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Localization theorem for holomorphic approximation on open Riemann surfaces

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§ 1. Introduction.

Let R be an open Riemann surface and K a compact subset of R. Let $C(K)$ be the class of complex valued continuous functions on K. A function f of $C(K)$ is said to be in $H(K)$, if f is the uniform limit on K of functions, each holomorphic in some neighborhood of K.

The localization theorem is the following

THEOREM A. *Let f be a function of $C(K)$. Suppose, for every point P of K, there is a neighborhood U_P of P such that $f|_{(\bar{U}_P\cap K)}\in H(\bar{U}_P\cap K)$. Then f is in $H(K)$.*

This theorem was proved in Bishop [2] and Kodama [5]. Garnett simplified the proof in the plane case [3].

In this note, we shall give two new proofs of Theorem A. The first proof is based on the solution of $\bar{\partial}$-problem with bounded estimate. The second one is a generalization of Garnett's method. Through both proofs, the elementary differential (Behnke-Stein [1]) plays the important role. In Section 2, we shall prove a generalization of Mergelyan's theorem for rational approximation [6] to open Riemann surface. In Section 8, we shall make a remark about the higher dimensional case.

§ 2. An approximation theorem.

Let $H(K, R)$ be the class of functions on K which are uniform limits on K of functions, each holomorphic on R. Let $A(K)$ be the class of functions of $C(K)$ which are holomorphic in the interior of K. As an application of Theorem A, we have the following

THEOREM B. *Let ρ be a metric on R. Suppose there is a positive constant k such that every component of $R\backslash K$ has ρ-diameter not less than k. Then $A(K)=H(K)$. In particular, if $R\backslash K$ has no relatively compact component, then $A(K)=H(K, R)$.*

PROOF. Let P be any point of K, U_P be a coordinate neighborhood of P

of ρ-diameter less than k and ψ be a coordinate map of U_P onto the disk $D=\{z \in \boldsymbol{C};\ |z|<1\}$ such that $\psi(P)=0$. Let V_P denote the neighborhood $\psi^{-1}\left(\left\{|z|<\frac{1}{2}\right\}\right)$. Then $U_P \backslash (\bar{V}_P \cap K)$ is connected and hence $\boldsymbol{C} \backslash \psi(\bar{V}_P \cap K)$ is also connected. By Mergelyan's theorem for polynomial approximation, we have $A(\psi(\bar{V}_P \cap K))=H(\psi(\bar{V}_P \cap K))$ and therefore $A(\bar{V}_P \cap K)=H(\bar{V}_P \cap K)$. Theorem A implies that $A(K)=H(K)$. The last statement follows from the following theorem.

THEOREM (Behnke-Stein [1]). *Suppose $R \backslash K$ has no relatively compact component. Then $H(K)=H(K, R)$.*

§ 3. Elementary differential.

We need the following result proved in [1]. There exists a differential $\omega(P, Q)$ on R satisfying the following conditions:

i) For any fixed point Q, $\omega(P, Q)$ is a meromorphic differential in P, which has its only pole at Q of residue $2\pi i$. If ψ is a coordinate map defined on a neighborhood V of P, and if $z=\psi(P)$, then we can write $\omega(P, Q)=k(z, Q)dz$.

ii) For fixed P and for fixed coordinate z near P, $k(z, Q)$ is a meromorphic function of Q on R with a pole only at $Q=P$.

Let G be a relatively compact open set of R whose boundary ∂G consists of a finite number of smooth Jordan curves. Let f be a function in $C^1(\bar{G})$. We write $\bar{\partial} f$ for a differential $f_{\bar{z}}\, d\bar{z}$. Then $\eta(P)=f(P)\cdot\omega(P, Q)$ is a differential in $C^1(\bar{G} \backslash \{Q\})$ and we have $d\eta(P)=\bar{\partial} f(P) \wedge \omega(P, Q)$. Therefore, by Stokes' theorem, we have the following generalized Green's formula:

$$f(Q)=\int_{\partial G} f(P)\omega(P, Q)-\int_G \bar{\partial} f(P) \wedge \omega(P, Q)\,. \tag{1}$$

In particular, if f is holomorphic in G, then we have

$$f(Q)=\int_{\partial G} f(P)\omega(P, Q)\,. \tag{2}$$

A differential $\gamma=g(z)dz$ of type (1, 0) defined on R is said to be in the class $\mathfrak{L}^1$, if, for any coordinate map ψ on an open neighborhood U and for any relatively compact subset V of U,

$$\int_V |g(z)|\cdot|d\bar{z} \wedge dz|<\infty$$

holds. This property is independent of the choice of U, ψ and V.

We note that, for fixed Q, $\omega(P, Q)$ is in $\mathfrak{L}^1$ as a differential in P, because of its behavior near Q.

The following lemma will be used in Section 7.

LEMMA 1. *Let ϕ be a coordinate map on a neighborhood V and P_0, Q_0 be distinct points in V. Set $z_0=\phi(P_0)$. If h is a function in $C^1(R)$ with compact support in V, then we have*

$$\int_R k(z_0, P)\cdot\bar\partial h(P)\wedge\omega(P, Q_0)=\{h(P_0)-h(Q_0)\}\,k(z_0, Q_0)\,.$$

PROOF. Let P be a point in V and set $z=\phi(P)$. From (1), we have

$$\int_R k(z_0, P)\bar\partial h(P)\wedge\omega(P, Q_0)$$

$$=-k(z_0, Q_0)h(Q_0)-\lim_{\varepsilon\to 0}\int_{|z-z_0|=\varepsilon} h(\phi^{-1}(z))k(z_0, \phi^{-1}(z))k(z, Q_0)dz\,.$$

By the property of $k(z, Q)$, this proves the lemma.

§ 4. The bounded solution of $\bar\partial$-problem.

Let u be a bounded function defined on a set S of $\boldsymbol{C}$ or R. We use the notation $\|u\|_S$ as the sup norm of u on S. The following lemma is well known.

LEMMA 2. *Let G be a bounded open set of $\boldsymbol{C}$ and G' any open subset of G. For every function v of $C^\infty(G')$ there exists a function u of $C^\infty(G')$ such that $\bar\partial u=vd\bar z$ in G' and*

$$(3)\qquad \|u\|_{G'}\leqq d(G)\|v\|_{G'}\,,$$

where $d(G)$ denotes the diameter of G.

Indeed, u is given by

$$(4)\qquad u(z)=\frac{1}{2\pi i}\int_{G'}\frac{v(\zeta)}{\zeta-z}d\zeta\wedge d\bar\zeta\,,$$

and (3) follows from

$$\int_{G'}\frac{1}{|\zeta-z|}|d\zeta\wedge d\bar\zeta|\leqq\int_G\frac{|d\zeta\wedge d\bar\zeta|}{|\zeta-z|}\leqq 2\pi d(G)\,.$$

In the next place, we shall generalize Lemma 2 to an open subset of R. Let G be a relatively compact open subset of R and α a differential of type (0, 1) defined on $\bar G$. We mean a finite covering $\mathfrak{A}$ of $\bar G$ by the system of finite number of pairs $\{(V_j, z_j)\}$ of open neighborhoods V_j covering $\bar G$ and local coordinates z_j defined on V_j, $j=1,\cdots,N$. For fixed $\mathfrak{A}$, we define the norm of α on any subset of G as follows: Let ϕ_j be the coordinate maps defining z_j. If α is written as $\alpha=a_j(z_j)d\bar z_j$ in $\phi_j(V_j\cap G)$ then the norm is defined by

$$\|\alpha\|_{S,\mathfrak{A}}=\sum_{j=1}^{N}\|a_j(z_j)\|_{\phi_j(V_j\cap S)}\,,$$

provided that the right hand side is finite.

LEMMA 3. *Let G be a relatively compact subset of R and $\mathfrak{A}=\{(V_j, z_j)\}_{j=1}^N$ be a finite covering of $\bar{G}$. Let G' be any open subset of G. For every differential α of type (0, 1) in $C^\infty(\bar{G}')$, there exists a function u in $C^\infty(G')$ such that $\bar{\partial}u=\alpha$, and*

(5) $$\|u\|_{G'} \leqq C\cdot\|\alpha\|_{G',\mathfrak{A}},$$

where C is a constant depending only on G and $\mathfrak{A}$.

PROOF. From the property of $\omega(P, Q)$, we have

$$\int_{V_j\cap G}|k(z_j, Q)|\cdot|dz_j\wedge d\bar{z}_j|\leqq M \qquad (j=1, \cdots, N),$$

for some constant M depending on G and $\mathfrak{A}$. Therefore, if $\alpha=a_j(z_j)d\bar{z}_j$ in $V_j\cap G'$, we have

$$\begin{aligned}\left|\int_{G'}\alpha(P)\wedge\omega(P, Q)\right| &\leqq \sum_{j=1}^N\int_{G'\cap V_j}|a_j(z_j)k(z_j, Q)|\,|dz_j\wedge d\bar{z}_j| \\ &\leqq \|\alpha\|_{G',\mathfrak{A}}\cdot\sum_{j=1}^N\int_{G'\cap V_j}|k(z_j, Q)|\,|dz_j\wedge d\bar{z}_j| \\ &\leqq N\cdot M\cdot\|\alpha\|_{G',\mathfrak{A}}.\end{aligned}$$

Thus, the required function u is given by

$$u(Q)=\int_{G'}\alpha(P)\wedge\omega(P, Q).$$

§ 5. The first proof of Theorem A.

We can choose N coordinate neighborhoods $U_1, \cdots, U_N$ such that $K\subset\bigcup_{j=1}^N U_j$ and $f|_{(\bar{U}_j\cap K)}\in H(\bar{U}_j\cap K)$, $j=1, \cdots, N$. Let the local coordinates z_j in U_j be fixed. For any positive number ε, there exist open sets $\Omega_j\supset\bar{U}_j\cap K$ and f_j holomorphic in Ω_j such that

(6) $$|f_j-f|<\varepsilon \qquad \text{on} \quad \bar{U}_j\cap K,\ j=1, \cdots, N.$$

Let G_0 be an open set such that $K\subset G_0\subset\bar{G}_0\subset\bigcup_{j=1}^N U_j$ and $\{\varphi_j\}_{j=1}^N$ be non negative functions on $C^\infty(R)$ such that each φ_j has the compact support in U_j and $\sum_{j=1}^N\varphi_j\equiv 1$ on G_0. Set $C_1=\sum_{j=1}^N\sup_{U_j}|\partial\varphi_j/\partial\bar{z}_j|$. Note that C_1 is independent of ε.

For every indices j and k, we define the function h_{jk} by $h_{jk}=\varphi_j(f_j-f_k)$ in $\Omega_j\cap\Omega_k$ and $h_{jk}=0$ in $\Omega_k\setminus\bar{U}_j$. Then h_{jk} is of class C^∞ in $\Omega'_{kj}=(\Omega_j\cap\Omega_k)\cup(\Omega_k\setminus\bar{U}_j)$. Set $\Omega'_k=\bigcap_{j=1}^N\Omega'_{kj}$. Since $\Omega_k\supset\bar{U}_k\cap K$, we have $\Omega'_k\supset\bar{U}_k\cap K$. Now set $h_k=\sum_{j=1}^N h_{jk}$, then h_k is in $C^\infty(\Omega'_k)$ and by (6) we have

(7) $$\|h_k\|_{\Omega'_k \cap K} < 2\varepsilon \quad \text{and} \quad \|\bar{\partial} h\|_{\Omega'_k \cap K, \mathfrak{A}} \leqq 2C_1 \cdot \varepsilon ,$$

where $\mathfrak{A} = \{(U_j, z_j)\}_{j=1}^{N}$.

Since $h_k - h_j = f_j - f_k$ in $\Omega'_j \cap \Omega'_k \cap G_0$, there is a differential of type (0, 1) in $C^\infty\left(G_0 \cap \left(\bigcup_{j=1}^{N} \Omega'_j\right)\right)$ such that $\alpha = -\bar{\partial} h_k$ in every $\Omega'_k \cap G_0$. By means of the continuity of α, we can find an open set G such that $K \subset G \subset G_0$ and $\|\alpha\|_{G, \mathfrak{A}} < 3C_1 \cdot \varepsilon$. By Lemma 3, there exists a function $u \in C^\infty(G)$ such that $\bar{\partial} u = \alpha$ and

(8) $$\|u\|_G < 3C_1 \cdot C \cdot \varepsilon ,$$

where C is dependent only on G_0 and $\mathfrak{A}$, and therefore not on ε.

Set $g_j = h_j + u$ on $\Omega'_j \cap G$. Then g_j is holomorphic in $\Omega'_j \cap G$, and by (7) and (8) we have

(9) $$|g_j| < (2 + 3C_1 \cdot C)\varepsilon \quad \text{on} \quad \Omega'_j \cap K .$$

Since $g_k - g_j = h_k - h_j = f_j - f_k$, we can find the global function F, holomorphic in G such that $F = f_j + g_j$ in $\Omega'_j \cap G$. By (9), we have

(10) $$|f - F| < |g_j| + |f - f_j| < 3(1 + C_1 C)\varepsilon \quad \text{on} \quad \Omega'_j \cap K .$$

Since C and C_1 are independent of ε and (10) is valid for all over K, we can conclude that $f \in H(K)$.

§ 6. Measure orthogonal to $H(K)$.

Let μ be a finite complex Borel measure on R with a compact support. Let V be a coordinate neighborhood and z a local coordinate in V. Then, by the property of $\omega(P, Q)$, we have

(11) $$\int_V \left(\int |k(z, Q)|\, d|\mu|(Q)\right) |d\bar{z} \wedge dz| < \infty .$$

In particular, $\int |k(z, Q)|\, d|\mu|(Q)$ is finite for almost every point P and fixed local coordinate z corresponding to P. (The term "almost every" is used here and hereafter in the sense of Lebesgue which is meaningful on R.) Thus the map T defined by

$$T\mu(P) = \int \omega(P, Q)\, d\mu(Q)$$

is a map of finite complex measures with compact supports into the class $\mathfrak{L}^1$. $T\mu(P)$ is holomorphic off the support of μ.

LEMMA 4. *Let μ be a complex measure with the support in K. If $T\mu(P) = 0$ for almost every $P \in R$, then $\mu = 0$.*

PROOF. Let g be a C^1-function with the compact support. Then we have by (1)

$$g(Q) = -\int_R \bar{\partial} g(P) \wedge \omega(P, Q) \qquad \text{for} \quad Q \in K.$$

Hence, by Fubini's theorem, we have

$$\int g(Q) d\mu(Q) = -\int \Big(\int_R \bar{\partial} g(P) \wedge \omega(P, Q) \Big) d\mu(Q)$$
$$= -\int_R \bar{\partial} g(P) \wedge \Big(\int \omega(P, Q)\, d\mu(Q) \Big) = 0 .$$

Approximating by C^1-functions with compact supports, we obtain $\int g\, d\mu = 0$ for any continuous function g and hence $\mu = 0$.

LEMMA 5. *A complex measure μ supported on K is orthogonal to $H(K)$ if and only if $T\mu(P) = 0$ for every point P of $R \backslash K$.*

PROOF. Fixing a point $P \in R \backslash K$ and a local coordinate z near P, $k(z, Q)$ is a holomorphic function of Q in a neighborhood of K. Therefore, if μ is orthogonal to $H(K)$, then $T\mu(P) = 0$.

Conversely, for any function f holomorphic in a neighborhood of K, we can choose an open set G containing K such that ∂G consists of a finite number of smooth curves and f is holomorphic on $\bar{G}$. If $Q \in K$, we have by (2)

$$f(Q) = \int_{\partial G} f(P) \omega(P, Q) .$$

By Fubini's theorem, we have

$$\int f(Q) d\mu(Q) = \int_{\partial G} f(P) T\mu(P) = 0 .$$

Thus, we have $\int f\, d\mu = 0$ for all $f \in H(K)$. The lemma is proved.

§ 7. The second proof of Theorem A.

Let μ be a finite complex measure with a compact support and h a continuous function on R. By $h\mu$ we mean the measure defined as a linear functional $f \rightarrow \int f h\, d\mu$ for any continuous function f on R. If P is a point such that $\int |\omega(P, Q)|\, d|\mu|(Q)$ is finite, then, by approximating $\omega(P, Q)$ by continuous functions on R, we have

$$T(h\mu)(P) = \int h(Q) \omega(P, Q) d\mu(Q) . \tag{11}$$

Therefore, (11) holds almost everywhere on R.

LEMMA 6. *Let μ be a complex measure with a compact support, U a coordinate neighborhood and h a function in $C^\infty(R)$ with its compact support* *in U. Then there exists a measure μ_1 supported in U such that $hT\mu=T\mu_1$ holds almost everywhere on R.*

PROOF. Set $d\nu=-\bar{\partial}h\wedge T\mu$, then ν is a measure supported in U. Let ϕ be a coordinate map defined on U. Let P, P_1 and Q be the points in U. Set $z=\phi(P)$. If P is any point such that (11) holds, then by Lemma 1 we have

$$\begin{aligned} T\nu(P) &= \int\omega(P, P_1)d\nu(P_1) \\ &= -\Big(\int k(z, P_1)\bar{\partial}h(P_1)\wedge\Big[\int\omega(P_1, Q)d\mu(Q)\Big]\Big)dz \\ &= -\Big(\int\Big[\int k(z, P_1)\bar{\partial}h(P_1)\wedge\omega(P_1, Q)\Big]d\mu(Q)\Big)dz \\ &= -\int[h(P)-h(Q)]\omega(P, Q)d\mu(Q) \\ &= T(h\mu)(P)-h(P)T\mu(P)\,. \end{aligned}$$

Setting $\mu_1=h\mu-\nu$, the lemma is proved.

Though the followings are similar to the proof in [3], we shall give the details for completeness.

LEMMA 7. *Let μ be a complex measure supported on K and orthogonal to $H(K)$. For any covering $\{U_j\}$ of K by the coordinate neighborhoods, we can choose the measures μ_j each supported on U_j and orthogonal to $H(K\cap\bar{U}_j)$ such that $\mu=\sum\mu_j$.*

PROOF. Let $\{h_j\}$ be a partition of unity subordinate to $\{U_j\}$. By Lemma 6, we can find μ_j supported on U_j such that $h_jT\mu=T\mu_j$ a.e. on R. Since μ is orthogonal to $H(K)$, we have, by Lemma 5, $T\mu(P)=0$ for all $P\in R\backslash K$. Since h_j vanishes off U_j, and $T\mu_j(P)$ is holomorphic off U_j, we have $T\mu_j(P)=0$ for all $P\in R\backslash(K\cap\bar{U}_j)$. Hence, by Lemma 5, μ_j is orthogonal to $H(K\cap\bar{U}_j)$. We have $T\mu=\sum h_jT\mu=\sum T\mu_j=T(\sum\mu_j)$, and therefore, $T(\mu-\sum\mu_j)=0$ a.e. on R. By Lemma 4, we have $\mu=\sum\mu_j$. The lemma is proved.

We note that μ_j are orthogonal to $H(K)$.

Now we are in a position to prove Theorem A. We can find a covering $\{U_j\}$ of K by a finite number of coordinate neighborhoods such that $f\in H(\bar{U}_j\cap K)$ for every j. If μ is orthogonal to $H(K)$, then, by Lemma 7, there are measures μ_j supported in U_j such that $\mu=\sum\mu_j$ and each μ_j is orthogonal to $H(\bar{U}_j\cap K)$. Since $f\in H(\bar{U}_j\cap K)$, $\int fd\mu_j=0$, and hence we have $\int fd\mu=0$. Since it holds for all measures μ orthogonal to $H(K)$, we conclude that $f\in H(K)$.

§ 8. A generalization.

In this section, we shall remark about the higher dimensional case.

Let X be a complex manifold of dimension n and K a compact subset of X. $H(K)$ will be defined similarly to the case of Riemann surface. Let $\mathfrak{A}=\{(V_j, z^{(j)})\}_{j=1}^N$ be a finite covering of K by the coordinate neighborhoods. We write $z^{(j)}=(z_1^{(j)}, \cdots, z_n^{(j)})$ and denote the coordinate maps defining $z^{(j)}$ by ψ_j.

Let α be a (0, 1)-form of class C^∞ on an open set G containing K. α is represented as

$$\alpha=\sum_{k=1}^n a_k^{(j)}(z^{(j)})d\bar{z}_k^{(j)} \qquad \text{in} \quad G\cap V_j\,.$$

We define the norm of α on a subset S of G with respect to $\mathfrak{A}$ by

$$\|\alpha\|_{S,\mathfrak{A}}=\sum_{j=1}^N\sum_{k=1}^n \sup_{\psi_j(S\cap V_j)} |a_k^{(j)}(z^{(j)})|\,.$$

DEFINITION. A compact subset K of X is said to be of class (δ), if there exists a sequence $\{D_m\}$ of open subsets of X satisfying the following conditions:

(i) $D_m\supset \bar{D}_{m+1}$ $(m=1, 2, \cdots)$ and $\bigcap_{m=1}^\infty D_m=K$.

(ii) For every finite covering $\mathfrak{A}$ of K, there exists a positive constant C such that, for any (0, 1)-form α of class $C^\infty(\bar{D}_m)$ satisfying $\bar\partial\alpha=0$, there is a function u of class $C^\infty(D_m)$ such that $\bar\partial u=\alpha$ and

$$\sup_{D_m} |u|\leqq C\cdot\|\alpha\|_{D_m,\mathfrak{A}}\,,$$

provided that $D_m\subset\bigcup_{j=1}^N V_j$.

By a slight modification of the first proof of Theorem A, we can conclude the following

THEOREM A′. *Let K be a compact subset of a complex manifold in the class (δ). Then the statement of Theorem A is true for K.*

Lemma 2 shows that, for the case of $X=R$, all compact subsets are of class (δ). We shall give some examples of the compact sets of class (δ) in $\boldsymbol{C}^n$ $(n>1)$. A bounded domain G of $\boldsymbol{C}^n$ with C^∞-boundary is said to be strictly pseudoconvex, if there is a function $\rho(z)$ of class $C^\infty(\bar{G})$ such that ρ is strictly plurisubharmonic in a neighborhood of ∂G and $G=\{z\in\boldsymbol{C}^n;\ \rho(z)<0\}$. We cite the following

THEOREM (Henkin [4]). *Let G be strictly pseudoconvex bounded domain with C^∞-boundary in $\boldsymbol{C}^n$. If $\alpha=\sum_{k=1}^n a_k d\bar{z}_k$ is a (0, 1)-form of class $C^\infty(\bar{G})$, with $\bar\partial\alpha=0$, then there exists a function u of class $C^\infty(G)$ such that $\bar\partial u=\alpha$ and*

$$\sup_G |u|\leqq C(G)\cdot\sum_{k=1}^n \sup_G |a_k|\,,$$

where $C(G)$ is a constant depending on the diameter of G and the function $\rho(z)$ defining G.

If we can take the sequence $\{D_m\}$ of open sets descending to K, so that each D_m consists of a finite number of bounded strictly pseudoconvex domains and the constants $C(D_m)$ in Henkin's theorem are bounded, then K is of class (δ). Especially, if there is a function $\rho_0(z)$, strictly plurisubharmonic in a neighborhood of K, such that D_m are represented as $\left\{\rho_0<\frac{1}{m}\right\}$, then K is of class (δ).

For example, if K is the closure of a bounded strictly pseudoconvex domain D with C^∞-boundary, then K is of class (δ). In this case, we can take the function defining D as $\rho_0(z)$. Another example is a finite or compact totally real C^∞-submanifold M of $\boldsymbol{C}^n$. In this case, ρ_0 is defined by $\rho_0(z)=\operatorname{dist}(z, M)^2$ (Nirenberg-Wells [7]).

The same method as our first proof had already been applied by I. Lieb in Math. Ann. 184 (1969) 56-60 in the case of the strictly pseudoconvex domain of $\boldsymbol{C}^n$, which the author did not know during this work. The author thanks the referee for his valuable suggestions and comments.

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