbol{Z}^d$ and $f, g \in C_0(\mathcal{X})$. Since $p_{j,k} = p_{k,j}$, we have

$$\begin{aligned}
(1.8) \qquad & p_{j,k}\{\langle (L_{j,k}+L_{k,j})f, g\rangle_\mu - \langle f, (L_{j,k}+L_{k,j})g\rangle_\mu\} \\
&= \int_{\mathcal{X}} \mu(d\xi) \sum_{x \in \xi \cap I_j} \int_{I_k} dy f(\xi^{x,y}) g(\xi) \chi(y \mid \xi \diagdown x) p(|x-y|) \\
&\quad + \int_{\mathcal{X}} \mu(d\xi) \sum_{x \in \xi \cap I_k} \int_{I_j} dy f(\xi^{x,y}) g(\xi) \chi(y \mid \xi \diagdown x) p(|x-y|) \\
&\quad - \int_{\mathcal{X}} \mu(d\xi) \sum_{x \in \xi \cap I_j} \int_{I_k} dy f(\xi) g(\xi^{x,y}) \chi(y \mid \xi \diagdown x) p(|x-y|) \\
&\quad - \int_{\mathcal{X}} \mu(d\xi) \sum_{x \in \xi \cap I_k} \int_{I_j} dy f(\xi) g(\xi^{x,y}) \chi(y \mid \xi \diagdown x) p(|x-y|).
\end{aligned}$$

Choose a compact set K satisfying $f(\xi_K) = f(\xi)$, $g(\xi_K) = g(\xi)$ and $B_{\hat{r}}(I_j \cup I_k) \subset K$, where $B_l(A)$ is an open l-neighborhood of $A \subset \boldsymbol{R}^d$. Then, by the definition of a Gibbs state we have

$$\int_{\mathscr{X}}\mu(d\xi)\sum_{x\in\xi\cap I_j}\int_{I_k}dyf(\xi^{x,y})g(\xi)\chi(y\,|\,\xi\setminus x)p(|x-y|)$$

$$=\sum_{n=0}^{\infty}\frac{z^n}{n!}\int_{K^n}dx_1\cdots dx_n\sigma_K(x_1\cdots x_n)\sum_{i=1}^{n}\mathbf{1}_{I_j}(x_i)$$

$$\times\int_{I_k}dyf(x_1\cdots x_n\cdot y\setminus x_i)g(x_1\cdots x_n)\chi(y\,|\,x_1\cdots x_n\setminus x_i)p(|x_i-y|)$$

$$=\sum_{n=1}^{\infty}\frac{z^n}{(n-1)!}\int_{K^{n-1}}dx_1\cdots dx_{n-1}\int_{I_j}dx_n\int_{I_k}dy\sigma_K(x_1\cdots x_n)$$

$$\times f(x_1\cdots x_{n-1}\cdot y)g(x_1\cdots x_n)\chi(y\,|\,x_1\cdots x_{n-1})p(|x_n-y|).$$

Let us note that $\sigma_K(x_1\cdots x_n)=\sigma_K(x_1\cdots x_{n-1})\chi(x_n\,|\,x_1\cdots x_{n-1})$ for $x_n\in I_j\cup I_k$. Using this relation, we have

$$\text{(1.9)}\qquad \int_{\mathscr{X}}\mu(d\xi)\sum_{x\in\xi\cap I_j}\int_{I_k}dyf(\xi^{x,y})g(\xi)\chi(y\,|\,\xi\setminus x)p(|x-y|)$$

$$=\sum_{n=1}^{\infty}\frac{z^n}{(n-1)!}\int_{K^{n-1}}dx_1\cdots dx_{n-1}\int_{I_j}dx_n\int_{I_k}dy\sigma_K(x_1\cdots x_{n-1})$$

$$\times f(x_1\cdots x_{n-1}\cdot y)g(x_1\cdots x_n)\chi(y\,|\,x_1\cdots x_{n-1})\chi(x_n\,|\,x_1\cdots x_{n-1})p(|x_n-y|)$$

$$=\int_{\mathscr{X}}\mu(d\xi)\sum_{x\in\xi\cap I_k}\int_{I_j}dyf(\xi)g(\xi^{x,y})\chi(y\,|\,\xi\setminus x)p(|x-y|).$$

Hence, from (1.8) and (1.9) we have

$$\langle(L_{j,k}+L_{k,j})f,\,g\rangle_\mu=\langle f,\,(L_{j,k}+L_{k,j})g\rangle_\mu\,,\quad\text{for } f,\,g\in C_0(\mathscr{X}),\ j,\,k\in\boldsymbol{Z}^d.$$

Therefore, $\langle Lf,\,g\rangle_\mu=\langle f,\,Lg\rangle_\mu$ for $f,\,g\in C_0(\mathscr{X})$ and consequently

$$\langle\bar{L}f,\,g\rangle_\mu=\langle f,\,\bar{L}g\rangle_\mu\,,\qquad\text{for } f,\,g\in\mathscr{D}(\bar{L}).$$

Since $\bar{L}$ is the generator for T_t, Lemma 1.2 is proved.

§ 2. Ergodicity of $(\xi_t,\,P_\mu)$.

The primary purpose of this section is to prove the following theorem.

THEOREM 2.1. *If $z>0$ is sufficiently small and if $\#\mathscr{G}(z)=1$ and $\mu\in\mathscr{G}(z)$, then the Markov process $(\xi_t,\,P_\mu)$ is ergodic.*

Let $\hat{T}_t$ be the strongly continuous semigroup on $L^2(\mathscr{X},\,\mu)$ associated with ξ_t and $\hat{L}$ be the generator of $\hat{T}_t$. To prove the ergodicity of the process $(\xi_t,\,P_\mu)$ it is enough to show the following condition (C.1).

(C.1) *If $f\in L_2(\mathscr{X},\,\mu)$ satisfies $\hat{T}_tf=f$ for any $t\geqq0$ then f is constant.*

We shall prove that the condition (C.1) holds for $(\xi_t,\,P_\mu)$, $\mu\in\mathscr{G}(z)$, if $z>0$

is sufficiently small and $\#\mathcal{G}(z)=1$. The following result ([1]) about Gibbs states is very useful for the proof of (C.1). Let $\mathcal{E}_\infty(\mathcal{X})$ be the σ-field defined by

$$\mathcal{E}_\infty(\mathcal{X}) = \mu\text{-completion of } \bigcap_{K:\text{compact}} \sigma(N_K, \mathcal{B}_{K^c}(\mathcal{X})),$$

where $\sigma(N_K, \mathcal{B}_{K^c}(\mathcal{X}))$ is the σ-field generated by $N_K(\xi)$ and $\mathcal{B}_{K^c}(\mathcal{X})$. If $\mu\in \operatorname{ex}\mathcal{G}(z)$ then $\mu(\Lambda)=0$ or 1 for any $\Lambda\in\mathcal{E}_\infty(\mathcal{X})$.

The following Lemma 2.1 follows from this result and Remark 1.2 immediately. For $m\in\boldsymbol{N}$ we put

$$K_m = \{x\in\boldsymbol{R}^d:\ |x|\leqq\sqrt{d}2^m r\}.$$

For $m, n\in\boldsymbol{N}$ and $\xi\in\mathcal{X}$, we denote by $\Lambda(K_m, n, \xi)$ the interior of the configuration space $\{\underline{x}\in\mathcal{M}(K_m, n): \chi(\underline{x}\,|\,\xi_{K_m^c})\neq 0\}$, that is,

$$\Lambda(K_m, n, \xi) = \{\{x_i\}_{i=1}^n:\ |x_i|<\sqrt{d}2^m r,\ d(\{x_i\}, \xi_{K_m^c})>r,$$
$$|x_i-x_j|>r,\ 1\leqq i<j\leqq n\}.$$

LEMMA 2.1. *Suppose $\#\mathcal{G}(z)=1$ and let μ be the unique Gibbs state of $\mathcal{G}(z)$. If $f\in L^2(\mathcal{X}, \mu)$ and if f satisfies*

$$\int_{\Lambda(K_m,n,\xi)} d^1\underline{x}\int_{\Lambda(K_m,n,\xi)} d^1\underline{y}\,|f(\underline{x}\cdot\xi_{K_m^c})-f(\underline{y}\cdot\xi_{K_m^c})| = 0, \tag{2.1}$$

for μ-almost all ξ and for all $m, n\in\boldsymbol{N}$ satisfying $n/|K_m|<\rho(z)+\varepsilon$ with some positive constant ε (independent of m, n), then f is constant.

Let us remark that the condition (2.1) implies that the function f is $\mathcal{E}_\infty(\mathcal{X})$-measurable.

By virtue of Lemma 2.1 the condition (C.1) follows from the following condition (C.2) with a positive constant $c>\rho(z)$:

(C.2) *If $f\in L^2(\mathcal{X}, \mu)$ satisfies $\hat{T}_t f=f$ for any $t\geqq 0$, then (2.1) holds for μ-almost all $\xi\in\mathcal{X}$ and for all $(m, n)\in\boldsymbol{N}\times\boldsymbol{N}$ satisfying $n/|K_m|<c$.*

Since $\rho(z)\downarrow 0$ as $z\downarrow 0$, Theorem 2.1 follows if we show that there exists a constant $c>0$ for which (C.2) holds. (In fact c will be chosen so as to depend only on h, r and d.)

To show (C.2) it is neccessary to develop a topological argument on the configuration space. We introduce a notion about the configuration space which is similar to (but weaker than) the connectedness.

DEFINITION 2.1. i) Two configurations $\xi\in\mathcal{X}$ and $\eta\in\mathcal{X}$ are said to be in h-communication (denoted by $\xi\leftarrow h\rightarrow\eta$), if there exist $x\in\xi$ and $y\in\eta$ such that $|x-y|\leqq h$ and $\xi^{x,y}=\eta$.

ii) Two subsets Γ and Λ of $\mathcal{X}$ are said to be in h-communication (denoted by $\Gamma\leftarrow h\rightarrow\Lambda$), if there exist $\xi\in\Gamma$ and $\eta\in\Lambda$ such that $\xi\leftarrow h\rightarrow\eta$.

iii) A family of subsets $\{\Lambda(j)\}_{j\in J}$ of $\mathcal{X}$ is said to be in h-communication, if for any j', $j''\in J$, there exists a sequence $\{j_1, j_2, \cdots, j_q\}$ such that

$$\Lambda(j')\leftarrow h\rightarrow\Lambda(j_1)\leftarrow h\rightarrow\Lambda(j_2)\leftarrow h\rightarrow\cdots\leftarrow h\rightarrow\Lambda(j_q)\leftarrow h\rightarrow\Lambda(j'').$$

Let $\{\Lambda_j\}_{j\in J}$ be the set of all connected components of $\Lambda(K_m, n, \xi)$. Then, our key lemma is the following.

LEMMA 2.2. *There exists a positive constant $c(r, h)$ such that for all $\xi\in\mathcal{X}$ and all $m, n\in\boldsymbol{N}$ satisfying $n/|K_m|<c(r, h)$ $\{\Lambda_j\}_{j\in J}$ are in h-communication.*

The proof of Lemma 2.2 is given in § 3.

We prove Theorem 2.1 by showing (C.2) for $c=c(r, h)$.

From the definition of L and Lemma 1.2, for $g\in C_0(\mathcal{X})$ we have

$$-2\langle Lg, g\rangle_\mu=\int_{\mathcal{X}}\mu(d\xi)\sum_{x\in\xi}\int_{R^d}\{g(\xi^{x,y})-g(\xi)\}^2\chi(y|\xi\setminus x)p(|x-y|)dy.$$

Since f is $\hat{T}_t$-invariant for any $t\geqq 0$, we see that $\hat{L}f=0$. Since $\hat{L}$ is the smallest closed extension of L, we have

$$\int_{\mathcal{X}}\mu(d\xi)\sum_{x\in\xi}\int_{R^d}\{f(\xi^{x,y})-f(\xi)\}^2\chi(y|\xi\setminus x)p(|x-y|)dy=0.$$

From the definition of Gibbs state and (1.3), we have

$$\int_{\Lambda(K_m,n,\xi)}d^1\underline{x}\sum_{x\in\underline{x}}\int_{B_{2h}(x)}dy|f(\underline{x}^{x,y}\cdot\xi_{K_m^c})-f(\underline{x}\cdot\xi_{K_m^c})|\mathbf{1}_{\Lambda(K_m,n,\xi)}(\underline{x}^{x,y})=0 \tag{2.2}$$

for all $m, n\in\boldsymbol{N}$ and almost all $\xi\in\mathcal{X}$. We define a non-negative function H on $\mathcal{M}(K_m, n)\times\mathcal{M}(K_m, n)$ by

$$H(\underline{x}, \underline{y})=\sum\sum\prod_{i=1}^{n}\mathbf{1}_{\Lambda(K_m,n,\xi)}(x_1\cdots x_{i-1}\cdot y_i\cdots y_n)\mathbf{1}_{B_{2h}(x_i)}(y_i).$$

The above sums run over all ordered n-tuples $(x_1, \cdots, x_n)$ and $(y_1, \cdots, y_n)$ such that $\{x_1, \cdots, x_n\}=\underline{x}$ and $\{y_1, \cdots, y_n\}=\underline{y}$. Employing this function and using (2.2) repeatedly

$$\int_{\Lambda(K_m,n,\xi)}d^1\underline{x}\int_{\Lambda(K_m,n,\xi)}d^1\underline{y}|f(\underline{x}\cdot\xi_{K_m^c})-f(\underline{y}\cdot\xi_{K_m^c})|H(\underline{x}, \underline{y})=0, \tag{2.3}$$

for all $m, n\in\boldsymbol{N}$ and for almost all $\xi\in\mathcal{X}$.

Since $\Lambda(K_m, n, \xi)$ is open, for any $\underline{x}\in\Lambda(K_m, n, \xi)$, we can choose $\varepsilon(\underline{x})\in(0, h)$ such that $I(\underline{x}, \varepsilon(\underline{x}))\subset\Lambda(\xi, K_m, n)$, where $I(\underline{x}, \varepsilon)=\{\{y_1, \cdots, y_n\}\in\mathcal{M}(K_m, n)$: $|y_i-x_i|<\varepsilon\}$. We abbreviate $I(\underline{x}, \varepsilon(\underline{x}))$ to $I(\underline{x})$. If $\underline{x}^1\leftarrow h\rightarrow\underline{x}^2$, then $H\geqq 1$ on $I(\underline{x}^1)\times I(\underline{x}^2)$. From (2.3), we have

$$\int_{I(\underline{x}^1)} d^1\underline{x} \int_{I(\underline{x}^2)} d^1\underline{y} \, |f(\underline{x}\cdot\xi_{K_m^c}) - f(\underline{y}\cdot\xi_{K_m^c})| = 0 . \tag{2.4}$$

Using the triangle inequality and Fubini's theorem, we have (2.4) for any $\underline{x}^1$, $\underline{x}^2 \in \Lambda(K_m, n, \xi)$ such that $\underline{x}^1 = \underline{y}^1 \leftarrow h \rightarrow \underline{y}^2 \leftarrow h \rightarrow \cdots \leftarrow h \rightarrow \underline{y}^k = \underline{x}^2$ for a sequence $\{\underline{y}^j\}_{1 \le j \le k}$ of configurations in $\Lambda(K_m, n, \xi)$. Then, we obtain

$$\int_{\Lambda_j} d^1\underline{x} \int_{\Lambda_{j'}} d^1\underline{y} \, |f(\underline{x}\cdot\xi_{K_m^c}) - f(\underline{y}\cdot\xi_{K_m^c})| = 0 \qquad \text{if } \Lambda_j \leftarrow h \rightarrow \Lambda_{j'} .$$

Therefore, it follows from Lemma 2.2 that the condition (C.2) holds for $c = c(r, h)$. This implies Theorem 2.1.

§ 3. Proof of Lemma 2.2.

First of all, we introduce some notions about configurations. Let B be a convex subset of K_m and k be a non-negative integer. A configuration $\{x_i\}_{i=1}^k$ in B is said to be standard in B if $d(B^c, \{x_i\}_{i=1}^k) > 2r$ and $|x_i - x_j| > 4r$, for $1 \leqq i < j \leqq k$. Note that a standard configuration in B_1 is also standard in B_2 if $B_1 \subset B_2$. Also a configuration $\underline{x} \in \Lambda(K_m, n, \xi)$ is said to be B-standard if $\underline{x} \cap B$ is standard in B. We abbreviate a K_m-standard configuration to a standard configuration.

Let $\underline{x}$ be a configuration. A point x of $\underline{x}$ represents the ball of diameter r and with center x. By the balls of $\underline{x}$ we mean all the balls of diameter r whose centers are points of $\underline{x}$. For a subset of B of $\boldsymbol{R}^d$ *a ball is said to be in* B (*or on* B) if *the center of the ball is in* B. It should be kept in mind that the phrase "a ball (which is represented by a point of configuration) is in B (or on B)" does not mean that the ball, as a set, is included in B.

If the number of balls in B (i. e. those whose centers are in B) is sufficiently large, then there is no B-standard configuration. For each convex set B let $\mathcal{N}(B)$ denote the largest number of balls in B for B-standard configurations, i. e.,

$$\mathcal{N}(B) = \max\{k \geqq 0 : \text{there exists a } B\text{-standard configuration } \underline{x} \text{ with } n_B(\underline{x}) = k\},$$

where $n_B(\underline{x})$ is the number of balls of $\underline{x}$ in B. If $k \leqq \mathcal{N}(B)$, there exists a B-standard configuration $\underline{x}$ with $n_B(\underline{x}) = k$. Since $\mathcal{N}(B_l(b))$ is determined by the radius of $B_l(b)$, we abbreviate $\mathcal{N}(B_l(b))$ to $\mathcal{N}(l)$. It is easily seen that

$$\mathcal{N}(l) \geqq \left(\frac{l - 2r}{2\sqrt{d}\, r}\right)^d . \tag{3.1}$$

For $\underline{x} \in \Lambda(K_m, n, \xi)$, let $\Lambda(\underline{x})$ be the connected component of $\Lambda(K_m, n, \xi)$ containing $\underline{x}$. In the following discussion B will always be a convex subset of K_m.

LEMMA 3.1. *Any standard configuration in B can be attained from any other standard configuration in B by moving balls continuously within B preserving the hard core condition and without being influenced by the boundary condition. To be precise, if $\underline{y}$ and $\underline{z}$ are standard configuration in B such that $n_B(\underline{y})=n_B(\underline{z})$, then*

$$\Lambda(\underline{y}\cdot\underline{w})=\Lambda(\underline{z}\cdot\underline{w}),$$

for any configuration $\underline{w}$ in $K_m \setminus B$.

This lemma will be proved later.

By Lemma 3.1 all standard configuration are contained in one connected component Λ^s of $\Lambda(K_m, n, \xi)$. For the connected components Γ_1 and Γ_2 of $\Lambda(K_m, n, \xi)$ we write $\Gamma_1 \leftrightarrow \Gamma_2$, if there exists a sequence of connected components $\Lambda_1, \Lambda_2, \cdots, \Lambda_k$ of $\Lambda(K_m, n, \xi)$ such that

$$\Gamma_1 \leftarrow h \rightarrow \Lambda_1 \leftarrow h \rightarrow \Lambda_2 \leftarrow h \rightarrow \cdots \leftarrow h \rightarrow \Lambda_k = \Gamma_2 .$$

We introduce several numbers j_0, l_0 and $c(r, h)$ which will be used in the proof of Lemma 2.2. Let j_0 be an integer such that $2^{j_0-1} \leqq r/h \vee 8\sqrt{d} < 2^{j_0}$. Put $l_0 = 2^{j_0} r$ and $c(r, h) = |B_{\sqrt{d}l_0}(0)|^{-1}$. Then, $j_0 \geqq 4$ and $c(r, h)|K_m| = 2^{d(m-j_0)}$.

We also denote the convex hull of $A \subset \boldsymbol{R}^d$ by $[A]$. For a given ordered sequence $B_1, B_2, \cdots, B_k$ of open balls, we define $B[i]$, $2 \leqq i \leqq k$ by $B[i] = [B_1 \cup B_2] \cup [B_2 \cup B_3] \cup \cdots \cup [B_{i-1} \cup B_i]$. See, e.g. Figure 1.

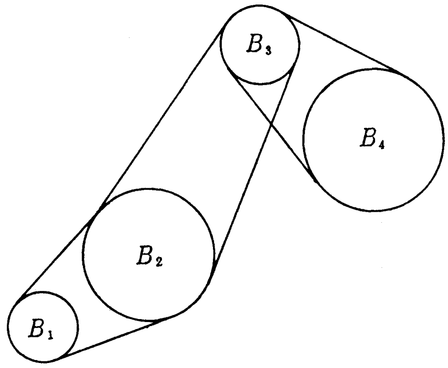

Figure 1. Example of $B[4]$.

To prove Lemma 2.2, it is enough to prove the following lemma.

LEMMA 2.2′. *Let $\xi \in \mathscr{X}$ and m be an integer with $m \geqq j_0$. If $n < 2^{d(m-j_0)}$, then for any $\underline{x} \in \Lambda(K_m, n, \xi)$ there exist a sequence of open balls $B_1, B_2, \cdots, B_k$ and a sequence of configurations $\underline{y}^1, \underline{y}^2, \cdots, \underline{y}^k \in \Lambda(K_m, n, \xi)$ such that for $1 \leqq i \leqq k$*

(i) $\underline{y}^i$ *is B_i-standard,*

(ii) $\underline{y}^i_{B[i]^c} = \underline{x}_{B[i]^c}$,

(iii) $n_{B[i]\setminus B_i}(\underline{y}^i)=0$ i.e. $n_{B[i]}(\underline{y}^i)=n_{B_i}(\underline{y}^i)$,

(iv) $\Lambda(\underline{x})\longleftrightarrow\Lambda(\underline{y}^1)\longleftrightarrow\Lambda(\underline{y}^2)\longleftrightarrow\cdots\longleftrightarrow\Lambda(\underline{y}^k)=\Lambda^s$.

We shall prove this lemma by induction. The following lemma plays an important role for the proof.

LEMMA 3.2. *Let $B_1=B_{l_1}(b_1)$, $B_2=B_{l_2}(b_2)\subset K_m$ for some $l_2\geqq l_1\geqq l_0$, $b_1, b_2\in\boldsymbol{R}^d$. Take a B_1-standard configuration $\underline{y}\in\Lambda(K_m, n, \xi)$. If $n_{[B_1\cup B_2]}(\underline{y})\leqq\mathfrak{N}(l_1-5r)$, then a B_2-standard configuration is obtained from $\underline{y}$ by moving balls in $[B_1\cup B_2]$ into B_2 by means of finitely many jumps of magnitude equal or less than h. To be precise, there exists a B_2-standard configuration $\underline{z}$ such that*

(i) $\Lambda(\underline{z})\longleftrightarrow\Lambda(\underline{y})$,

(ii) $n_{B_2}(\underline{z})=n_{[B_1\cup B_2]}(\underline{z})=n_{[B_1\cup B_2]}(\underline{y})$,

(iii) $\underline{z}_{B_2^c}=\underline{z}_{[B_1\cup B_2]^c}=\underline{y}_{[B_1\cup B_2]^c}$.

PROOF OF LEMMA 2.2′. We construct an increasing sequence $E_0, E_1, \cdots, E_{m-j_0}$ of cubes in K_m satisfying the following condition:

$$n_{E_j}(\underline{x})<2^{dj},\quad \text{for } 0\leqq j\leqq m-j_0. \tag{3.2}$$

First, we put

$$E_{m-j_0}=\{(a_1,\cdots,a_d)\in\boldsymbol{R}^d:\ -2^m r<a_i\leqq 2^m r,\ 1\leqq i\leqq d\}.$$

From the assumption of Lemma 2.2′ we have $n_{E_{m-j_0}}(\underline{x})\leqq n<2^{d(m-j_0)}$. We decompose E_{m-j_0} into the disjoint union of congruent 2^d cubes with edge length $2^m r$. Pick up one of the cubes having the smallest number of balls of $\underline{x}$ and denote it by E_{m-j_0-1}. Then, $n_{E_{m-j_0-1}}(\underline{x})\leqq 2^{d(m-1-j_0)}$. Repeating this procedure, we can construct a sequence $E_0\subset E_1\subset\cdots\subset E_{m-j_0}$ satisfying the condition (3.2).

Let B_i be the open ball inscribed in E_i for $0\leqq i\leqq m-j_0$ and B_{m+1-j_0} be the interior of K_m. The radius of B_i is $2^i l_0$ for $0\leqq i\leqq m-j_0$. Taking (3.1) into account and using (3.2) we have

$$n_{B[i]}(\underline{x})<2^{id}<\left(\frac{2^{j_0+i-1}-7}{2\sqrt{d}}\right)^d\leqq\mathfrak{N}(2^{i-1}l_0-5r),\quad \text{for } 1\leqq i\leqq m+1-j_0. \tag{3.3}$$

From (3.3) $\underline{x}$ is a B_0-standard configuration satisfying $n_{[B_0\cup B_1]}(\underline{x})\leqq\mathfrak{N}(l_0-5r)$. Then, we can apply Lemma 3.2 and see that there exists a B_1-standard configuration $\underline{y}^1$ satisfying

$$\Lambda(\underline{y}^1)\longleftrightarrow\Lambda(\underline{x}),$$

$$n_{B_1}(\underline{y}^1)=n_{[B_0\cup B_1]}(\underline{y}^1)=n_{[B_1\cup B_0]}(\underline{x}), \tag{3.4}$$

$$\underline{y}^1_{B_1^c}=\underline{y}^1_{[B_0\cup B_1]^c}=\underline{x}_{[B_0\cup B_1]^c}. \tag{3.5}$$

From (3.4) and (3.5) we have

$$n_{[B_1\cup B_2]}(\underline{y}^1) = n_{B_1}(\underline{y}^1)+n_{[B_1\cup B_2]\setminus B_1}(\underline{y}^1)$$
$$= n_{[B_0\cup B_1]}(\underline{x})+n_{[B_1\cup B_2]\setminus[B_0\cup B_1]}(\underline{x}) = n_{B[2]}(\underline{x})\leqq \mathcal{N}(2l_0-5r).$$

Then, we can apply Lemma 3.2 to obtain a B_2-standard configuration $\underline{y}^2$ satisfying

$$\Lambda(\underline{y}^2) \longleftrightarrow \Lambda(\underline{y}^1),$$
$$n_{B_2}(\underline{y}^2) = n_{[B_1\cup B_2]}(\underline{y}^2) = n_{[B_1\cup B_2]}(\underline{y}^1),$$
$$\underline{y}^2_{B_2^c} = \underline{y}^2{}_{[B_1\cup B_2]^c} = \underline{y}^1{}_{[B_1\cup B_2]^c}.$$

Repeating this procedure, we construct the sequence of configurations $\underline{y}^1, \underline{y}^2, \cdots, \underline{y}^{m+1-j_0}$. Since $\Lambda(K_m, n, \xi)$ is open, there is no ball on the boundary ∂K_m of K_m, therefore, $\underline{y}^{m+1-j_0}$ is a standard configuration. Now, we have a sequence $\underline{y}^1, \underline{y}^2, \cdots, \underline{y}^{m+1-j_0}$ of configurations satisfying the conditions (i)~(iv) in Lemma 2.2′.

PROOF OF LEMMA 3.1. This lemma is trivial when $\underline{y}=\underline{z}$ or $n_B(\underline{y})=1$. So, we assume that $\underline{y}\neq\underline{z}$ and $n_B(\underline{y})\geqq 2$. Suppose that $y\in\underline{y}$ and $z\in\underline{z}$ satisfy $|y-z|\leqq 2r$. Since $B_{3r}(z)\cap B_r(z')=\emptyset$ for any $z'\in\underline{z}\setminus z$ and $[B_r(y)\cup B_r(z)]\subset B_{3r}(z)$, we have

$$[B_r(y)\cup B_r(z)]\cap B_r(z') = \emptyset \qquad \text{for any } z'\in\underline{z}\setminus z. \tag{3.6}$$

On the other hand, from the convexity of B we have

$$d([B_r(y)\cup B_r(z)],\ B^c) > r.$$

Hence, we can move the ball z to the position y continuously without being influenced by balls with centers $(\underline{z}\setminus z)\cdot\underline{w}$. Therefore, $\Lambda(\underline{z})=\Lambda((\underline{z}\setminus z)\cdot y)$. Suppose that $(y', z')\in\underline{y}\times\underline{z}$ satisfies $|y'-z'|\leqq 2r$ and $(y', z')\neq(y, z)$. Then, from (3.6) $y'\neq y$. In the same way as we showed (3.6), we have

$$[B_r(y')\cup B_r(z')]\cap B_r(y) = \emptyset.$$

Thus, we can move the ball z' to the position y' continuously without being influenced by balls with centers $(\underline{z}\setminus z')\cdot y'\cdot\underline{w}$. Therefore, $\Lambda((\underline{z}\setminus z)\cdot y)=\Lambda((\underline{z}\setminus\{z, z'\})\cdot\{y, y'\})$. Repeating this procedure, we obtain the configuration $\{y_1, \cdots, y_q, z_{q+1}, \cdots, z_k\}$ such that $\{y_1, \cdots, y_q\}\subset\underline{y}$, $\{z_{q+1}, \cdots, z_k\}\subset\underline{z}$,

$$\Lambda(y_1\cdots y_q\cdot z_{q+1}\cdots z_k\cdot\underline{w}) = \Lambda(\underline{z}\cdot\underline{w}), \tag{3.7}$$

and

$$|z'-z''| > 2r, \qquad \text{for all } z', z''\in\underline{y}\cdot\{z_{q+1}, \cdots, z_k\} \text{ with } z'\neq z'', \tag{3.8}$$

where $q=\#\{y\in\underline{y}: d(y, \underline{z})\leqq 2r\}$. If $q=k$, we obtain Lemma 3.1 from (3.7). When $0\leqq q<k$, we write $\underline{y}\setminus\{y_1, \cdots, y_q\}=\{y_{q+1}, \cdots, y_k\}$. Then, from (3.8) for any i with $q+1\leqq i\leqq k$ we can obtain the configuration $\{y_1, \cdots, y_i, z_{i+1}, \cdots, z_k\}$

from $\{y_1, \cdots, y_{i-1}, z_i, \cdots, z_k\}$ by moving the ball z_i to y_i continuously within B and without being influenced by the boundary condition. To be precise,

$$\Lambda(y_1 \cdots y_i \cdot z_{i+1} \cdots z_k \cdot \underline{w}) = \Lambda(y_1 \cdots y_{i-1} \cdot z_i \cdots z_k \cdot \underline{w}), \tag{3.9}$$

for $q+1 \leqq i \leqq k-1$ and

$$\Lambda(\underline{y} \cdot \underline{w}) = \Lambda(y_1 \cdots y_{k-1} \cdot z_k \cdot \underline{w}). \tag{3.10}$$

Combining (3.7), (3.9) and (3.10) we complete the proof.

For the proof of Lemma 3.2, we prepare the following two lemmas.

LEMMA A-1. *Let $B=B_l(b)$ with $l \geqq l_0$. For $x \in \partial B$ put $x(\alpha)=x-\alpha(x-b)/l$, $\alpha \in [0, l]$. Then,*

$$d(B^c \cap B_r(x)^c, x(\alpha)) > r, \qquad \text{if } \alpha > r^2/l.$$

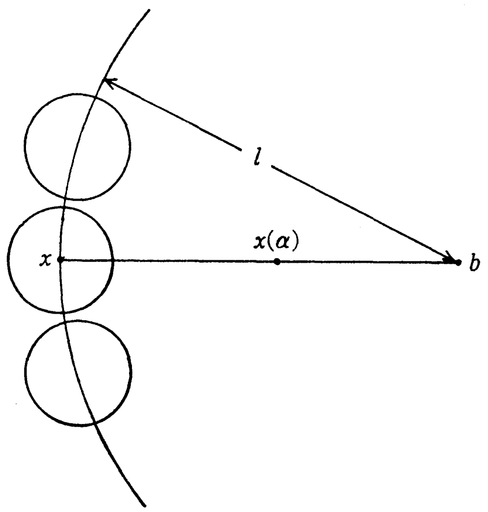

Figure 2.

Lemma A-1 implies that even if balls are arranged closely on ∂B as in Figure 2, we can move the ball at x to $x(\alpha)$ by means of a jump of range α preserving the hard core condition. The proof of this lemma is easy, so we omit the proof.

LEMMA A-2. *Let $B=B_l(b)$ with $l \geqq l_0$. Let $\underline{y}$ and $\underline{z}$ be standard configurations in B with $n_B(\underline{y})=n_B(\underline{z})-1$ and $\underline{w}$ be a configuration in $K_m \setminus B$ with $\underline{w} \cap \partial B \neq \emptyset$ and $\underline{y} \cdot \underline{w} \in \Lambda(K_m, n, \xi)$. If $n_B(\underline{y}) \leqq \mathfrak{N}(l-5r)$,*

$$\Lambda(\underline{y} \cdot \underline{w}) \longleftarrow h \longrightarrow \Lambda(\underline{z} \cdot \underline{w} \setminus x), \qquad \text{for any } x \in \underline{w} \cap \partial B.$$

PROOF. By Lemma 3.1 and the premise $n_B(\underline{y}) \leqq \mathfrak{N}(l-5r)$ we can assume $\underline{y}$ is a standard configuration in $B_{l-5r}(b)$.

For $x \in \underline{w} \cap \partial B$ put $x(\alpha)=x-\alpha(x-b)/l$, $\alpha \in [0, l]$. Since, $l_0 > r^2/h \vee 8\sqrt{d}r$, we have $h \wedge r > r^2/l$. Then, it follows from Lemma A-1 that

$$d(\underline{w} \setminus x,\ x(h\wedge r)) > d(B^c \cap B_r(x)^c,\ x(h\wedge r)) > r\,,$$

so that

$$\underline{y} \cdot \underline{w}^{x,\, x(h\wedge r)} \in \Lambda(K_m,\ n,\ \xi)\,.$$

Since $|x-x(h\wedge r)| \leqq h$, we have

$$\underline{y} \cdot \underline{w} \longleftarrow h \longrightarrow \underline{y} \cdot \underline{w}^{x,\, x(h\wedge r)}\,. \tag{3.11}$$

Since $\underline{y}$ is the standard configuration in $B_{l-5r}(b)$, there is no ball of $\underline{y}$ outside $B_{l-7r}(b)$. Then, we have

$$d(\underline{y} \cdot \underline{w} \setminus x,\ x(\alpha)) > r \vee \alpha\,, \qquad \text{for } \alpha \in [h\wedge r,\ 3r]\,, \tag{3.12}$$

and

$$d(\underline{y},\ x(3r)) > 4r\,. \tag{3.13}$$

The condition (3.12) implies that we can move the ball at $x(h\wedge r)$ to the position $x(3r)$ continuously along the line segment connecting these two points, so that

$$\Lambda(\underline{y} \cdot \underline{w}^{x,\, x(h\wedge r)}) = \Lambda(\underline{y} \cdot \underline{w}^{x,\, x(3r)})\,. \tag{3.14}$$

Also the condition (3.13) implies that the configuration $\underline{y} \cdot x(3r)$ is standard in B, so we have the following relation (3.14) from Lemma 3.1

$$\Lambda(\underline{y} \cdot \underline{w}^{x,\, x(3r)}) = \Lambda(\underline{z} \cdot \underline{w} \setminus x)\,. \tag{3.15}$$

Combining (3.11), (3.14) and (3.15) we complete the proof.

PROOF OF LEMMA 3.2. For $\alpha \in [0, 1]$ put

$$B(\alpha) = B_{l(\alpha)}(b(\alpha))\,,$$

where $b(\alpha) = \alpha b_2 + (1-\alpha) b_1$ and $l(\alpha) = \alpha l_2 + (1-\alpha) l_1$. Since $[B_1 \cup B_2] \setminus B_1 \subset \bigcup_{\alpha \in [0,1]} \partial B(\alpha)$ there exists $\alpha \in [0, 1)$ such that $x \in \partial B(\alpha)$, for any $x \in \underline{y} \cap ([B_1 \cup B_2] \setminus B_1)$. First, we put

$$\alpha_1 = \min\{\alpha \geqq 0:\ d(\underline{y}_{[B_1 \cup B_2] \setminus B_1},\ \partial B(\alpha))=0\},$$

and pick up one of the balls of $\underline{y}_{[B_1 \cap B_2] \setminus B_1}$ on $\partial B(\alpha_1)$ and denote it by x_1. Next, we put

$$\alpha_2 = \min\{\alpha \geqq \alpha_1:\ d(\underline{y}_{[B_1 \cup B_2] \setminus B_1} \setminus \{x_1\},\ \partial B(\alpha))=0\},$$

and pick up one of the balls of $\underline{y}_{[B_1 \cup B_2] \setminus B_1} \setminus \{x_1\}$ on $\partial B(\alpha_2)$ and denote it by x_2. Repeating this procedure, we can take $x_1, x_2, \cdots, x_k$ and $0=\alpha_0 \leqq \alpha_1 \leqq \cdots \leqq \alpha_k \leqq \alpha_{k+1}=1$ such that

$$\{x_1, x_2, \cdots, x_k\} = \underset{\sim}{y}_{[B_1\cup B_2]\setminus B_1},$$
$$\{x_i, x_{i+1}, \cdots, x_k\} \subset [B_1\cup B(\alpha_i)]^c, \qquad 1\leq i\leq k,$$
$$x_i \in \partial B(\alpha_i), \qquad 1\leq i\leq k.$$

Let $\underset{\sim}{y}^i$, $0\leq i\leq k$ and $\underset{\sim}{z}^i$, $1\leq i\leq k+1$ be $B(\alpha_i)$-standard configurations such that

$$\underset{\sim}{y}^i_{B(\alpha_i)^c} = \underset{\sim}{y}^i_{[B_1\cup B(\alpha_i)]^c} = \{x_{i+1}, \cdots, x_k\}\cdot \underset{\sim}{y}_{[B_1\cup B_2]^c},$$
$$\underset{\sim}{z}^i_{B(\alpha_i)^c} = \underset{\sim}{z}^i_{[B_1\cup B(\alpha_i)]^c} = \{x_i, \cdots, x_k\}\cdot \underset{\sim}{y}_{[B_1\cup B_2]^c};$$

and let $\underset{\sim}{y}^0=\underset{\sim}{y}$ and $\underset{\sim}{z}^{k+1}$ be a B_2-standard configuration satisfying (ii) and (iii) in Lemma 3.2. Then, it is enough to observe the following two relations (3.15) and (3.16) to finish the proof of Lemma 3.2.

$$\Lambda(\underset{\sim}{y}^i) = \Lambda(\underset{\sim}{z}^{i+1}), \qquad \text{for } 0\leq i\leq k, \tag{3.15}$$
$$\Lambda(\underset{\sim}{z}^i) \longleftarrow h \longrightarrow \Lambda(\underset{\sim}{y}^i), \qquad \text{for } 1\leq i\leq k. \tag{3.16}$$

Since $\underset{\sim}{y}^i$ and $\underset{\sim}{z}^{i+1}$ are $[B_1\cup B(\alpha_i)]$-standard, (3.15) follows from Lemma 3.1, and (3.16) follows from Lemma A-2 directly.

§ 4. Asymptotics for a tagged particle.

In this section, we study the behavior of a tagged particle in our process. In order to follow the motion of a tagged particle it is convenient to regard the process ξ_t as a Markov process (x_t, η_t) on the locally compact space $\boldsymbol{R}^d\times\mathcal{X}_0$, where

$$\mathcal{X}_0 = \{\eta\in\mathcal{X}: \eta\cap B_r(0)=\emptyset\}.$$

x_t is the position of the tagged particle and η_t is the entire configuration seen from the tagged particle. We can see that η_t is a Markov process whose generator $\bar{\mathcal{L}}$ is the smallest closed extension of the operator given by

$$\mathcal{L}f(\eta) = \int_{\boldsymbol{R}^d}\{f(\tau_{-u}\eta)-f(\eta)\}\chi(u\,|\,\eta)p(|u|)du$$
$$+\sum_{z\in\eta}\int_{\boldsymbol{R}^d\setminus B_r(0)}\{f(\eta^{z,y})-f(\eta)\}\chi(y\,|\,\eta\setminus z)p(|z-y|)dy,$$

where

$$\tau_u\eta = \{x_i+u\}, \qquad \text{if } \eta=\{x_i\}.$$

We denote by S_t the semigroup with generator $\bar{\mathcal{L}}$ and by (η_t, P^0_ν) the associated process with initial distribution ν.

By Remark 1.2, if $z\in(0, z_0)$, then $\#\mathcal{G}(z)=1$ and so $\mu\in\mathcal{G}(z)$ is shift invariant and rotation invariant. For such a μ we put

$$\mu_0(d\eta) = \text{const.}\,\chi(0\,|\,\eta)\mu(d\eta),$$

where const. is the normalizing constant which makes μ_0 a probability measure on $\mathcal{X}_0$.

Using the same argument as Lemma 1.2 and Theorem 2.1, we have the following lemma.

LEMMA 4.1. *If* $z\in(0, z_0)$ *is sufficiently small (hence* $\#\mathcal{G}(z)=1$) *and if* $\mu\in\mathcal{G}(z)$, *then* $(\eta_t, P_{\mu_0}^0)$ *is an ergodic reversible Markov process.*

The process x_t is driven by the process η_t in the following way. Let $A\in\mathcal{B}(\boldsymbol{R}^d)$ and put

$$\begin{aligned}\Lambda_0 &= \{\eta\in\mathcal{X}_0:\ \eta=\tau_{-u}\eta \text{ for some } u\in\boldsymbol{R}^d\setminus\{0\}\},\\ \Delta &= \{(\eta, \eta):\ \eta\in\mathcal{X}_0\}\cup(\Lambda_0\times\Lambda_0),\\ \Gamma_A &= \{(\eta, \zeta)\in(\mathcal{X}_0\times\mathcal{X}_0)\setminus\Delta:\ \zeta=\tau_{-u}\eta \text{ for some } u\in A\}.\end{aligned}$$

We can prove that Λ_0 and Γ_A are measurable subsets of $\mathcal{X}_0$ and $\mathcal{X}_0\times\mathcal{X}_0$, respectively. Put

$$\begin{aligned}\mathcal{F}_t &= \bigcap_{\varepsilon>0}\{P_{\mu_0}^0\text{-completion of } \sigma(\eta_s:\ s\in[0, t+\varepsilon])\},\\ N((0, t]\times A) &= \sum_{s\in(0, t]}\mathbf{1}_{\Gamma_A}(\eta_{s-}, \eta_s) \qquad \text{for } t>0.\end{aligned}$$

Then $N((0, t]\times A)$ is an $\mathcal{F}_t$-adapted σ-finite random measure and

$$x_t = x_0+\int_{(0, t]}\int_{\boldsymbol{R}^d} uN(dsdu).$$

Using the same argument as for Theorem 2.4 of [3], we have the following result.

THEOREM 4.1. *If* $z>0$ *is sufficiently small and if* $\#\mathcal{G}(z)=1$ *and* $\mu\in\mathcal{G}(z)$, *then*

$$\lambda x_{t/\lambda^2} \longrightarrow \sigma B_t \qquad \text{as } \lambda\to 0$$

in the sense of distribution in the Skorohod space, where B_t *is a d-dimensional Brownian motion and* σ *is a non-negative constant given by*

$$\begin{aligned}\sigma^2 &= \int_{\boldsymbol{R}^d}du\int_{\mathcal{X}_0}d\mu_0u_1^2\chi(u\,|\,\cdot)p(|u|)-2\int_{[0,\infty)}dt\langle S_tF, F\rangle_{\mu_0},\\ F(\eta) &= \int_{\boldsymbol{R}^d}duu_1p(|u|)\chi(u\,|\,\eta).\end{aligned}$$

REMARK 4.1. An application of the method of [3] yields that the limiting process is of the form DB_t where D is a symmetric and nonnegative definite $d\times d$ matrix; however, by the rotation invariance of x_t the matrix D must be a constant multiple of the unit matrix. Unfortunately, we have not proved the strict positivity of the constant σ.

ACKNOWLEDGEMENT. The author would like to thank Professor H. Tanaka and Dr. K. Kuroda for their valuable suggestion and constant encouragement. The author also wishs to thank the referee for valuable comments, in particular, the clarification of a part (p. 688-689) of the proof of Theorem 2.1.

References

[1] H.O. Georgii, Canonical and Grand Canonical Gibbs States for continuum systems, Comm. Math. Phys., **48** (1976), 31-51.

[2] H.O. Georgii, Canonical Gibbs Measures, Lecture Notes in Math., **760**, Springer, 1979.

[3] C. Kipnis and S.R.S. Varadhan, Central limit theorem for additive functional of reversible Markov processes and applications to simple exclusions, Comm. Math. Phys., **104** (1986), 1-9.

[4] D. Klein, Dobrushin uniqueness technique and the decay of correlations in continuum statistical mechanics, Comm. Math. Phys., **86** (1982), 227-246.

[5] T.M. Liggett, Existence theorems for infinite particle systems, Trans. Amer. Math. Soc., **165** (1972), 471-481.

[6] T.M. Liggett, Interacting Particle Systems, Springer, 1985.

[7] D. Ruelle, Statistical Mechanics, Benjamin, 1969.

[8] H. Tanemura, Stochastic process for an infinite hard core particle system in $\boldsymbol{R}^d$, Proc. Fifth Japan-USSR Symp. on Probability Theory, 1986, Lecture Notes in Math., **1299**, Springer, 1988, pp. 502-510.

Hideki TANEMURA
Department of Mathematics
Faculty of Science and Technology
Keio University
Yokohama 223
Japan