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On meromorphic maps into a compact complex manifold

By Hirotaka FUJIMOTO

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§ 1. Introduction.

In [8], the author has shown that, for any given hyperplanes $H_1, \cdots, H_{N+2}$ in $P^N(\boldsymbol{C})$ located in general position and effective divisors $E_1, \cdots, E_{N+2}$ on $\boldsymbol{C}^n$, the set $\mathcal{F} := \mathcal{F}\begin{pmatrix} H_1, \cdots, H_{N+2} \\ E_1, \cdots, E_{N+2} \end{pmatrix}$ of all non-degenerate meromorphic maps of $\boldsymbol{C}^n$ into $P^N(\boldsymbol{C})$ such that the pull-backs $f^*(H_i)$ $(1 \leqq i \leqq N+2)$ of divisors H_i are equal to E_i respectively is finite. The purpose of this paper is partly to prove that the number of maps in the above set $\mathcal{F}$ is bounded by a constant depending only on N and mainly to generalize this result to the case of meromorphic maps into a compact complex manifold.

Let M be an N-dimensional connected compact complex manifold and L be a line bundle over M. We denote by $H^0(M, \mathcal{O}(L))$ the set of all holomorphic sections of L and by (ϕ) the divisor of zeros of a non-zero section $\phi \in H^0(M, \mathcal{O}(L))$. Set

$$|L| = \{(\phi);\ \phi \in H^0(M, \mathcal{O}(L)),\ \phi \not\equiv 0\}.$$

DEFINITION 1.1. Let $D_1, \cdots, D_m \in |L|$ such that $D_i = (\phi_i)$ $(1 \leqq i \leqq m)$ for $\phi_i \in H^0(M, \mathcal{O}(L))$. We define $\phi_1, \cdots, \phi_m$ (or $D_1, \cdots, D_m$) to be algebraically independent if there exists no non-zero homogeneous polynomial $P(w_1, \cdots, w_m)$ satisfying the relation

$$P(\phi_1, \cdots, \phi_m) \equiv 0$$

in $H^0(M, \mathcal{O}(L^d))$, where $d = \deg P$.

DEFINITION 1.2. A meromorphic map $f: \boldsymbol{C}^n \to M$ is said to be *algebraically non-degenerate* with respect to L if there exists no non-zero holomorphic section $\phi \in H^0(M, \mathcal{O}(L^d))$ $(d > 0)$ such that $f(\boldsymbol{C}^n) \subseteqq \{\phi = 0\}$.

Take $N+2$ divisors $D_1, \cdots, D_{N+2} \in |L|$ and effective divisors $E_1, \cdots, E_{N+2}$ on $\boldsymbol{C}^n$. Let $\mathcal{F}\begin{pmatrix} D_1, \cdots, D_{N+2} \\ E_1, \cdots, E_{N+2} \end{pmatrix}$ denote the set of all meromorphic maps of $\boldsymbol{C}^n$ into M which are algebraically non-degenerate with respect to L such that the

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pull-backs $f^*(D_i)$ $(1 \leqq i \leqq N+2)$ are equal to E_i respectively. The main result is stated as follows.

MAIN THEOREM. *In the above situation, if* $D_1, \cdots, D_{i-1}, D_{i+1}, \cdots, D_{N+2}$ *are algebraically independent for every* $i=1, 2, \cdots, N+2$ *and have no common component, then the number of maps in* $\mathcal{F}\begin{pmatrix} D_1, \cdots, D_{N+2} \\ E_1, \cdots, E_{N+2} \end{pmatrix}$ *is bounded by a constant depending only on* L.

Needless to say, in the case of the hyperplane bundle over $P^N(\boldsymbol{C})$, $N+2$ hyperplanes in $P^N(\boldsymbol{C})$ located in general position satisfy the assumption of Main Theorem. The following proposition given by Aihara-Mori provides other examples satisfying the assumption of Main Theorem.

PROPOSITION ([1], Lemma 1). *Let* L *be a very ample line bundle over an* N*-dimensional smooth projective algebraic variety* M. *If* $D_1, \cdots, D_{N+1} \in |L|$ *satisfy the condition*

$$\mathrm{Supp}\, D_1 \cap \mathrm{Supp}\, D_2 \cap \cdots \cap \mathrm{Supp}\, D_{N+1} = \emptyset ,$$

then $D_1, \cdots, D_{N+1}$ *are algebraically independent.*

COROLLARY. *Let* L *be a positive line bundle over an* N*-dimensional smooth projective algebraic variety* M, *let* $D_1, \cdots, D_{N+2} \in |L|$ *and let* $E_1, \cdots, E_{N+2}$ *be effective divisors on* $\boldsymbol{C}^n$. *If*

$$\mathrm{Supp}\, D_1 \cap \cdots \cap \mathrm{Supp}\, D_{i-1} \cap \mathrm{Supp}\, D_{i+1} \cap \cdots \cap \mathrm{Supp}\, D_{N+2} = \emptyset$$

for every $i=1, 2, \cdots, N+2$, *then the number of algebraically non-degenerate meromorphic maps* $f: \boldsymbol{C}^n \to M$ *with* $f^*(D_i)=E_i$ $(1 \leqq i \leqq N+2)$ *is bounded by a constant depending only on* L.

This is the case where Aihara-Mori gave some degeneracy theorems in [1].

Corollary is an immediate consequence of Main Theorem and Proposition. Because, L^l is very ample for some positive integer l. On the other hand,

$$\#\mathcal{F}\begin{pmatrix} D_1, \cdots, D_{N+2} \\ E_1, \cdots, E_{N+2} \end{pmatrix} \leqq \#\mathcal{F}\begin{pmatrix} lD_1, \cdots, lD_{N+2} \\ lE_1, \cdots, lE_{N+2} \end{pmatrix}$$

and the right hand side is bounded by a constant depending only on L, where $\# A$ denotes the number of elements of a set A.

We prove Main Theorem first in the case where L is the hyperplane bundle over $P^N(\boldsymbol{C})$ in § 2. For the proof, we need a lemma concerning on monomials, which is proved in § 3. After giving some algebraic lemmas, we complete the proof of Main Theorem in § 5.

§ 2. The case of meromorphic maps into $P^N(C)$.

First, we consider Main Theorem for the case where $M=P^N(\boldsymbol{C})$ and L is the hyperplane bundle over $P^N(\boldsymbol{C})$.

Let $H_1, \cdots, H_{N+2}$ be hyperplanes in $P^N(\boldsymbol{C})$ located in general position which may be regarded as divisors on $P^N(\boldsymbol{C})$. For any effective divisors $E_1, \cdots, E_{N+2}$ on $\boldsymbol{C}^n$ we consider the set $\mathcal{F}:=\mathcal{F}\binom{H_1, \cdots, H_{N+2}}{E_1, \cdots, E_{N+2}}$ of all meromorphic maps f of $\boldsymbol{C}^n$ into $P^N(\boldsymbol{C})$ which are non-degenerate, namely $f(\boldsymbol{C}^n)\nsubseteq H$ for any hyperplane H in $P^N(\boldsymbol{C})$, and satisfy the condition $f^*(H_i)=E_i$ $(1\leqq i\leqq N+2)$. We shall prove the following theorem.

THEOREM 2.1. *The number of maps in* $\mathcal{F}$ *is bounded by a constant depending only on* N.

To prove this, we assume $\mathcal{F}$ contains mutually distinct maps $f^1, f^2, \cdots, f^q$. Our task is to seek a number q_N with $q\leqq q_N$ depending only on N. This is given by the induction on N. For the case $N=1$, we can take $q_1=2$ as was shown by H. Cartan and R. Nevanlinna (cf., [**3**], [**10**], [**7**], p. 79). We assume Theorem 2.1 is true and choose numbers $q_1, \cdots, q_{N-1}$ with the above property for each case N is replaced by $1, 2, \cdots, N-1$ respectively.

For convenience' sake, we identify $P^N(\boldsymbol{C})$ with the subspace

$$\{w_1+w_2+\cdots+w_{N+2}=0\}$$

in $P^{N+1}(\boldsymbol{C})$, where $(w_1:w_2:\cdots:w_{N+2})$ are homogeneous coordinates on $P^{N+1}(\boldsymbol{C})$. We may assume here

$$H_i=\{w_i=0\}\cap P^N(\boldsymbol{C}) \qquad (1\leqq i\leqq N+2).$$

Using these coordinates, we express each f^j as

$$f^j=(f_1^j:f_2^j:\cdots:f_{N+2}^j) \qquad (1\leqq j\leqq q)$$

with holomorphic functions f_i^j on $\boldsymbol{C}^n$, where each expression may be assumed to be reduced, namely

$$\operatorname{codim}\{f_1^j=f_2^j=\cdots=f_{N+2}^j=0\}\geqq 2\,.$$

Take holomorphic functions k_i on $\boldsymbol{C}^n$ such that $(k_i)=E_i$ $(1\leqq i\leqq N+2)$. Let $\boldsymbol{H}^*$ denote the set of all nowhere zero holomorphic functions on $\boldsymbol{C}^n$. By the assumption $f^*(H_i)=E_i$, we see

$$h_{ij}:=f_i^j/k_i\in\boldsymbol{H}^* \tag{1}$$

and they satisfy the condition

$$h_{1j}k_1+h_{2j}k_2+\cdots+h_{N+2j}k_{N+2}\equiv 0$$

for every $j=1, 2, \cdots, q$. It then follows that

$$\det(h_{ij_l};\ 1\leqq i,\ l\leqq p)\equiv 0 \tag{2}$$

for every $j_1, j_2, \cdots, j_p$ with $1\leqq j_1, \cdots, j_p\leqq q$, where $p=N+2$.

In this situation, there is no loss of generality in performing the following operations;

(a) changing the order of the indices $i=1, 2, \cdots, p$ or $j=1, 2, \cdots, q$,

(b) multiplying a row or column of the matrix $(h_{ij};\ 1\leqq i\leqq p,\ 1\leqq j\leqq q)$ by a common element in $\boldsymbol{H}^*$,

because we can replace f_i^j by $h^j f_i^j$ and k_i by $h_i k_i$ for suitable $h^j, h_i\in\boldsymbol{H}^*$.

LEMMA 2.2. *Let h_{ij} $(1\leqq i\leqq p:=N+2,\ 1\leqq j\leqq q)$ be functions given by* (1). *For some r with $2\leqq r\leqq p$, if $e_{ij}:=h_{ij}\equiv const.$ for $i=1, 2, \cdots, r,\ j=1, 2, \cdots, q$ and*

$$\mathrm{rank}(e_{ij};\ i=1, 2, \cdots, r,\ j=1, 2, \cdots, q)<r,$$

then $q\leqq q_{N-1}$.

PROOF. By the assumption, changing indices if necessary, we can choose $\lambda_2, \cdots, \lambda_r\in\boldsymbol{C}$ such that

$$e_{1j}=\textstyle\sum_{i=2}^{r}\lambda_i e_{ij} \qquad (j=1, 2, \cdots, q).$$

Setting $\tilde{k}_i:=k_i+\lambda_i k_1$ for $i=2, \cdots, r$ and $\tilde{k}_i:=k_i$ for $i=1, r+1, \cdots, p$, we define meromorphic maps

$$\tilde{f}^j=(e_{2j}\tilde{k}_2:\cdots: e_{rj}\tilde{k}_r: h_{r+1j}\tilde{k}_{r+1}:\cdots: h_{pj}\tilde{k}_p)$$

of $\boldsymbol{C}^n$ into $P^{N-1}(\boldsymbol{C})=\{(w_2:\cdots: w_{N+2})\in P^N(\boldsymbol{C});\ w_2+\cdots+w_{N+2}=0\}$. Obviously, $\tilde{f}^{j_1}\not\equiv\tilde{f}^{j_2}$ for any mutually distinct j_1, j_2. We have $q\leqq q_{N-1}$ by the induction hypothesis. q. e. d.

For our purpose, we need the following generalization of a classical theorem of E. Borel.

LEMMA 2.3. *Let $h_1, \cdots, h_t\in\boldsymbol{H}^*$ satisfy the condition that $h_1^{l_1}h_2^{l_2}\cdots h_t^{l_t}\notin\boldsymbol{C}^*:=\boldsymbol{C}-\{0\}$ for any $(l_1, \cdots, l_t)\in\boldsymbol{Z}^t-\{0\}$. Then $h_1, \cdots, h_t$ are algebraically independent, namely*

$$P(h_1, \cdots, h_t)\not\equiv 0$$

for any non-zero polynomial $P(w_1, \cdots, w_t)$.

For the proof, see [4], Proposition 4.5.

We may regard $\boldsymbol{C}^*$ as a subgroup of a multiplicative group $\boldsymbol{H}^*$. Let us consider the factor group $G=\boldsymbol{H}^*/\boldsymbol{C}^*$. For $h\in\boldsymbol{H}^*$ we denote the class containing h by $[h]$. Take $\eta_1, \cdots, \eta_t\in\boldsymbol{H}^*$ such that $[\eta_1], \cdots, [\eta_t]$ are linearly independent over $\boldsymbol{Z}$ and generate a subgroup G containing all $[h_{ij}]$'s $(1\leqq i\leqq p,\ 1\leqq j\leqq q)$. Then, we can write each h_{ij} as

$$h_{ij}=c_{ij}\eta_1^{l_{ij}^1}\eta_2^{l_{ij}^2}\cdots\eta_t^{l_{ij}^t} \tag{3}$$

uniquely, where $c_{ij}\in\boldsymbol{C}^*$ and $l_{ij}^1, \cdots, l_{ij}^t$ are integers. Moreover, $\eta_1, \cdots, \eta_t$ are algebraically independent by virtue of Lemma 2.3. We choose integers $p_1, \cdots, p_t$ such that, setting

$$l_{ij} := l_{ij}^1 p_1 + l_{ij}^2 p_2 + \cdots + l_{ij}^t p_t ,$$

we have $l_{ij} \neq l_{i'j'}$ whenever $(l_{ij}^1, \cdots, l_{ij}^t) \neq (l_{i'j'}^1, \cdots, l_{i'j'}^t)$ (cf. [4], (2.2)). We can assume that $l_{ij} \geqq 0$ for all i, j by performing operation (b) suitably. With each h_{ij} we associate monomial $P_{ij}(u) = c_{ij} u^{l_{ij}}$ in one variable u. Then we have

$$\det(P_{ij}(u)\,;\ i=1, \cdots, p,\ j=j_1, \cdots, j_p) \equiv 0 \tag{4}$$

for all $j_1, \cdots, j_p$. In fact, by (2) and (3),

$$\det(c_{ij} \eta_1^{l_{ij}^1} \cdots \eta_t^{l_{ij}^t}\,;\ i=1, \cdots, p,\ j=j_1, \cdots, j_p) \equiv 0 . \tag{5}$$

Since $\eta_1, \cdots, \eta_t$ are algebraically independent, (5) remains valid if we substitute $\eta_i = u^{p_i}$ $(1 \leqq i \leqq p)$. This gives (4).

We give here a lemma concerning on monomials which will be proved in § 3. We consider $p \times q$ matrices $(P_{ij}(u)\,;\ 1 \leqq i \leqq p,\ 1 \leqq j \leqq q)$ with monomials $P_{ij}(u) = c_{ij} u^{l_{ij}}$ as entries, where $c_{ij} \in \boldsymbol{C}^*$ and l_{ij} are non-negative integers for various p, q. By $\mathrm{rank}(P_{ij})$ we mean the rank of the matrix in the field $\boldsymbol{C}(u)$ of rational functions.

MAIN LEMMA. *For each q_0 $(\geqq 1)$ there exists some constant $Q(p, q_0)$ depending only on p and q_0 with the following property:*

If $q > Q(p, q_0)$ and $\mathrm{rank}(P_{ij}(u)\,;\ 1 \leqq i \leqq p,\ 1 \leqq j \leqq q) < p$, then there exists an integer r depending on (P_{ij}) and satisfying $2 \leqq r \leqq p$ such that, after performing operation (a) *suitably, we have*

$$l_{i1} - l_{i'1} = l_{i2} - l_{i'2} = \cdots = l_{iq_0} - l_{i'q_0} \tag{6}$$

for all i, i' with $1 \leqq i \leqq i' \leqq r$ and

$$\mathrm{rank}(P_{ij}(u)\,;\ 1 \leqq i \leqq r,\ 1 \leqq j \leqq q_0) < r . \tag{7}$$

Apply Main Lemma to the above-mentioned monomials $P_{ij}(u) = c_{ij} u^{l_{ij}}$ and $q_0 := q_{N-1} + 1$. Set $q_N := Q(p, q_0)$. Suppose that $q > q_N$. We have then conclusions (6) and (7). This shows that h_{ij} $(1 \leqq i \leqq p,\ 1 \leqq j \leqq q_0)$ satisfy the assumption of Lemma 2.2. So, we have an absurd conclusion q_0 $(= q_{N-1} + 1) \leqq q_{N-1}$. This concludes $q \leqq q_N$ and completes the proof of Theorem 2.1.

§ 3. Proof of Main Lemma.

To prove Main Lemma, we give first

LEMMA 3.1. *Assume that $P_{ij}(u) = c_{ij} u^{l_{ij}} \in \boldsymbol{C}[u]$ $(1 \leqq i \leqq p,\ 1 \leqq j \leqq q)$ with $c_{ij} \in \boldsymbol{C}^*$, $l_{ij} \geqq 0$ such that*

(C_1) $\mathrm{rank}(P_{ij}\,;\ i=1, \cdots, p,\ j=1, \cdots, q) < p$ *in the field $\boldsymbol{C}(u)$ of all rational functions of u,*

(C_2) *there exist some indices $a_1, \cdots, a_s$ with $1 \leqq a_1 < \cdots < a_s = p$* (let $a_0 = 0$)

such that, whenever $a_{\sigma-1}<i_1\leqq i_2\leqq a_\sigma$ $(1\leqq\sigma\leqq s)$,

$$l_{i_2 1}-l_{i_1 1}=l_{i_2 2}-l_{i_1 2}=\cdots=l_{i_2 q}-l_{i_1 q},$$

(C$_3$) $\det(P_{ij};\ 1\leqq i,\ j\leqq p-1)\not\equiv 0$,

(C$_4$) $\det(P_{ij};\ 1\leqq i,\ j\leqq a_1)\not\equiv 0$.

Then, after a suitable change of indices $j=p, p+1, \cdots, q$, *there exist some* q_1 $(\geqq p-1)$ *and* q_2 *such that* $q_1+q_2\leqq q$, $q_2(p!)^2\geqq q-q_1$ *and*

(α) $\operatorname{rank}(P_{ij};\ i=a_1+1, \cdots, p,\ j=p, \cdots, q_1)<p-a_1$,

(β) $l_{a_\tau q_1+1}-l_{a_{\tau'} q_1+1}=\cdots=l_{a_\tau q_1+q_2}-l_{a_{\tau'} q_1+q_2}$

for some distinct τ, τ'.

REMARK. In Lemma 3.1, the case $q_1=p-1$ of the conclusion means that the only case (β) occurs.

PROOF OF LEMMA 3.1. For each $j_0=p, p+1, \cdots, q$, we have by condition (C$_1$)

$$\det(P_{ij};\ i=1, 2, \cdots, p,\ j=1, 2, \cdots, p-1, j_0)\equiv 0.$$

For each $\iota=1, 2, \cdots, p$ set

$$\Phi_\iota(u)=(-1)^{\iota-1}\det(P_{ij};\ i=1, \cdots, \iota-1, \iota+1, \cdots, p,\ j=1, \cdots, p-1).$$

Then,

$$c_{1j_0}u^{l_{1j_0}}\Phi_1(u)+\cdots+c_{pj_0}u^{l_{pj_0}}\Phi_p(u)\equiv 0.$$

Let

$$\Psi_{j_0}(u):=\textstyle\sum_{\iota=1}^{a_1}c_{\iota j_0}u^{l_{\iota j_0}}\Phi_\iota(u)$$

and $q_1:=\#\{j_0;\ \Psi_{j_0}\equiv 0\}+p-1$. We may assume

$$\Psi_p(u)\equiv\Psi_{p+1}(u)\equiv\cdots\equiv\Psi_{q_1}(u)\equiv 0$$

by a suitable change of indices $j=p, p+1, \cdots, q$.

First we shall show conclusion (α). There is nothing to prove for the case $q_1=p-1$. Let $q_1\geqq p$. Choose indices $j_1, \cdots, j_{p-a_1}$ out of $1, 2, \cdots, q_1$. Our task is to show

$$\det(P_{ij};\ i=a_1+1, a_1+2, \cdots, p,\ j=j_1, \cdots, j_{p-a_1})\equiv 0.$$

We may assume $p\leqq j_1<\cdots<j_{p-a_1}\leqq q_1$ and, moreover, $j_1=p$, $j_2=p+1, \cdots, j_{p-a_1}=2p-a_1-1$ after a suitable change of indices. We set

$$P_{1j}^*=\cdots=P_{a_1 j}^*=0,\quad P_{a_1+1 j}^*=P_{a_1+1 j}, \cdots, P_{pj}^*=P_{pj}$$

for $j=p, p+1, \cdots, 2p-a_1-1$. Since

$$\textstyle\sum_{\iota=a_1+1}^{p}c_{\iota j}u^{l_{\iota j}}\Phi_\iota(u)\equiv 0,$$

we see

$$\det(P_{i1}, \cdots, P_{ip-1}, P_{ij}^*;\ i=1, 2, \cdots, p)\equiv 0$$

for $j=p, p+1, \cdots, 2p-a_1-1$. By condition ($C_3$), ${}^t(P_{1j}^*, \cdots, P_{pj}^*)$ can be expressed as a linear combination of ${}^t(P_{11}, \cdots, P_{p1}), \cdots, {}^t(P_{1p-1}, \cdots, P_{pp-1})$ over $\boldsymbol{C}(u)$. Therefore,

$$\operatorname{rank}(P_{i1}, \cdots, P_{ip-1}, P_{ip}^*, \cdots, P_{i2p-a_1-1}^*;\ 1\leqq i\leqq p)<p.$$

Particularly,

$$\begin{aligned}&\det(P_{i1}, \cdots, P_{ia_1}, P_{ip}^*, \cdots, P_{i2p-a_1-1}^*;\ 1\leqq i\leqq p)\\&=\det(P_{ij};\ 1\leqq i,\ j\leqq a_1)\det(P_{ij};\ i=a_1+1, \cdots, p,\ j=p, \cdots, 2p-a_1-1)\\&\equiv 0.\end{aligned}$$

By condition (C_4), we obtain the desired conclusion

$$\det(P_{ij};\ i=a_1+1, \cdots, p,\ j=p, \cdots, 2p-a_1-1)\equiv 0.$$

Next let us show conclusion (β). We may assume $q>q_1$. Set

$$\Phi_\iota(u)=:d_{\iota 1}u^{m_{\iota 1}}+d_{\iota 2}u^{m_{\iota 2}}+\cdots+d_{\iota t_\iota}u^{m_{\iota t_\iota}}\quad (a_1+1\leqq\iota\leqq p),$$

and

$$\nu_i:=l_{i1}-l_{a_\sigma 1}\ (=l_{i2}-l_{a_\sigma 2}=\cdots=l_{iq}-l_{a_\sigma q})$$

if $a_{\sigma-1}+1\leqq i\leqq a_\sigma$ (let $a_0=0$), where $d_{\iota 1}, \cdots, d_{\iota t_\iota}\in\boldsymbol{C}^*$ and $0\leqq t_\iota\leqq(p-1)!$. Take arbitrarily a quadruple of indices $(i_1, i_2, \tau_1, \tau_2)$ such that $1\leqq i_1\leqq a_1$, $a_{\sigma-1}+1\leqq i_2\leqq a_\sigma$ for some $\sigma\geqq 2$, and $1\leqq\tau_1\leqq t_{i_1}$, $1\leqq\tau_2\leqq t_{i_2}$. Set

$$A_{i_1i_2\tau_1\tau_2}:=\{j;\ l_{a_1j}-l_{a_\sigma j}=\nu_{i_2}-\nu_{i_1}+m_{i_2\tau_2}-m_{i_1\tau_1},\ q_1+1\leqq j\leqq q\}.$$

For each j with $q_1+1\leqq j\leqq q$, since

$$\Psi_j(u)+\textstyle\sum_{\iota=a_1+1}^{p}c_{\iota j}u^{l_{\iota j}}\Phi_\iota(u)\equiv 0$$

and $\Psi_j\not\equiv 0$, we can find a term of $\Psi_j(u)$ which has the same degree as a term of $\sum_{\iota=a_1+1}^{p}c_{\iota j}u^{l_{\iota j}}\Phi_\iota(u)$ has. Therefore, there exist indices i_1, i_2, τ_1, τ_2 with $1\leqq i_1\leqq a_1$, $a_{\sigma-1}+1\leqq i_2\leqq a_\sigma$ ($\sigma\geqq 2$), $1\leqq\tau_1\leqq t_{i_1}$, $1\leqq\tau_2\leqq t_{i_2}$ such that

$$l_{i_1j}+m_{i_1\tau_1}=l_{i_2j}+m_{i_2\tau_2}$$

and so

$$\nu_{i_1}+l_{a_1j}+m_{i_1\tau_1}=\nu_{i_2}+l_{a_\sigma j}+m_{i_2\tau_2}.$$

This shows $j\in A_{i_1i_2\tau_1\tau_2}$. We have thus

$$\bigcup_{(i_1, i_2, \tau_1, \tau_2)}A_{i_1i_2\tau_1\tau_2}=\{q_1+1, q_1+2, \cdots, q\}.$$

Set $q_2:=\max_{(i_1, i_2, \tau_1, \tau_2)}\sharp A_{i_1i_2\tau_1\tau_2}$. Then,

$$q_2(p\,!)^2\geqq\sum_{(i_1, i_2, \tau_1, \tau_2)}\sharp A_{i_1i_2\tau_1\tau_2}\geqq q-q_1,$$

because there are at most p! possibilities of choices of quadruples $(i_1, i_2, \tau_1, \tau_2)$. If we choose indices so that $\{q_1+1, \cdots, q_1+q_2\}=A_{i_1i_2\tau_1\tau_2}$ for some $(i_1, i_2, \tau_1, \tau_2)$, we have the desired conclusion. q. e. d.

PROOF OF MAIN LEMMA. We shall prove Main Lemma by induction on p. In the case $p=2$, we easily see $l_{2j_1}-l_{1j_1}=l_{2j_2}-l_{1j_2}$ for each j_1, j_2 with $1\leqq j_1<j_2\leqq q$ by the assumption. For each q_0 $(\geqq 1)$, $Q(2, q_0)=q_0-1$ has the desired property. Assume that Main Lemma is true for the case $\leqq p-1$ and so there exist $Q(2, q_0), \cdots, Q(p-1, q_0)$ with the property in the conclusion of Main Lemma. For any given q_0 $(\geqq 1)$ we set

$$q^*:=\max(q_0, Q(2, q_0), \cdots, Q(p-1, q_0)).$$

Moreover, we define q'_s $(1\leqq s\leqq p)$ by $q'_1=q^*$ and $q'_s:=q^*+p+q'_{s-1}(p\,!)^2$ inductively. We shall prove that $Q(p, q_0):=q'_p$ has the desired property. To this end, we shall show the following by downward induction on s $(p\geqq s\geqq 1)$.

(3.2) Either the conclusion of Main Lemma is valid, or there exist indices $a_1, a_2, \cdots, a_s$ with $1\leqq a_1<\cdots<a_s=p$ (let $a_0=0$) such that, for each i_1, i_2 with $a_{\sigma-1}+1\leqq i_1\leqq i_2\leqq a_\sigma$ $(1\leqq\sigma\leqq s)$

$$l_{i_21}-l_{i_11}=l_{i_22}-l_{i_12}=\cdots=l_{i_2q'_s}-l_{i_1q'_s}$$

after performing operation (a) suitably.

In the case $s=1$, the conclusion of (3.2) means that Main Lemma is true when we take $r=p$. Therefore, we can conclude Main Lemma from (3.2).

If $s=p$, (3.2) is trivial because we can take $a_1=1, \cdots, a_p=p$. Suppose that (3.2) is true for the case $\geqq s$ and particularly the conclusion of (3.2) is valid. Then, P_{ij} $(1\leqq i\leqq p,\ 1\leqq j\leqq q'_s)$ satisfy conditions (C_1) and (C_2). Moreover, we may assume that they satisfy also conditions (C_3) and (C_4) after a suitable change of indices. In fact, if (C_3) does not hold for any choice of indices, we have

$$\operatorname{rank}(P_{ij};\ i=1, 2, \cdots, p-1,\ j=1, 2, \cdots, q)<p-1.$$

Then, monomials P_{ij} $(1\leqq i\leqq p-1,\ 1\leqq j\leqq q)$ satisfy the assumption of Main Lemma. By the induction hypothesis concerning on p for Main Lemma, Main Lemma is true. Moreover, by the same reason, we may assume that conclusion (C_4) is also satisfied after a suitable change of indices $j=1, 2, \cdots, p-1$.

Apply Lemma 3.1 to P_{ij} $(1\leqq i\leqq p-1,\ 1\leqq j\leqq q'_s)$. There exists an index q_1 $(\geqq p-1)$ such that (α) and (β) hold. If $q_1\geqq p+q^*$, then

$$q_1-(p-1)=q_1-p+1>q^*\geqq Q(p-a_1, q_0)$$

and P_{ij} $(a_1+1\leqq i\leqq p,\ p\leqq j\leqq q_1)$ satisfy the assumption of Main Lemma for the case $p-a_1$ $(<p)$ because of (α). So, the conclusion of Main Lemma is valid. Assume that $q_1<p+q^*$. By (β), there exist τ, τ' with $\tau\neq\tau'$ and q_2 with $q_1+q_2\leqq q'_s$ and $q_2(p\,!)^2\geqq q'_s-q_1$ such that

$$l_{a_\tau q_1+1}-l_{a_{\tau'}q_1+1}=\cdots=l_{a_\tau q_1+q_2}-l_{a_{\tau'}q_1+q_2}.$$

We see here $q_2>q'_{s-1}$ because

$$q_2(p!)^2>q'_s-q^*-p=q'_{s-1}(p!)^2.$$

Then, we obtain easily the conclusion (3.2) for the case $s-1$ after a suitable change of indices. This completes the proof of Main Lemma.

§ 4. Some algebraic lemmas.

For the proof of Main Theorem, we need some preparations.

DEFINITION 4.1. Let M_1 and M_2 be irreducible complex analytic spaces. A set-valued map $f: M_1\to M_2$ is called to be meromorphic if there is an irreducible analytic set G^f in $M_1\times M_2$ such that $f(x)=\pi_2\pi_1^{-1}(x)$ and

(i) $\pi_1: G^f\to M_1$ is proper,

(ii) $\pi_1|\pi_1^{-1}(M_1^*): \pi_1^{-1}(M_1^*)\to M_1^*$ is a biholomorphic map for an open dense subset M_1^* of M_1, where $\pi_i: G^f\to M_i$ $(i=1, 2)$ denote the canonical projections into M_i. The set G^f is called the graph of f.

We have easily

(4.2) *Let L be a line bundle over an irreducible compact complex analytic space M and $\phi_1, \phi_2, \cdots, \phi_{m+1}\in H^0(M, \mathcal{O}(L))$, where $m\geqq 1$ and $\phi_{i_0}\not\equiv 0$ for some i_0. Consider the set $G^\Phi:=$the closure of $\{(x, (\phi_1(x):\cdots:\phi_{m+1}(x)))\,;\,(\phi_1(x), \cdots, \phi_{m+1}(x))\neq(0, \cdots, 0)\}$ in $M\times P^m(\boldsymbol{C})$. Then, a meromorphic map $\Phi: M\to P^m(\boldsymbol{C})$ whose graph is G^Φ can be defined uniquely.*

We denote the map Φ defined as above by $(\phi_1:\cdots:\phi_{m+1})$ in the following.

LEMMA 4.3. *Let $P(w_1, \cdots, w_{m+1})$ be a homogeneous polynomial of degree d $(\geqq 1)$ which is expanded as*

$$P(w)=\sum_{\sigma=1}^{s+1}P_\sigma(w)$$

with non-zero monomials $P_\sigma(w)$ and define a meromorphic map $F=(P_1:\cdots:P_{s+1}): P^m(\boldsymbol{C})\to P^s(\boldsymbol{C})$. Assume that

$$\pi_i(\{(w_1:\cdots:w_{m+1})\in P^m(\boldsymbol{C})\,;\,P(w_1, \cdots, w_{m+1})=0\})=P^{m-1}(\boldsymbol{C})$$

for every $i=1, 2, \cdots, m+1$, where $\pi_i: P^m(\boldsymbol{C})\to P^{m-1}(\boldsymbol{C})$ are meromorphic maps defined by $\pi_i((w_1:\cdots:w_{m+1}))=(w_1:\cdots:w_{i-1}:w_{i+1}:\cdots:w_{m+1})$. Then, $\sharp F^{-1}F(w)\leqq d^m$ for every point $w\in G:=\{(w_1:\cdots:w_{m+1})\,;\,w_1w_2\cdots w_{m+1}\neq 0\}$.

PROOF. The proof is given by induction on m. In the case $m=1$, we set

$$P(w_1, w_2)=a_0w_1^d+a_1w_1^{d-1}w_2+\cdots+a_dw_2^d\quad(a_i\in\boldsymbol{C}).$$

By the assumption, there are at least two indices i_1, i_2 $(0\leqq i_1<i_2\leqq d)$ with $a_{i_1}\neq 0$, $a_{i_2}\neq 0$. For each $c=(c_1:c_2)\in G$, take $w=(w_1:w_2)\in F^{-1}F(c)$ arbitrarily. Then,

$$a_{i_1}w_1^{d-i_1}w_2^{i_1}/a_{i_2}w_1^{d-i_2}w_2^{i_2}=a_{i_1}c_1^{d-i_1}c_2^{i_1}/a_{i_2}c_1^{d-i_2}c_2^{i_2}$$

and so $(w_1/w_2)^{i_2-i_1}=(c_1/c_2)^{i_2-i_1}$. This gives $\sharp F^{-1}F(c)\leqq i_2-i_1\leqq d$.

Assume that Lemma 4.3 is true for the case $\leqq m-1$. We express $P(w)$ as

$$P(w)=A_0(w_1, \cdots, w_m)w_{m+1}^d+A_1(w_1, \cdots, w_m)w_{m+1}^{d-1}+\cdots+A_d(w_1, \cdots, w_m),$$

where $A_i(w_1, \cdots, w_m)$ are homogeneous polynomials of degree $d-i$ or vanish identically. We may assume $A_d(w_1, \cdots, w_m)\not\equiv 0$. For, otherwise, we may replace $P(w)$ by $\tilde{P}(w)=P(w)w_{m+1}^{-l}$ with $w_{m+1}\nmid\tilde{P}(w)$ $(l>0)$. Moreover, we see $A_{i_0}(w_1, \cdots, w_m)\not\equiv 0$ for some i_0 with $0\leqq i_0\leqq d-1$ by the assumption. Consider a hypersurface $V=\{(w_1:\cdots:w_m);\ A_d(w_1, \cdots, w_m)=0\}$ in $P^{m-1}(\boldsymbol{C})$. For each point $\tilde{w}:=(w_1:\cdots:w_{i-1}:w_{i+1}:\cdots:w_m)\in P^{m-2}(\boldsymbol{C})$, there exists a point $w^*:=(w_1:\cdots:w_{i-1}:w_i:w_{i+1}:\cdots:w_m:0)\in P^m(\boldsymbol{C})$ such that $P(w^*)=0$ by the assumption. Since $P(w^*)=A_d(w^*)=0$, we get $\tilde{w}^*:=(w_1:\cdots:w_{i-1}:w_i:w_{i+1}:\cdots:w_m)\in V$ and $\tilde{\pi}_i(\tilde{w}^*)=\tilde{w}$ for the map $\tilde{\pi}_i: P^{m-1}(\boldsymbol{C})\to P^{m-2}(\boldsymbol{C})$ defined by $\tilde{\pi}_i((w_1:\cdots:w_m))=(w_1:\cdots:w_{i-1}:w_{i+1}:\cdots:w_m)$. So, $\tilde{\pi}_i(V)=P^{m-2}(\boldsymbol{C})$. This shows that the homogeneous polynomial $A_d(w_1, \cdots, w_m)$ in m variables satisfies the assumption of Lemma 4.3. By the induction hypothesis, if we expand A_d as

$$A_d(w_1, \cdots, w_m)=\sum_{\tau=1}^{t+1}\tilde{P}_\tau(w_1, \cdots, w_m)$$

with non-zero monomials $\tilde{P}_\tau$ and define a meromorphic map $\tilde{F}=(\tilde{P}_1:\cdots:\tilde{P}_{t+1}): P^{m-1}(\boldsymbol{C})\to P^t(\boldsymbol{C})$, then $\sharp\tilde{F}^{-1}\tilde{F}(\tilde{c})\leqq d^{m-1}$ for each $\tilde{c}=(\tilde{c}_1:\cdots:\tilde{c}_m)$ with $\tilde{c}_1\tilde{c}_2\cdots\tilde{c}_m\neq 0$. For a point $c=(c_1:\cdots:c_{m+1})\in G$, take $w=(w_1:\cdots:w_{m+1})\in F^{-1}F(c)$ arbitrarily. Since $\{\tilde{P}_1, \cdots, \tilde{P}_{t+1}\}$ is a subset of $\{P_1, \cdots, P_{s+1}\}$, we have

$$(\tilde{P}_1(w):\cdots:\tilde{P}_{t+1}(w))=(\tilde{P}_1(c):\cdots:\tilde{P}_{t+1}(c)).$$

Set $\tilde{c}=(c_1, \cdots, c_m)$ and $\tilde{F}^{-1}\tilde{F}(\tilde{c})=\{\tilde{c}^{(1)}, \cdots, \tilde{c}^{(a)}\}$, where $\tilde{c}=\tilde{c}^{(1)}$ and $1\leqq a\leqq d^{m-1}$. The point $\tilde{w}=(w_1:\cdots:w_m)$ coincides with some $\tilde{c}^{(\alpha)}$, say $\tilde{c}^{(\alpha_0)}$. Since $F(c)=F(w)$,

$$\begin{aligned}&A_{i_0}(c_1, \cdots, c_m)c_{m+1}^{d-i_0}/A_d(c_1, \cdots, c_m)\\&=A_{i_0}(w_1, \cdots, w_m)w_{m+1}^{d-i_0}/A_d(w_1, \cdots, w_m)\\&=A_{i_0}(c_1^{(\alpha_0)}, \cdots, c_m^{(\alpha_0)})w_{m+1}^{d-i_0}/A_d(c_1^{(\alpha_0)}, \cdots, c_m^{(\alpha_0)}).\end{aligned}$$

For each fixed α_0 there are at most $d-i_0$ $(\leqq d)$ complex numbers w_{m+1}'s satisfying this condition. Thus, we conclude $\sharp F^{-1}F(c)\leqq d^m$.

LEMMA 4.4. *Let M be an N-dimensional connected compact complex manifold and L a line bundle over M. Then there exists a positive constant d_L depending only on L satisfying the condition that for any $N+2$ holomorphic sections $\phi_1, \cdots, \phi_{N+2}\in H^0(M, \mathcal{O}(L))$ we can find a homogeneous polynomial of degree at most d_L such that*

$$P(\phi_1, \phi_2, \cdots, \phi_{N+2})=0.$$

For the proof, see L. Siegel [**11**].

LEMMA 4.5. *Let L be a line bundle over an N-dimensional connected compact complex manifold which has at least one system of $N+1$ algebraically independent holomorphic sections. Then, there exists a positive constant k_L depending only on L such that for algebraically independent $\phi_1, \cdots, \phi_{N+1} \in H^0(M, \mathcal{O}(L))$ the meromorphic map $\Phi=(\phi_1:\phi_2:\cdots:\phi_{N+1}): M \to P^N(\boldsymbol{C})$ satisfies the condition that $\sharp\Phi^{-1}(w) \leqq k_L$ for every point w in a Zariski open dense subset G of $P^N(\boldsymbol{C})$.*

PROOF. Take a basis $\{\psi_1, \cdots, \psi_{m+1}\}$ of $H^0(M, \mathcal{O}(L))$ and define a meromorphic map $\Psi=(\psi_1:\psi_2:\cdots:\psi_{m+1}): M \to P^m(\boldsymbol{C})$. The image $\Psi(M)$ is an algebraic subset of $P^m(\boldsymbol{C})$ and we can find a positive number d_1 such that $\sharp\Psi^{-1}(w)=d_1$ for each w in a Zariski open dense subset of $\Psi(M)$. Obviously, d_1 and the degree d_2 of $\Psi(M)$ are determined independently of a choice of a basis of $H^0(M, \mathcal{O}(L))$. Let $\phi_1, \cdots, \phi_{N+1}$ be arbitrary algebraically independent holomorphic sections of L. We can choose $\phi_{N+2}, \cdots, \phi_{m+1}$ such that $\phi_1, \cdots, \phi_{N+1}, \phi_{N+2}, \cdots, \phi_{m+1}$ constitute a basis of $H^0(M, \mathcal{O}(L))$. Let $\Phi=(\phi_1:\cdots:\phi_{N+1})$, $\tilde{\Phi}=(\phi_1:\cdots:\phi_{m+1})$ and $\pi: P^m(\boldsymbol{C}) \to P^N(\boldsymbol{C})$ be defined by $\pi((w_1:\cdots:w_{m+1}))=(w_1:\cdots:w_{N+1})$. Since generic fibers of $\pi|\tilde{\Phi}(M): \tilde{\Phi}(M) \to P^N(\boldsymbol{C})$ consist of at most d_2 points, generic fibers of $\pi\circ\tilde{\Phi}: M \to P^N(\boldsymbol{C})$ consist of at most $d_1 d_2$ points. The number $k_L := d_1 d_2$ has the desired property.

§ 5. Proof of Main Theorem.

As in § 1, let L be a line bundle over an N-dimensional connected compact complex manifold M, and let $D_1, \cdots, D_{N+2}$ be divisors on M such that $D_i=(\phi_i)$ for $\phi_i \in H^0(M, \mathcal{O}(L))$ and $D_1, \cdots, D_{i-1}, D_{i+1}, \cdots, D_{N+2}$ are algebraically independent for each i. Moreover, let $E_1, \cdots, E_{N+2}$ be effective divisors on $\boldsymbol{C}^n$ and $\mathcal{F}\begin{pmatrix} D_1, \cdots, D_{N+2} \\ E_1, \cdots, E_{N+2} \end{pmatrix}$ the set of all meromorphic maps f of $\boldsymbol{C}^n$ into M such that f is algebraically non-degenerate with respect to L and $f^*(D_i)=E_i$ $(1 \leqq i \leqq N+2)$. Consider the meromorphic map $\Phi=(\phi_1:\cdots:\phi_{N+2}): M \to P^{N+1}(\boldsymbol{C})$ and set $V=\Phi(M)$, which is an irreducible algebraic set in $P^{N+1}(\boldsymbol{C})$. We define a map $\pi_i: P^{N+1}(\boldsymbol{C}) \to P^N(\boldsymbol{C})$ by $\pi_i((w_1:\cdots:w_{N+2}))=(w_1:\cdots:w_{i-1}:w_{i+1}:\cdots:w_{N+2})$. If $\pi_i(V) \subsetneqq P^N(\boldsymbol{C})$, there exists a non-zero homogeneous polynomial R such that $\pi_i(V) \subseteqq \{R=0\}$. Then

$$R(\phi_1, \cdots, \phi_{i-1}, \phi_{i+1}, \cdots, \phi_{N+2}) \equiv 0,$$

which contradicts the assumption. Therefore $\pi_i(V)=P^N(\boldsymbol{C})$ for each $i=1, 2, \cdots, N+2$. Since

$$N \geqq \dim \Phi(M) \geqq \dim \pi_i \Phi(M) = N,$$

we have $\dim V=N$. Take an irreducible non-zero homogeneous polynomial P such that $P(\phi_1, \cdots, \phi_{N+2})=0$, where $d := \deg P$ is not larger than a constant d_L

depending only on L by Lemma 4.4. Then,

$$V=\{(w_1:\cdots:w_{N+2})\in P^{N+1}(\boldsymbol{C});\ P(w_1:\cdots:w_{N+2})=0\}$$

and $P(w)$ satisfies the assumption of Lemma 4.3. We expand $P(w)$ as

$$P(w)=\sum_{\sigma=1}^{s+1}P_\sigma(w)$$

with non-zero monomials P_σ and consider the meromorphic map $F=(P_1:\cdots:P_{s+1}):P^{N+1}(\boldsymbol{C})\rightarrow P^s(\boldsymbol{C})$. By Lemmas 4.3 and 4.5, $\sharp F^{-1}F(w)\leqq d^{N+1}$ for each point $w\in G:=\{w_1w_2\cdots w_{N+2}\neq 0\}$ and $\sharp\Phi^{-1}(w)$ does not exceed a constant k_L depending only on L for each point w in a Zariski open dense subset of $\Phi(M)$. Consequently, for the meromorphic map $\Psi:=F\circ\Phi: M\rightarrow P^s(\boldsymbol{C})$

$$\sharp\Psi^{-1}\Psi(w)\leqq k_L d^{N+1}\leqq k_L d_L^{N+1}$$

for each point w in an open dense subset M^* of M, where $M-M^*$ is the set of zeros of a holomorphic section of L^d $(d>0)$. We consider hyperplanes

$$H_{s+2}=\{u_1+u_2+\cdots+u_{s+1}=0\} \qquad (\cong P^{s-1}(\boldsymbol{C})),$$
$$H_i=\{u_i=0\}\cap H_{s+2} \qquad (1\leqq i\leqq s+1)$$

in $P^s(\boldsymbol{C})$, where $(u_1:\cdots:u_{s+1})$ are homogeneous coordinates on $P^s(\boldsymbol{C})$. With each $f\in\mathfrak{F}\begin{pmatrix}D_1, \cdots, D_{N+2}\\ E_1, \cdots, E_{N+2}\end{pmatrix}$ we associate $\tilde{f}:=F\circ\Phi\circ f:\ \boldsymbol{C}^n\rightarrow P^{s-1}(\boldsymbol{C})$. Then, $\tilde{f}$ is non-degenerate. For, if there is a hyperplane $H=\{a_1u_1+\cdots+a_{s+1}u_{s+1}=0\}$ such that $H\neq H_{s+2}$ and $\tilde{f}(\boldsymbol{C}^n)\subseteqq H$, then

$$f(\boldsymbol{C}^n)\subseteqq\{a_1P_1(\phi_1, \cdots, \phi_{N+2})+\cdots+a_{s+1}P_{s+1}(\phi_1, \cdots, \phi_{N+2})=0\}$$

but $a_1P_1+\cdots+a_{s+1}P_{s+1}\not\equiv 0$. This contradicts the assumption. Let

$$P_\sigma(w_1, \cdots, w_{N+2})=c_\sigma w_1^{l_{\sigma 1}}\cdots w_{N+2}^{l_{\sigma N+2}}$$

for $\sigma=1, 2, \cdots, s+1$. We set

$$\tilde{D}_\sigma:=l_{\sigma 1}D_1+\cdots+l_{\sigma N+2}D_{N+2}$$
$$\tilde{E}_\sigma:=l_{\sigma 1}E_1+\cdots+l_{\sigma N+2}E_{N+2}.$$

Since $(\phi_1), \cdots, (\phi_{N+2})$ and $P_1(w), \cdots, P_{s+1}(w)$ have no common component respectively, we have $(F\circ\Phi)^*(H_\sigma)=\tilde{D}_\sigma$, $f^*(\tilde{D}_\sigma)=\tilde{E}_\sigma$ and therefore

$$\tilde{f}^*(H_\sigma)=\tilde{E}_\sigma \quad (\sigma=1, 2, \cdots, s+1).$$

Set

$$\tilde{\mathfrak{F}}:=\{\tilde{f};\ \tilde{f}=F\circ\Phi\circ f,\ f\in\mathfrak{F}\}.$$

Since $H_1, \cdots, H_{s+2}$ are located in general position, $\sharp\tilde{\mathfrak{F}}$ is bounded by a constant depending only on s. On the other hand, $s+1\leqq\binom{d_L+N+1}{N+1}$. So, $\sharp\tilde{\mathcal{F}}$ is bounded by a constant q_1 depending only on L. Take a map $f_0\in\mathcal{F}$. We shall show

$$\sharp\{f\in\mathcal{F}\,;\ \Psi\circ f=\Psi\circ f_0\}\leqq k_L d_L^{N+1},$$

which gives the desired conclusion because this gives

$$\sharp\mathcal{F}\leqq k_L d_L^{N+1}q_1.$$

Suppose that there are mutually distinct $q:=k_L d_L^{N+1}+1$ meromorphic maps $f^1, \cdots, f^q\in\mathcal{F}$ such that $\Psi\circ f^i=\Psi\circ f$. Set $G^*:=\{z\in \boldsymbol{C}^n\,;\ f^i(z)\in M^*$ for all i and $f^i(z)\neq f^j(z)$ if $1\leqq i<j\leqq q\}$. By the assumption of non-degeneracy of f^i, G^* is an open dense subset of $\boldsymbol{C}^n$. For a point $z_0\in G^*$, we have $w_0=f_0(z_0)\in M^*$ and

$$\{f^1(z_0), \cdots, f^q(z_0)\}\subseteqq\Psi^{-1}\Psi(w_0),$$

whence $\sharp\Psi^{-1}\Psi(w_0)\geqq q$. This is a contradiction. Thus, the proof of Main Theorem is completed.

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Hirotaka FUJIMOTO
Department of Mathematics
Nagoya University
Chikusa-ku, Nagoya 464
Japan