J. Math. Soc. Japan
Vol. 26, No. 4, 1974

# On the shape of decomposition spaces

By Yukihiro KODAMA

(Received April 18, 1973)
(Revised Nov. 21, 1973)

## § 1. Introduction.

In [2] K. Borsuk defined shapes of compacta (= compact metric spaces). He generalized in [3, 5] this concept to general metric spaces by defining their weak shapes. The notion of shape or weak shape gives a classification of metric spaces coarser than the homotopy type. For a pair of a metric space and its subset K. Borsuk [5] has introduced the concept of position which is closely related to its weak shape. In some sense a classification of pairs of spaces by the position is similar to one by the weak shape or the homotopy type but they are generally different. In a certain case the position gives a classification finer than the homotopy type (cf. [5, p. 150]). For 0-dimensional metric spaces, we shall show that a classification by the weak shape or the position is equal to one by the homeomorphism. The main theorem in this paper is the following.

THEOREM. *Let* $(X, A)$ *and* $(Y, B)$ *be pairs of metric spaces and arbitrary subsets respectively. Let* $f:(X, A)\to(Y, B)$ *be a perfect map such that* $f(X-A)=Y-B$ *and* $f(A)=B$. *If* $\dim Y\leqq n$ *and* $f^{-1}(y)$ *is approximatively k-connected for each* $y\in Y$ *and* $k=0,\cdots,n$, *then* $\mathrm{Sh}_W(X)\geqq\mathrm{Sh}_W(Y)$ *and* $\mathrm{Pos}(X, A)\geqq\mathrm{Pos}(Y, B)$. *In addition, if* $\dim X\leqq n$, *then the equalities in these relations hold.*

Notations and definitions are given in the next section. Since the position $\mathrm{Pos}(X, A)$ is equal to the weak shape $\mathrm{Sh}_W(X)$ in case $A=\emptyset$ and $\mathrm{Sh}_W(X)$ is equal to the shape $\mathrm{Sh}(X)$ for a compactum $X$, there are several corollaries for shapes among which are theorems by Borsuk [4, Theorem (6.1)] and Sher [10, Theorem 11]. The main theorem and the corollaries are proved in § 3.

Throughout this paper all of spaces are metric and maps are continuous. By an *AR* and an *ANR* we mean always those for metric spaces.

## § 2. $\mathrm{Sh}_W(X)$ and $\mathrm{Pos}(X, A)$.

Let $M$ and $N$ be *AR*'s and let $X$ and $Y$ be closed subsets of $M$ and $N$ respectively. According to Borsuk [5, p. 142] a sequence of maps $f_k: M\to N$, $k=1, 2, \cdots$, is said to be a *W-sequence* if it satisfies the following condition:

(2.1) For every compactum $C\subset X$ there is a compactum $D\subset Y$ such that for every neighborhood $V$ of $D$ in $N$ there is a neighborhood $U$ of $C$ in $M$ such that $f_k|U\cong f_{k+1}|U$ in $V$ for almost all $k$.

We denote this $W$-sequence by $\{f_k, X, Y\}_{M,N}$, or shortly by $\underline{f}$, and we write $\underline{f}: X\to Y$ in $M, N$. Every compactum $D$ satisfying (2.1) is said to be *$\underline{f}$-assigned* to the compactum $C$.

A $W$-sequence $\underline{f}=\{f_k, X, Y\}_{M,N}$ is said to be *generated* by a map $f: X\to Y$ if $f_k(x)=f(x)$ for every $x\in X$ and $k=1, 2, \cdots$. If $X=Y$ and $M=N$ and if $i$ is the identity map of $X$ onto itself, then $\{i, X, X\}_{M,M}$ is the *identity $W$-sequence* for $X$ in $M$ and denoted by $\underline{i}_{X,M}$.

Two $W$-sequences $\underline{f}=\{f_k, X, Y\}_{M,N}$ and $\underline{f}'=\{f'_k, X, Y\}_{M,N}$ are said to be *homotopic* (notation: $\underline{f}\cong\underline{f}'$) if the following condition is satisfied:

(2.2) For every compactum $C\subset X$ there is a compactum $D\subset Y$ such that for every neighborhood $V$ of $D$ in $N$ there is a neighborhood $U$ of $C$ in $M$ such that $f_k|U\cong f'_k|U$ in $V$ for almost all $k$.

Every compactum $D$ satisfying (2.2) is said to be *$(\underline{f}, \underline{f}')$-assigned* to the compactum $C$.

LEMMA 1. *If* $\dim Y=0$, *then for every $W$-sequence $\underline{f}=\{f_k, X, Y\}_{M,N}$ there is a unique map $f: X\to Y$ which generates a $W$-sequence homotopic to $\underline{f}$.*

PROOF. By (2.1), for every $x\in X$, there is a compactum $D_x$ of $Y$ which is $\underline{f}$-assigned to $\{x\}$. From the compactness of $D_x$ and (2.1) it follows that the set $\{f_k(x): k=1, 2, \cdots\}$ has an accumulation point in $D_x$. Since $\dim Y=0$, if $y_x$ is an arbitrary accumulation point of $\{f_k(x)\}$, then it is known by (2.1) that the sequence $\{f_k(x)\}$ converges to $y_x$ and hence $y_x$ is uniquely determined by the point $x$. Define $f: X\to Y$ by $f(x)=y_x$ for $x\in X$. To show the continuity of $f$, let $W$ be an open neighborhood of $f(x)$ in $N$. Since $\dim Y=0$, we can find disjoint open sets $V_1$ and $V_2$ of $N$ such that $D_x\subset V_1\cup V_2$, $D_x-W\subset V_1$ and $f(x)\in V_2\subset \bar{V}_2\subset W$. By (2.1) there is a neighborhood $U$ of $x$ such that $f_k|U\cong f_{k+1}|U$ in $V=V_1\cup V_2$ for almost all $k$. Since $\{f_k(x)\}$ converges to $f(x)\in V_2$ and $V_1\cap V_2=\emptyset$, we know that there is a neighborhood $U'\subset U$ of $x$ such that $f_k(U')\subset V_2\subset W$ for almost all $k$. From (2.1) if follows that $f(U')\subset W$. Thus $f$ is continuous. Let $f': M\to N$ be an extension of $f$ and define $\underline{f}'=\{f'_k, X, Y\}$ by $f'_k=f'$ for every $k$. Then $\underline{f}'$ is generated by $f$. For a compactum $C$ of $X$, let $D$ be a compactum of $Y$ which is $\underline{f}$-assigned to $C$. By the definition of $\underline{f}'$ and the compactness of $D$ it is known that $D$ is $(\underline{f}, \underline{f}')$-assigned to $C$ and $\underline{f}\cong\underline{f}'$. Let $\underline{f}'$ and $\underline{f}''$ be $W$-sequences generated by maps $f'$ and $f''$ respectively, and let $\underline{f}'\cong\underline{f}''$. Since $\dim Y=0$, we know $f'=f''$ by (2.2). This completes the proof.

Let $X$ and $Y$ be closed sets in $AR$'s $M$ and $N$ respectively. The sets $X$ and $Y$ are said to be *W-equivalence* in $M$ and $N$ (notation: $X \cong_W Y$ in $M, N$) if there exist two $W$-sequences $\underline{f} = \{f_k, X, Y\}_{M,N}, \underline{g} = \{g_k, Y, X\}_{N,M}$ satisfying the following conditions:

$$\underline{g}\underline{f} \cong \underline{i}_{X,M} \quad \text{and} \quad \underline{f}\underline{g} \cong \underline{i}_{Y,N} .$$

If only the condition $\underline{f}\underline{g} \cong \underline{i}_{Y,N}$ is satisfied, then we say that $X$ *W-dominates* $Y$ in $M, N$ and we write $X \geqq_W Y$ in $M, N$.

Let $A$ and $B$ be arbitrary subsets of $X$ and $Y$ respectively. We say that $(X, A)$ and $(Y, B)$ are *W-similar* in $M$ and $N$ (notation: $(X, A) \leftrightarrow (Y, B)$ in $M, N$) if there exist two sequences of maps $f_k: M \to N$ and $g_k: N \to M$, $k=1, 2, \cdots$, such that

$$\text{(2.3)} \qquad \left.\begin{array}{l} \underline{f}' = \{f_k, A, B\}_{M,N}, \quad \underline{f}'' = \{f_k, X-A, Y-B\}_{M,N} \\ \underline{g}' = \{g_k, B, A\}_{N,M}, \quad \underline{g}'' = \{g_k, Y-B, X-A\}_{N,M} \end{array}\right. \quad \text{are } W\text{-sequences}$$

and that

$$\text{(2.4)} \qquad \underline{g}'\underline{f}' \cong \underline{i}_{A,M}, \qquad \underline{g}''\underline{f}'' \cong \underline{i}_{(X-A),M},$$

$$\text{(2.5)} \qquad \underline{f}'\underline{g}' \cong \underline{i}_{B,N}, \qquad \underline{f}''\underline{g}'' \cong \underline{i}_{(Y-B),N}.$$

If only (2.3) and (2.5) are satisfied, then we say that $(X, A)$ *dominates W-similarly* $(Y, B)$ and we write $(X, A) \underset{W}{\rightrightarrows} (Y, B)$ in $M, N$.

Borsuk [**5**, Theorem 6.7] proved that the relations of $W$-equivalence, $W$-domination, $W$-similarity and $W$-similar domination of spaces do not depend on choice of $AR$'s containing those spaces as closed subsets. Thus we may write $(X, A) \underset{W}{\rightrightarrows} (Y, B)$ instead of $(X, A) \underset{W}{\leftrightarrows} (Y, B)$ in $M, N$. Similarly we omit in the relation "$\underset{W}{\rightrightarrows}$", "$\cong_W$" and "$\geqq_W$" the word "in $M, N$" respectively. The relations $\underset{W}{\leftrightarrows}$ and $\cong_W$ are equivalence relations [**5**, Theorem 7.1]. The *position* $\operatorname{Pos}(X, A)$ of a pair of a metric space $X$ and its subset $A$ is defined as the equivalence class containing $(X, A)$ under the relation $\underset{W}{\leftrightarrows}$. Similarly the *weak shape* $\operatorname{Sh}_W(X)$ of a space $X$ is the equivalence class containing $X$ under $\cong_W$. If $(X, A) \underset{W}{\rightrightarrows} (Y, B)$ then we write $\operatorname{Pos}(X, A) \geqq \operatorname{Pos}(Y, B)$. Similarly if $X \geqq_W Y$ then we write $\operatorname{Sh}_W(X) \geqq \operatorname{Sh}_W(Y)$. If $A$ is empty, then $\operatorname{Pos}(X, \emptyset)$ is equal to $\operatorname{Sh}_W(X)$ and if we consider only the category of all compacta then $\operatorname{Pos}(X, \emptyset) = \operatorname{Sh}_W(X) = \operatorname{Sh}(X)$, where $\operatorname{Sh}(X)$ is the shape of a compactum $X$ (cf. [**2**, **3**]).

The following is known for shape of compact spaces [**9**, Theorem 20].

THEOREM 1. *Let $(X, A)$ and $(Y, B)$ be pairs of metric spaces and subsets and let* $\dim X = \dim Y = 0$. *Then* $\operatorname{Pos}(X, A) \geqq \operatorname{Pos}(Y, B)$ *if and only if there is an imbedding $i: Y \to X$ and a retraction $r: X \to i(Y)$ such that $i(B) \subset A$ and*

$r(A)=i(B)$. *In particular,* $\mathrm{Pos}(X, A)=\mathrm{Pos}(Y, B)$ *if and only if there is a homeomorphism of* $(X, A)$ *onto* $(Y, B)$, *and* $\mathrm{Sh}_W(X)=\mathrm{Sh}_W(Y)$ *if and only if there is a homeomorphism of* $X$ *onto* $Y$.

The proof is obvious by Lemma 1.

According to Borsuk [**4**, p. 266] a compactum $X$ is said to be *approximatively k-connected* if there is an imbedding $i$ of $X$ into an $AR$ $M$ satisfying the condition: For every neighborhood $V$ of $i(X)$ in $M$ there is a neighborhood $V_0$ of $i(X)$ such that every map of a $k$-sphere $S^k$ into $V_0$ is null-homotopic in $V$. (In this definition we do not consider any base point.) As shown by Borsuk, it is known that the approximative $k$-connectedness of $X$ does not depend on choice of an $AR$ containing $X$ and it is a shape invariant.

## § 3. Shape of a decomposition space.

Let $X$ be a metric space lying in an $AR$ $M$ and let $\mathfrak{D}$ be an upper semicontinuous decomposition of $X$ consisting of compacta. By $X_{\mathfrak{D}}$ we mean the decomposition space of $X$ by $\mathfrak{D}$. Let $f: X \to X_{\mathfrak{D}}$ be the decomposition map. Then $f$ is a perfect map. By a *cover* of $\mathfrak{D}$ we imply a collection $\mathfrak{U}$ of open sets in $M$ such that $X \subset \cup \{U : U \in \mathfrak{U}\}$ and for each element $U$ of $\mathfrak{U}$ $U \cap X$ is non-empty and saturated, i.e. $U \cap X = f^{-1}f(U \cap X)$. For two collections $\mathfrak{U}$ and $\mathfrak{V}$ of subsets of $X$ or $M$, we mean by $\mathfrak{U} > \mathfrak{V}$ that $\mathfrak{U}$ is a refinement of $\mathfrak{V}$ and by $\mathfrak{U} \overset{*}{>} \mathfrak{V}$ that $\mathfrak{U}$ is a star refinement of $\mathfrak{V}$.

LEMMA 2. *Let $X$ be a metric space and let $M$ be an $AR$ containing $X$ as a closed set. Let $\mathfrak{D}$ be an upper semicontinuous decomposition of $X$ each element of which is a compactum and approximatively $j$-connected for $j=0, 1, \cdots, k$. For every positive integer $n$ and every cover $\mathfrak{U}$ of $\mathfrak{D}$ there is a cover $\mathfrak{V}$ of $\mathfrak{D}$ satisfying the following conditions:*

(3.1) *There is a sequence $\mathfrak{V}_0, \mathfrak{V}_1', \mathfrak{V}_1, \cdots, \mathfrak{V}_n, \mathfrak{V}_{n+1}', \mathfrak{V}_{n+1}$ of covers of $\mathfrak{D}$ such that $\mathfrak{V}=\mathfrak{V}_0$, $\mathfrak{U}=\mathfrak{V}_{n+1}$, $\mathfrak{V}_i > \mathfrak{V}_{i+1}'$ and $\mathfrak{V}_{i+1}' \overset{*}{>} \mathfrak{V}_{i+1}$.*

(3.2) *For each element $V$ of $\mathfrak{V}_i$, $i=0, 1, \cdots, n$, there is a $V' \in \mathfrak{V}_{i+1}'$ such that every map $f: S^j \to V$, $j=0, 1, \cdots, k$, is null-homotopic in $V'$.*

For a cover $\mathfrak{U}$ of $\mathfrak{D}$, a cover $\mathfrak{V}$ satisfying the conditions of the lemma is said to be a *$(k, n)$-refinement* of $\mathfrak{U}$.

PROOF OF LEMMA 2. Let $\mathfrak{U}$ be an open collection of $M$ such that each element of $\mathfrak{D}$ is contained in some element of $\mathfrak{U}$. It is enough to prove that there is a cover $\mathfrak{V}$ of $\mathfrak{D}$ such that $\mathfrak{V} \overset{*}{>} \mathfrak{U}$. Let $\mathfrak{D}'$ be a decomposition of $M$ consisting of all compacta in $\mathfrak{D}$ and every point in $M-X$. Let $f: M \to M_{\mathfrak{D}'}$ be the decomposition map. Obviously $f$ is a perfect map and hence $M_{\mathfrak{D}'}$ is a metric space. The collection $\mathfrak{W}=\{M_{\mathfrak{D}'}-f(M-U): U\in\mathfrak{U}\}$ is an open collection which covers $f(X)$. Let $\mathfrak{V}'$ be an open collection of $M_{\mathfrak{D}'}$ which is a star refinement of $\mathfrak{W}$ and covers $f(X)$. Put $\mathfrak{V}=\{f^{-1}(V): f^{-1}(V)\cap X\neq\emptyset, V\in\mathfrak{V}'\}$. Then $\mathfrak{V}$ is a cover of $\mathfrak{D}$ and $\mathfrak{V}\overset{*}{>}\mathfrak{U}$.

The following lemma is an immediate consequence of the definition of $(n, n)$-refinements.

LEMMA 3. *Under the same hypothesis as in Lemma* 2, *let* $\mathfrak{U}$ *be a cover of* $\mathfrak{D}$ *and let* $\mathfrak{V}$ *be an* $(n, n)$*-refinement of* $\mathfrak{U}$.

(3.3) *Let $K$ be an $(n+1)$-dimensional simplicial complex and $K^0$ the set of its vertices. If $f: K^0\to M$ is a map such that for each closed simplex $\sigma$ of $K$ there is a $V\in\mathfrak{V}$ containing $f(\sigma\cap K^0)$, then $f$ has an extension $g: K\to M$ such that for closed simplex $\sigma$ of $K$ there is a $U\in\mathfrak{U}$ containing $g(\sigma)$.*

(3.4) *Let $K$ be an $n$-dimensional simplicial complex. If $f$ and $g$ are maps of $K$ into $M$ such that for each closed simplex $\sigma$ of $K$ there is a $V\in\mathfrak{V}$ containing $f(\sigma)\cup g(\sigma)$ then there is a homotopy $H: K\times I\to M$ connecting $f$ and $g$ such that for each closed simplex $\sigma$ $H(\sigma\times I)$ is contained in some $U\in\mathfrak{U}$.*

REMARK 1. In Lemmas 2 and 3, let $\{\mathfrak{V}_i, \mathfrak{V}_i'\}$ be covers in the definition of $(n, n)$-refinements such that $\mathfrak{V}=\mathfrak{V}_0$ and $\mathfrak{U}=\mathfrak{V}_{n+1}$. If $\sigma$ is an $i$-simplex of $K$, then we can construct a map $g$ and a homotopy $H$ such that $g(\sigma)$ is in some element of $\mathfrak{V}_i'$ and $H(\sigma\times I)$ is in some element of $\mathfrak{V}_{i+1}'$ for each $i=1, 2, \cdots, n$. In particular, if $\sigma$ is an $n$-simplex, then we can assume that $g(\sigma)$ is in some element of $\mathfrak{V}_n'$.

Let $X$ and $Y$ be spaces and $\mathfrak{U}$ an open collection of $Y$. If $f, g: X\to Y$ are maps, then we say that $f$ and $g$ are $\mathfrak{U}$-*close* provided that for each $x\in X$ there exists a $U\in\mathfrak{U}$ which contains $f(x)$ and $g(x)$. If $H: X\times I\to Y$ is a homotopy, then we say that $H$ is *limited by* $\mathfrak{U}$ provided that for each $x\in X$ there exists a $U\in\mathfrak{U}$ which contains $H(\{x\}\times I)$. If $f, g: X\to Y$ and there is a homotopy connecting $f$ and $g$ which is limited by $\mathfrak{U}$, then we write $f\cong g: X\to Y$ lim.$\mathfrak{U}$ or, shortly $f\cong g$ lim.$\mathfrak{U}$. The following lemma is well known and easily proved.

LEMMA 4. *Let $N$ be an ANR. Let $Y$ be a subset of $N$ and let $\mathfrak{W}$ be an open collection in $N$ which covers $Y$. Then there is an open collection $\mathfrak{T}$ which covers $Y$, refines $\mathfrak{W}$ and satisfies the following condition:*

(3.5) *If $f, g$ are maps of a space $T$ into $N$ which are $\mathfrak{T}$-close then* $f\cong g: T\to N$ lim.$\mathfrak{W}$.

LEMMA 5. *Let $M$ be an ANR and let $\mathfrak{U}$ be an open cover of $M$. If $f$, $g$*

*are maps of a closed set* $A$ *of a space* $X$ *into* $M$ *such that* $f \cong g: A \to M$ lim.$\mathfrak{U}$ *and* $f$ *has an extension* $f': X \to M$, *then there is an extension* $g': X \to M$ *of* $g$ *such that* $f' \cong g'$ lim.$\mathfrak{U}$.

This is proved by a standard technique (cf. [**1**, Chapter IV, (8.1)]).

THEOREM 2. *Let* $(X, A)$ *and* $(Y, B)$ *be pairs of metric spaces and subsets and let* $f: (X, A) \to (Y, B)$ *be a perfect map such that* $f(X-A) = Y-B$ *and* $f(A) = B$. *If* $\dim Y \leqq n$ *and* $f^{-1}(y)$ *is approximatively* $k$*-connected for each* $y \in Y$ *and* $k = 0, 1, \cdots, n$, *then* $\mathrm{Sh}_W(X) \geqq \mathrm{Sh}_W(Y)$ *and* $\mathrm{Pos}(X, A) \geqq \mathrm{Pos}(Y, B)$. *In addition, if* $\dim X \leqq n$, *then* $\mathrm{Sh}_W(X) = \mathrm{Sh}_W(Y)$ *and* $\mathrm{Pos}(X, A) = \mathrm{Pos}(Y, B)$.

PROOF. For the proof we shall use an argument as in the proof of [**7**, Theorem 1]. Let $M$ and $N$ be $AR$'s containing $X$ and $Y$ as closed sets respectively. Extend $f$ to a map of $M$ into $N$ and denote it by $f$ again. Since $M$ is perfectly normal and $X$ is closed in $M$, if necessary, by replacing $N \times [0, \infty)$ in place of $N$ we can assume that

$$f^{-1}(Y) = X \quad \text{and} \quad f^{-1}(B) = A. \tag{3.6}$$

We denote by $d$ a metric of $M$. Let $\mathfrak{D}$ be the decomposition $\{f^{-1}(y): y \in Y\}$ of $X$. We take an arbitrary open cover $\mathfrak{S}_j$, $j = 1, 2, \cdots$, of $N$ such that

$$\operatorname{mesh} \mathfrak{S}_j < 2^{-j}. \tag{3.7}$$

Let $\mathfrak{T}_1$ be an open collection of $N$ such that $Y \subset \cup \{T: T \in \mathfrak{T}_1\}$ and $\mathfrak{T}_1$ satisfies the condition (3.5) of Lemma 4 for $\mathfrak{S}_1$. Let $\mathfrak{P}_1$ be an open collection of $N$ such that $Y \subset \cup \{P: P \in \mathfrak{P}_1\}$ and $\mathfrak{P}_1 \overset{*}{>} \mathfrak{T}_1$. For each point $y \in Y$, take an open set $V_y$ in $M$ such that $f^{-1}(y) \subset V_y \cap X$, $d(f^{-1}(y), M - V_y) < 1/2$ and $V_y$ is contained in some element of $f^{-1}(\mathfrak{P}_1)$ (cf. (3.6)). Let $\mathfrak{U}_1$ be a cover of $\mathfrak{D}$ such that $\mathfrak{U}_1 \overset{*}{>} \{V_y: y \in Y\}$ and put $M_1 = \cup \{U: U \in \mathfrak{U}_1\}$. Let $\mathfrak{V}_1'$ be a cover of $\mathfrak{D}$ which is an $(n, n)$-refinement of $\mathfrak{U}_1$ (Lemma 2). Since $f|X$ is perfect and $\dim Y \leqq n$, there is an open cover $\mathfrak{W}_1'$ of $Y$ such that $f^{-1}\mathfrak{W}_1' \overset{*}{>} \mathfrak{V}_1'$, $\mathfrak{W}_1' \overset{*}{>} \mathfrak{P}_1$, $\operatorname{mesh} \mathfrak{W}_1' < 1/2$ and $\operatorname{ord} \mathfrak{W}_1'$ (=the order of $\mathfrak{W}_1'$) $\leqq n+1$. Since $Y$ is a metric space, there exists an open collection $\mathfrak{W}_1$ of $N$ such that

$$\mathfrak{W}_1 \cap Y = \mathfrak{W}_1' \quad \text{and} \quad \mathfrak{W}_1 \overset{*}{>} \mathfrak{P}_1, \tag{3.8}$$

$$\mathfrak{W}_1 \text{ is similar to } \mathfrak{W}_1'. \tag{3.9}$$

Put $\mathfrak{V}_1 = \mathfrak{V}_1' \wedge f^{-1}(\mathfrak{W}_1)$. Then $\mathfrak{V}_1$ is an $(n, n)$-refinement of $\mathfrak{U}_1$. Take an open neighborhood $W_1$ of $Y$ in $N$ such that $\overline{W}_1 \subset \cup \{W: W \in \mathfrak{W}_1\}$. Let $K_1$ be the nerve of $\mathfrak{W}_1 \cap \overline{W}_1$ with weak topology and $\phi_1: \overline{W}_1 \to K_1$ a canonical map. For each vertex $w$ of $K$, choose a point $v \in f^{-1}(\phi_1^{-1} w \cap Y)$ and define $h_1': K_1^0 \to X$ $(\subset M)$ by $h_1'(w) = v$, where $K_1^0$ is the set of vertices of $K_1$. If $\sigma$ is a simplex of $K_1$ and $w_i$, $i=0, \cdots, k$, are its vertices, then $W_{i_0}\cap \cdots \cap W_{i_k}\cap Y\neq\emptyset$ by (3.9) and hence $h_1'(\sigma\cap K_1^0)\subset \bigcup_{j=0}^{k} f^{-1}(W_{i_j})\cap X\subset$ some element of $\mathfrak{V}_1'$. Thus there is an extension $h_1: K_1\to M_1$ of $h_1'$ such that $h_1(\sigma)$ is in some element of $\mathfrak{U}_1$ for each simplex $\sigma$ of $K_1$, where $M_1=\cup\{U: U\in\mathfrak{U}_1\}$. Put $g_1=h_1\phi_1: \overline{W}_1\to M_1$. Note that if $i_N: N\to N$ is the identity map then $i_N|\overline{W}_1$ and $fg_1$ are $\mathfrak{T}_1$-close and hence $i_N|\overline{W}_1\cong fg_1: \overline{W}_1\to N\,\mathrm{lim}.\mathfrak{S}_1$. Now, by induction, for each $j=1, 2, \cdots$, we are going to construct a neighborhood $M_j$ of $X$ in $M$, covers $\mathfrak{U}_j$ and $\mathfrak{V}_j$ of $\mathfrak{D}$, open collections $\mathfrak{T}_j$, $\mathfrak{P}_j$ and $\mathfrak{W}_j$ of $N$ which cover $Y$, a neighborhood $W_j$ of $Y$ such that $\overline{W}_j\subset\cup\{W: W\in\mathfrak{W}_j\}$ and a map $h_j$ of the nerve $K_j$ of $\mathfrak{W}_j\cap\overline{W}_j$ into $M_j$, which satisfy the following conditions:

(3.10) (i) $\mathfrak{T}_j$ satisfies the condition (3.5) for $\mathfrak{W}_{j-1}\wedge\mathfrak{S}_j$ and $\mathfrak{T}_j>\mathfrak{W}_{j-1}\wedge\mathfrak{S}_j$.

(ii) $\mathfrak{P}_j\overset{*}{>}\mathfrak{T}_j$, $\mathfrak{U}_j\overset{*}{>}f^{-1}\mathfrak{P}_j\wedge\mathfrak{V}_{j-1}$ and $\mathfrak{W}_j\overset{*}{>}\mathfrak{W}_{j-1}\wedge\mathfrak{P}_j$.

(iii) $M_j=\cup\{U: U\in\mathfrak{U}_j\}$ and $\bar{M}_j\subset M_{j-1}$.

(iv) For an arbitrary subset $L$ of $Y$, $d(f^{-1}L, M-\cup\{U: U\cap f^{-1}(L)\neq\emptyset, U\in\mathfrak{U}_j\})<2^{j+1}$.

(v) $\mathfrak{V}_j$ is an $(n, n)$-refinement of $\mathfrak{U}_j$ and $\mathfrak{V}_j\overset{*}{>}f^{-1}\mathfrak{W}_j$.

(vi) $\mathfrak{W}_j$ and $\mathfrak{W}_j\cap Y$ are similar, and $\mathrm{ord}\,\mathfrak{W}_j\leqq n+1$ and $f^{-1}(\mathfrak{W}_j\cap Y)\overset{*}{>}\mathfrak{U}_j$.

(vii) $Y\subset W_j\subset\overline{W}_j\subset W_{j-1}$.

(viii) Let $g_j=h_j\ \phi_j: \overline{W}_j\to M_j$, where $\phi_j: \overline{W}_j\to K_j$ is a canonical map. For each element $W\in\mathfrak{W}_j\cap\overline{W}_j$, $g_j(W)\cap f^{-1}(W)\neq\emptyset$ and for each $i$ and $j$, $2<i<j$, $g_i|\overline{W}_j\cong g_j: \overline{W}_j\to M_{i-2}\,\mathrm{lim}.\mathfrak{U}_{i-2}$.

(ix) $i_N|\overline{W}_j$ and $fg_j$ are $\mathfrak{T}_j$-close and hence $i_N|\overline{W}_j\cong fg_j: \overline{W}_j\to N\,\mathrm{lim}.\mathfrak{S}_j$.

(x) If $\dim X\leqq n$ and $2<j$, then there is a neighborhood $U_j$ of $X$ in $M$ such that $\bar{U}_j\subset f^{-1}(W_j)$ and $i_M|\bar{U}_j\cong g_jf|\bar{U}_j: \bar{U}_j\to M_{j-2}$ $\mathrm{lim}.\mathfrak{U}_{j-2}$, where $i_M$ is the identity map of $M$.

We already constructed $\mathfrak{U}_j, \cdots, h_j$ satisfying the conditions of (3.10) for $j=1$. Suppose that for each $j<k$, $k>1$, $\mathfrak{U}_j, \cdots, h_j$ satisfying the conditions of (3.10) are constructed. Let us construct $\mathfrak{U}_k, \cdots, h_k$. By Lemma 4 there is a cover $\mathfrak{T}_k$ of $\mathfrak{D}$ satisfying (i) in (3.10). By the same argument as in the construction of $\mathfrak{W}_1$ and $\mathfrak{U}_1$ and by using Lemma 2 we can find open collections $\mathfrak{W}_k$, $\mathfrak{P}_k$ and $\mathfrak{U}_k$ in $N$ and $M$ satisfying (ii), (iii), (iv), (v), (vi) and (vii) in (3.10) for $j=k$. Take an open neighborhood $W_k$ of $Y$ such that $\overline{W}_k\subset W_{k-1}$. Denote by $K_k$ the nerve of $\mathfrak{W}_k\cap\overline{W}_k$ with weak topology. By a similar way to the construction of $h_1$ we can construct a map $h_k: K_k\to M_k$ such that $h_k(\sigma)$ is in some element of $\mathfrak{U}_k$ for each simplex $\sigma$ of $K_k$ and $h_k\phi_k(W)\cap f^{-1}(W)\neq\emptyset$ for each $W\in\mathfrak{W}_k\cap\overline{W}_k$, where $M_k=\cup\{U: U\in\mathfrak{U}_k\}$ and $\phi_k: \overline{W}_k\to K_k$ is a canonical map. Let $2<i<k$. Since $\mathfrak{W}_k$ is a refinement of $\mathfrak{W}_i$, there is a projection

$\pi: K_k \to K_i$. Since for each $x \in \overline{W}_k$, $\pi\phi_k(x)$ and $\phi_i(x)$ are contained in some closed simplex of $K_i$, we know $h_i\pi\phi_k \cong h_i\phi_i | \overline{W}_k \lim.\mathfrak{U}_i$. Consider the maps $h_k$ and $h_i\pi: K_k \to M_i$. By (ii), (iv), (v) and (vi), for each closed simplex $\sigma$ of $K_k$ there is a $V$ of $\mathfrak{V}_{i-1}$ containing $h_k(\sigma) \cup h_i\pi(\sigma)$. From Lemma 3 it follows that $h_k \cong h_i\pi \lim.\mathfrak{U}_{i-1}$. Thus $h_i\phi_i | \overline{W}_k \cong h_k\phi_k \lim.\mathfrak{U}_{i-2}$ and (viii) of (3.10) holds for $j=k$. For each point $y' \in \overline{W}_k$, it is known by the construction of $\mathfrak{P}_k$, $\mathfrak{U}_k$ and $g_k$ that there is a point $y \in Y$ such that the open star $\mathrm{St}(y, \mathfrak{P}_k) = \cup \{P : P \in \mathfrak{P}_k, y \in P\}$ contains both the points $y'$ and $fg_k(y')$. Thus $i_N | \overline{W}_k$ and $fg_k$ are $\mathfrak{T}_k$-close and hence (ix) is true for $j=k$. Finally, let $\dim X \leqq n$. By using the same argument as in the construction of the map $g_k$, we can find an open collection $\mathfrak{V}_k$ of $M$ which covers $X$, an open neighborhood $U_k$ of $X$ in $M$ such that $\bar{U}_k \subset \cup \{V : V \in \mathfrak{V}_k\}$, a map $\xi_k$ from the nerve $A_k$ of $\mathfrak{V}_k \cap \bar{U}_k$ into $M$ satisfying the conditions:

(3.11) (i) $\mathfrak{V}_k$ and $\mathfrak{V}_k \cap X$ are similar, $\mathrm{ord}\,\mathfrak{V}_k \leqq n+1$, $\mathrm{mesh}\,\mathfrak{V}_k < 2^{-k}$ and $\mathfrak{V}_k \overset{*}{>} f^{-1}\mathfrak{W}_k$.

(ii) $i_M | \bar{U}_k \cong \xi_k\phi_k : \bar{U}_k \to M \lim.\mathfrak{U}_k$ and $g_k f \xi_k \phi_k \cong g_k f | \bar{U}_k \lim.\mathfrak{U}_k$, where $\phi_k : \bar{U}_k \to A_k$ is a canonical map.

(iii) For each closed simplex $\sigma$ of $A_k$ there is a $V$ of $\mathfrak{V}_{k-1}$ containing both $\xi_k(\sigma)$ and $g_k f \xi_k(\sigma)$.

By Lemma 3 the condition (iv) means $\xi_k \cong g_k f \xi_k \lim.\mathfrak{U}_{k-1}$. Thus $i_M | \bar{U}_k \cong g_k f | \bar{U}_k : \bar{U}_k \to M_{k-2} \lim.\mathfrak{U}_{k-2}$ and hence (3.10) (x) holds for $j=k$. We proved that there exist $\mathfrak{U}_j, \cdots, h_j$ for each $j=1, 2, \cdots$ satisfying the conditions (i)—(x) in (3.10).

To complete the proof, for each $j$, $3<j$, consider the maps $g_j : \overline{W}_j \to M_j$ ($\subset M_{j-3}$) and $g_{j-1} : \overline{W}_{j-1} \to M_{j-1}$ ($\subset M_{j-3}$). Since $g_{j-1} | \overline{W}_j \cong g_j : \overline{W}_j \to M_{j-3} \lim.\mathfrak{U}_{j-3}$ by (3.10) (viii) and $M_{j-3}$ is an $ANR$, by Lemma 5 there is a map $g_j^{j-1} : \overline{W}_{j-1} \to M_{j-3}$ such that $g_j^{j-1} | \overline{W}_j = g_j$, $g_j^{j-1} | \overline{W}_{j-1} - W_{j-1} = g_{j-1}$ and $g_{j-1} \cong g_j^{j-1} \lim.\mathfrak{U}_{j-3}$. Define $\bar{g}_j : \overline{W}_3 \to M_1$ by $\bar{g}_j | \overline{W}_{i-1} - W_i = g_i^{i-1} | \overline{W}_{i-1} - W_i$ for each $i$, $3<i\leqq j$. Note that, for each $i$, $\bar{g}_i | \overline{W}_i \cong \bar{g}_{i+2} | \overline{W}_i \lim \mathrm{St}(\mathfrak{U}_{i-2}, \mathfrak{U}_{i-1})$, where $\mathrm{St}(\mathfrak{U}_{i-2}, \mathfrak{U}_{i-1}) = \{\mathrm{St}(U, \mathfrak{U}_{i-1}) : U \in \mathfrak{U}_{i-2}\}$. Hence, for each $i$ and $j$, $3<i\leqq j$, $\bar{g}_i | \overline{W}_i \cong \bar{g}_j | \overline{W}_i \lim.\mathfrak{U}_{i-3}$. Extend $\bar{g}_j : \overline{W}_3 \to M_1$, $j=3, 4, \cdots$, to a map of $N$ into $M$ and denote it by $\bar{g}_j$ again. Let $\bar{g}_i$, $i=1, 2$, be an extension of $g_i$ from $N$ to $M$. Then the sequence $\{\bar{g}_k : k=1, 2, \cdots\}$ of $N$ into $M$ satisfies the following conditions:

(3.12) For each $i$ and $j$, $3<i\leqq j$, $\bar{g}_i | \overline{W}_i \cong \bar{g}_j | \overline{W}_i \lim.\mathfrak{U}_{i-3}$.

It remains to prove that $\{f_k = f : k=1, 2, \cdots\}$ and $\{\bar{g}_k : k=1, 2, \cdots\}$ satisfy (2.2) and (2.4), and (2.3) in case $\dim X \leqq n$. Let us prove that $\underline{g}'' = \{\bar{g}_k, Y-B, X-A\}_{N,M}$ is a $W$-sequence and the first relation of (2.4) $\underline{g}'\underline{f}' \cong \underline{i}_{A,M}$ holds in case $\dim X \leqq n$. The other parts are similarly proved. Let $C$ be a compact set in $Y-B$. Put $D = f^{-1}(C)$. Then $D$ is a compact set in $X-A$. We shall prove that $D$ is $\underline{g}''$-assigned to $C$. Let $G$ be an open neighborhood of $D$ in $M$. Since $d(D, M-G)>0$, by (3.10) (iv), there is an $i$ such that $\mathrm{St}(D, \mathfrak{U}_{i-4})\subset G$. Put $H=\mathrm{St}(D, \mathfrak{W}_i \cap \overline{W}_i)$. Then $H$ is a neighborhood of $D$. From (3.12) it follows that for every $j$, $i\leqq j$, $\bar{g}_i|H\cong\bar{g}_j|H$ in $G$. Thus $\underline{g}''$ is a $W$-sequence. To prove the first relation of (2.4), let $C$ be a compact set of $A$. Put $D=f^{-1}f(C)$. We shall prove that $D$ is $\underline{g}'\underline{f}'$-assigned to $C$. Let $G$ be an open neighborhood of $D$ in $M$. Take an $i$ such that $\mathrm{St}(D, \mathfrak{U}_{i-5})\subset G$ and put $H=\mathrm{St}(D, \mathfrak{B}_i)$, where $\mathfrak{B}_i$ is the open collection given in (3.11). Then for each $j$, $i\leqq j$, $\bar{g}_j f|H\cong i_M|H$ in $G$, because $i_M|H\cong\bar{g}_i f|H \lim.\mathfrak{U}_{i-2}$ by (3.10) (x), $\bar{g}_i f|H\cong\bar{g}_j f|H \lim.\mathfrak{U}_{i-3}$ by (3.12) and hence $i_M|H\cong\bar{g}_j f|H \lim.\mathfrak{U}_{i-4}$ for each $j$, $i\leqq j$. This completes the proof.

A simple example shows that we can not omit the perfectness of a map $f$ in Theorem 2.

The shape of a set consisting of one point is said to be *trivial* and denoted by Sh(1). As known by [**4**, **6**, **8**], a compactum is of *trivial shape* if and only if either $X$ is an *FAR* in the sense of Borsuk [**4**] or there is an imbedding of $X$ into the Hilbert cube $Q$ whose image has a complete neighborhood system $\{X_k : k=1, 2, \cdots\}$ in $Q$ such that $X_k$ is homeomorphic to $Q$.

The following corollary is a generalization of Borsuk [**4**, Theorem (6.1)].

COROLLARY 1. *An $n$-dimensional compactum $X$ is of trivial shape if and only if $X$ is approximatively $k$-connected for $k=0, 1, \cdots, n$.*

For the proof, it is enough to assume in Theorem 2 that $X$ is an $n$-dimensional compactum and $Y$ is a point.

From Theorem 2 and Corollary 1 follows:

COROLLARY 2. *Let $X$ and $Y$ be finite dimensional metric spaces and let $A$ and $B$ be subsets of $X$ and $Y$ respectively. If there is a perfect map $f:(X, A)\to(Y, B)$ such that $f(X-A)=Y-B$ and $f(A)=B$ and $f^{-1}(y)$ is of trivial shape for each $y\in Y$, then* $\mathrm{Sh}_W(X)=\mathrm{Sh}_W(Y)$ *and* $\mathrm{Pos}(X, A)=\mathrm{Pos}(Y, B)$.

If $X$ is compactum, then we have $\mathrm{Sh}_W(X)=\mathrm{Sh}(X)$. Hence, in case $X$ is compact and $A=\emptyset$ in Corollary 2, we obtain the following theorem by Sher [**10**].

COROLLARY 3 (R. B. Sher). *If $X$ and $Y$ are finite dimensional compacta and $f$ is a map of $X$ onto $Y$ such that $f^{-1}(y)$ is of trivial shape for each $y\in Y$, then* $\mathrm{Sh}(X)=\mathrm{Sh}(Y)$.

As seen in [**10**, Remark, p. 88], R. D. Anderson proved a strengthened version of Corollary 3 by weakening the hypothesis that $X$ and $Y$ are finite dimensional to the case in which $X$ and $Y$ are countable dimensional. It is open whether Corollary 2 is weakened in a similar form or not. The following is a generalization of Sher's theorem [**10**, Theorem 12].

COROLLARY 4. *Let $(X, x_0)$ and $(Y, y_0)$ be pointed metric spaces and let $f$ be a perfect map of $(X, x_0)$ onto $(Y, y_0)$. If $f^{-1}(y)$ is approximatively $k$-connected* *for each* $y\in Y$ *and* $k=0, 1, \cdots, n$, *then the induced homomorphism* $f_*:\underline{\pi}_k(X, x_0)\rightarrow\underline{\pi}_k(Y, y_0)$ *is an isomorphism onto for* $k=1, 2, \cdots, n-1$, *where* $\underline{\pi}_*$ *is the fundamental group defined by Borsuk* [3, § 31].

Note that there are no assumptions for dimensionality of $X$ and $Y$. The proof is given by a suitable modification of the proof of Theorem 2. For each $k=1, 2, \cdots$, we construct $\mathfrak{U}_k, \cdots, h_k$ satisfying (3.10) (i)—(vii) without regard to the order of collections $\mathfrak{W}_k$ and we define $h_k$ as a map from the $n$-skeleton $K_k^n$ of $K_k$ into $M_k$ by the same way as in the proof of Theorem 2. If $K$ is a finite polyhedron with $\dim K\leqq n$ and $\eta: K\rightarrow \overline{W}_k$ is a map, then we can find a map $\xi: K\rightarrow K_k$ such that $f\xi$ and $\eta$ are $2^{-k+1}$-homotopic. In this case a map $\xi$ is defined as a composition $h_k\xi'$ of maps $\xi': K\rightarrow K_k^n$ and $h_k: K_k^n\rightarrow M_k$, where $K_k^n$ is the $n$-skeleton of $K_k$. From this consideration it follows that the induced homomorphism $f_*:\underline{\pi}_i(X, x_0)\rightarrow\underline{\pi}_i(Y, y_0)$ is $1:1$ for $i<n$. That $f_*$ is onto is shown similarly.

For a compactum $X$, denote by $\square(X)$ the set of all components of $X$. We consider $\square(X)$ as the decomposition space of $X$. Then it is a compactum. As an application of Theorem 2 we obtain a simple proof of the following theorem by Borsuk [3, Theorem (8.1)].

COROLLARY 5 (K. Borsuk). *Let $X$ and $Y$ be compacta in the Hilbert cube $Q$. Then for every fundamental sequence $\underline{f}: X\rightarrow Y$ there is a unique (continuous) map $\Lambda_f:\square X\rightarrow\square Y$ such that for every component $X_0$ of $X$ $\underline{f}: X_0\rightarrow\Lambda_f(X_0)$ is a fundamental sequence. Moreover $\Lambda_f$ depends only on the fundamental class $\underline{f}$ and this dependence is covariant, that is, if $\underline{g}: Y\rightarrow Z$ is a fundamental sequence then $\Lambda_{gf}=\Lambda_g\Lambda_f$.*

PROOF. Let $\pi_X: X\rightarrow\square X$ and $\pi_Y: Y\rightarrow\square Y$ be the decomposition maps. Since $\pi_X^{-1}(x)$ is a continuum for each $x\in\square X$, it is approximatively 0-connected. Since $\dim\square X=0$, by Theorem 2 there is a fundamental sequence $\underline{h}:\square X\rightarrow X$ such that $\underline{\pi}_X\underline{h}\cong i_{\square X,Q}$, where $\underline{\pi}_X$ is the fundamental sequence generated by $\pi_X$. Consider $\underline{\pi}_Y\underline{fh}:\square X\rightarrow\square Y$. By Lemma 1 $\underline{\pi}_Y\underline{fh}$ is generated by a map $\Lambda_f:\square X\rightarrow\square Y$. Obviously $\Lambda_f$ satisfies the corollary.

By the proof it is known that Corollary 5 holds in metric spaces $X$ and $Y$ for which the decomposition maps $\pi_X: X\rightarrow\square X$ and $\pi_Y: Y\rightarrow\square Y$ are perfect.

For a metric space $X$ the *weak fundamental dimension* $Fd_W(X)$ is defined as the minimum of dimensions of all metric spaces $Y$ with $\mathrm{Sh}_W(Y)\geqq\mathrm{Sh}_W(X)$, that is, $Fd_W(X)=\mathrm{Min}\,\{\dim Y:\mathrm{Sh}_W(Y)\geqq\mathrm{Sh}_W(X)\}$ (cf. [3, p. 31]). The following corollary follows from Theorem 2.

COROLLARY 6. *If $f$ is a perfect map of a space $X$ onto an $n$-dimensional space $Y$ and $f^{-1}(y)$ is approximatively $k$-connected for each $y\in Y$ and $k=0, 1, \cdots, n$, then $Fd_W(X)\geqq Fd_W(Y)$. In addition, if $\dim X\leqq n$, then $Fd_W(X)=Fd_W(Y)$.*

Let $(X, x_0)$ and $(Y, y_0)$ be pointed spaces. According to Borsuk [3, § 33], by a *sum* of $(X, x_0)$ and $(Y, y_0)$ we mean a pointed space $(Z, z_0)$ such that $Z=Z'\cup Z''$, where $Z'$, $Z''$ are closed subsets of $Z$ such that $Z'\cap Z''=\{z_0\}$ and that there exist two homeomorphisms: $h' : (X, x_0)\rightarrow(Z', z_0)$ and $h'' : (Y, y_0)\rightarrow(Z'', z_0)$. We write $(Z, z_0)=(X, x_0)\underset{\text{top}}{+}(Y, y_0)$. By [3, Theorem (33.1)] it is known that the weak shape of the pointed space $(Z, z_0)=(X, x_0)\underset{\text{top}}{+}(Y, y_0)$ depends only on the weak shapes of pointed spaces $(X, x_0)$ and $(Y, y_0)$. We write $\mathrm{Sh}_W(Z, z_0)=\mathrm{Sh}_W(X, x_0)+\mathrm{Sh}_W(Y, y_0)$. A weak shape $\mathrm{Sh}_W(X, x_0)$ is said to be *simple* if $\mathrm{Sh}_W(X, x_0)=\mathrm{Sh}_W(X', x_0')+\mathrm{Sh}_W(X'', x_0'')$ means either $\mathrm{Sh}_W(X', x_0')$ is trivial or $\mathrm{Sh}_W(X', x_0')=\mathrm{Sh}_W(X, x_0)$.

COROLLARY 7. *Let $(X, x_0)$ be a finite dimensional pointed space whose weak shape is simple. Suppose that there is a finite dimensional compactum $N$ of $X$ with trivial shape such that $X-N$ is a union of non-empty and disjoint open sets $U$ and $V$ and $x_0\in N$. Then either $\mathrm{Sh}_W(X-V, x_0)$ or $\mathrm{Sh}_W(X-U, x_0)$ is trivial.*

PROOF. Denote the quotient spaces $X/N$, $X-V/N$ and $X-U/N$ by $Y$, $Y_1$ and $Y_2$ respectively. Let $y_0$ be the point of $Y$ corresponding to $N$. Obviously $Y$ is finite-dimensional. From the proof of Theorem 2 we can find that $\mathrm{Sh}_W(X, x_0)=\mathrm{Sh}_W(Y, y_0)$, $\mathrm{Sh}_W(X-V, x_0)=\mathrm{Sh}_W(Y_1, y_0)$ and $\mathrm{Sh}_W(X-U, x_0)=\mathrm{Sh}_W(Y_2, y_0)$. Since $\mathrm{Sh}_W(Y, y_0)=\mathrm{Sh}_W(Y_1, y_0)+\mathrm{Sh}_W(Y_2, y_0)$, we have $\mathrm{Sh}_W(X, x_0)=\mathrm{Sh}_W(X-V, x_0)=\mathrm{Sh}_W(X-U, x_0)$. This completes the proof.

## References


[1] K. Borsuk, Theory of retracts, Warszawa, 1967.
[2] K. Borsuk, Concerning homotopy properties of compacta, Fund. Math., **62** (1968), 223-254.
[3] K. Borsuk, Theory of shape, Lecture Notes Series, Aarhus University, 1971.
[4] K. Borsuk, A note on the theory of shape of compacta, Fund. Math., **67** (1970), 265-278.
[5] K. Borsuk, On positions of sets in spaces, Fund. Math., **79** (1973), 141-158.
[6] D.M. Hyman, On decreasing sequences of compact absolute retracts, Fund. Math., **64** (1969), 91-97.
[7] Y. Kodama, On a closed mapping between ANR's, Fund. Math., **45** (1958), 217-228.
[8] S. Mardešić, Decreasing sequences of cubes and compacta of trivial shape, General Topology and its Applications, **2** (1972), 17-23.
[9] S. Mardešić and J. Segal, Shapes of compacta and ANR-systems, Fund. Math., **72** (1972), 41-59.
[10] R.B. Sher, Realizing cell-like maps in euclidean space, General Topology and its Applications, **2** (1972), 75-89.



Yukihiro KODAMA
Department of Mathematics
Faculty of Science
Tokyo University of Education
Otsuka, Bunkyo-ku
Tokyo, Japan