

## On the fundamental conjecture of *GLC* II.

By Gaisi TAKEUTI

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This paper is a continuation of our former papers [1], [2]. In [1] we have extended Gentzen's logic calculus *LK* (Gentzen [3]) to *GLC* (generalized logic calculus) and shown that the "fundamental conjecture" of *GLC* would mean the consistency of the analysis. Here and in the following, we shall mean by the "fundamental conjecture" (abbrev. *FC*) of a logical system the theorem of the type: Every provable sequence in the logical system in question is provable without cut in the same logical system. Now *G<sup>1</sup>LC* is a subsystem of *GLC* obtained from *LK* by introducing *f*-variables with *n* argument-places  $\alpha[*_1, \dots, *_n]$  ( $n \geq 1$ ) (for terminology see [2]). It was also shown in [1] that *FC* of *G<sup>1</sup>LC* would imply the consistency of the theory of real numbers. In [2] we have proved *FC* on a certain subsystem of *G<sup>1</sup>LC* (containing of course *LK* of Gentzen), from which the consistency of the theory of natural numbers follows.

The purpose of the present paper is to prove *FC* on two more subsystems of *G<sup>1</sup>LC*, which will be called *PL* and *QL*. *PL* is obtained from *LK* by introducing *f*-variables without argument-place  $\alpha, \beta, \dots$ \*) *QL* is a subsystem of *G<sup>1</sup>LC* containing only proof-figures having no inference  $\forall$  on variables of type (0).

We give the proof only for the case of *QL*, but it is easy to see that our proof holds also for *PL*; one has only to put the number *n* of argument-places of *f*-variables to 0 everywhere in the proof, and to notice that the essential point of the proof depends on the following circumstance. Suppose there appears an inference  $\forall$  on *f*-variable in a proof-figure. We shall denote with  $\mathfrak{S}$  this inference.

\*) *f*-variables without argument-place  $\alpha, \beta, \dots$  may be considered as  $\alpha'[1], \beta'[1], \dots$ , where  $\alpha', \beta', \dots$  are *f*-variables with one argument-place and 1 is a fixed special variable. Thus *PL* is a subsystem of *G<sup>1</sup>LC*.

Let  $A(X_1, \dots, X_n)$  be any original formula of  $\mathfrak{F}$ . Then none of  $X_1, \dots, X_n$  is bound on the way of going down from  $A(X_1, \dots, X_n)$  to  $\mathfrak{F}$  in the proof-figure. This circumstance, which will be denoted by  $(C)$ , takes place both in  $PL$  and  $QL$ .

To prove  $FC$  of these systems, we shall use a reduction schema of a new type, essentially different from Gentzen's, which might also be used, as it seems to the author, to prove  $FC$  for a larger class of logical systems.

The reader is referred to [2] as to the definitions of concepts such as fibre, original formula, explicitness, end-place and suitable cut.

### § 1. Separative proof-figure.

#### 1.1. Equivalence of formulas in a proof-figure.

We shall speak of a 'formula in a proof-figure', when the formula is considered together with the place which it occupies in a proof-figure.

Let  $A$  and  $B$  be formulas in a proof-figure. ' $A$  is equivalent to  $B$ ' is defined recursively as follows:

1.1.1.  $A$  is equivalent to  $A$  itself.

1.1.2. If  $A$  is a successor of  $B$  and  $A$  is not a chief formula of a logical inference, then  $A$  is equivalent to  $B$ .

1.1.3. If  $A$  is equivalent to  $B$ , then  $B$  is equivalent to  $A$ .

1.1.4. If  $A$  is equivalent to  $B$  and  $B$  is equivalent to  $C$ , then  $A$  is equivalent to  $C$ .

#### 1.2. Leading formula.

Let  $A$  be a formula in a proof-figure and  $\mathfrak{T}$  be a fibre through  $A$ . Then ' $B$  is the leading formula of  $A$  in  $\mathfrak{T}$ ' means that  $B$  is the uppermost formula in  $\mathfrak{T}$ , which is equivalent to  $A$ . Therefore  $B$  is an ancestor of  $A$  or  $A$  itself.

' $B$  is a leading formula of  $A$ ' means that there exists such a fibre  $\mathfrak{T}$  that  $B$  is the leading formula of  $A$  in  $\mathfrak{T}$ . If  $B$  is a leading formula of  $A$ , then we see easily that  $B$  is a beginning formula or a weakening formula or a chief formula of a logical inference.

### 1.3. Inseparative number.

Let  $F(H)$  be a formula in a proof-figure and  $H$  be a ( $f$ -) variety of the form  $H(*, \dots, *)$  with  $n$  argument-places. ( $f$ -variety means that  $H(X_1, \dots, X_n)$  is a formula, where  $X_1, \dots, X_n$  are terms. And if  $n$  is zero,  $H$  is a formula.) Now, we consider a range of  $n$  terms  $(X_1, \dots, X_n)$ . We call this range inseparated, if and only if, there exist two original formula  $A$  and  $B$  of the indication  $F(H)$ , and  $A$  and  $B$  are of the form  $H(X_1, \dots, X_n)$  and not a weakening formula, and  $A$  is in left side of a sequence and  $B$  is in right side of a sequence. 'The inseparative number of  $F(H)$ ' is defined as the number of inseparated ranges of  $n$  terms.

$F(H)$  is called 'separative' if the inseparative number of  $F(H)$  is zero, and in other cases  $F(H)$  is called 'be inseparative'.

The inseparative number or the separativity of an inference

$$\frac{F(H), \Gamma \rightarrow \Delta}{\forall \varphi F(\varphi), \Gamma \rightarrow \Delta}$$

is defined as the inseparative number or the separativity of  $F(H)$  in the upper sequence of the inference.

### 1.4. Separativity of a proof-figure.

A proof-figure  $\mathfrak{P}$  is called 'separative', if and only if all the beginning formulas with logical symbols in  $\mathfrak{P}$  are divided into two classes, which are called the first and the second class, and the following conditions are fulfilled.

1.4.1. Every inference  $\forall$  left on  $f$ -variable contained in  $\mathfrak{P}$  is separative.

1.4.2. If  $A$  is a beginning formula belonging to the first class and  $B$  is related to  $A$  (See 2.5. in [2]), then any leading formula of  $B$  is neither a beginning formula belonging to the second class nor a chief formula in the left side of a sequence.

1.4.3. If  $A$  is a beginning formula belonging to the second class and  $B$  is related to  $A$ , then any leading formula of  $B$  is neither a beginning formula belonging to the first class nor a chief formula in the right side of a sequence.

1.4.4. Let  $D \rightarrow D$  be a beginning sequence. Two formulas of this sequence are simultaneously of the first class or of the second class.

## § 2. The first reduction.

In this section we consider only the proof-figures, whose beginning formulas have no logical symbols. We shall reduce these proof-figures to separative proof-figures.

2.1. Ordinal number in the first reduction.

First, we assign to each sequence of a proof-figure a natural number called 'the ordinal number of the sequence' as follows:

2.1.1. The ordinal number of a beginning sequence is one.

2.1.2. If  $\mathfrak{J}$  is an inference except  $\wedge$  right,  $\forall$  left on  $f$ -variable and cut, and  $\mathfrak{S}_1$  is the upper sequence of  $\mathfrak{J}$  and  $\mathfrak{S}_2$  is the lower sequence of  $\mathfrak{J}$ , then the ordinal number of  $\mathfrak{S}_2$  is equal to the ordinal number of  $\mathfrak{S}_1$ .

2.1.3. If  $\mathfrak{S}_1$  and  $\mathfrak{S}_2$  are two upper sequences of a cut or a  $\wedge$  right and  $\mathfrak{S}$  is the lower sequence of the inference, then the ordinal number of  $\mathfrak{S}$  is the sum of two ordinal numbers of  $\mathfrak{S}_1$  and  $\mathfrak{S}_2$ .

2.1.4. If  $\mathfrak{J}$  is an inference  $\forall$  left on  $f$ -variable and  $\mathfrak{S}_1$  and  $\mathfrak{S}_2$  are the upper and the lower sequence of  $\mathfrak{J}$  respectively, then the ordinal number of  $\mathfrak{S}_2$  is  $3^k\alpha$ , where  $k$  is the inseparative number of  $\mathfrak{J}$  and  $\alpha$  is the ordinal number of  $\mathfrak{S}_1$ .

The ordinal number of a proof-figure is defined as the ordinal number of the end-sequence of the proof-figure.

2.2. Let a proof-figure  $\mathfrak{P}$  be not separative. Since beginning formulas in  $\mathfrak{P}$  have no logical symbol, the conditions 1.4.2, 1.4.3 and 1.4.4 are trivial. Hence there exists an inseparative inference  $\forall$  left on  $f$ -variable

$$\frac{F(H), \Gamma \rightarrow \Delta}{\forall \varphi F(\varphi), \Gamma \rightarrow \Delta} \mathfrak{J}$$

in  $\mathfrak{P}$ . We denote the proof-figure to the lower sequence of  $\mathfrak{J}$  by  $\mathfrak{Q}$ , the inseparative number of  $\mathfrak{J}$  by  $k$ , and the ordinal number of the upper sequence of  $\mathfrak{J}$  by  $\alpha$ . There exists an inseparative range of  $n$  terms  $(X_1, \dots, X_n)$ .

2.3. Let  $\Gamma_i \rightarrow \Delta$ ,  $H(X_1, \dots, X_n)$ ,  $\Lambda_i (i=1, 2, \dots)$  be all the sequences which

have an original formula of  $F(H)$  of the form  $H(X_1, \dots, X_n)$  in the right side and it is not a weakening formula. Now we can modify by the circumstance (C) (See introduction) the proof-figure  $\mathfrak{Q}$  to a proof-figure  $\mathfrak{Q}_1$  of the following form:

$$\begin{array}{c}
 \downarrow \\
 \Gamma_i \rightarrow \Sigma, \Delta_j, H(X_1, \dots, X_n), \Lambda_i \\
 \text{Some exchanges} \\
 \Gamma_i \rightarrow \Sigma, H(X_1, \dots, X_n), \Delta_j, \Lambda_i \\
 \text{Some contractions} \\
 \Gamma_i \rightarrow H(X_1, \dots, X_n), \Delta_j, \Lambda_i \\
 \text{Weakening and some exchanges} \\
 \Gamma_i \rightarrow H(X_1, \dots, X_n), \Delta_j, H(X_1, \dots, X_n), \Lambda_i \\
 \downarrow \\
 \frac{F(H), \Gamma \rightarrow H(X_1, \dots, X_n), \Delta}{\forall \varphi F(\varphi), \Gamma \rightarrow H(X_1, \dots, X_n), \Delta} \mathfrak{S}_1
 \end{array}$$

where  $\Sigma$  is void if there are no sequences of the form  $\Gamma_j \rightarrow \Delta_j, H(X_1, \dots, X_n), \Lambda_j$  over the sequence  $\Gamma_i \rightarrow \Delta_j, H(X_1, \dots, X_n), \Lambda_j$ , and  $\Sigma$  is  $H(X_1, \dots, X_n)$  if there exists such a sequence.

Clearly we see that the inseparative number of the inference  $\mathfrak{S}_1$  is  $k-1$ , and therefore, the ordinal number of  $\mathfrak{Q}_1$  is  $3^{k-1}\alpha$ .

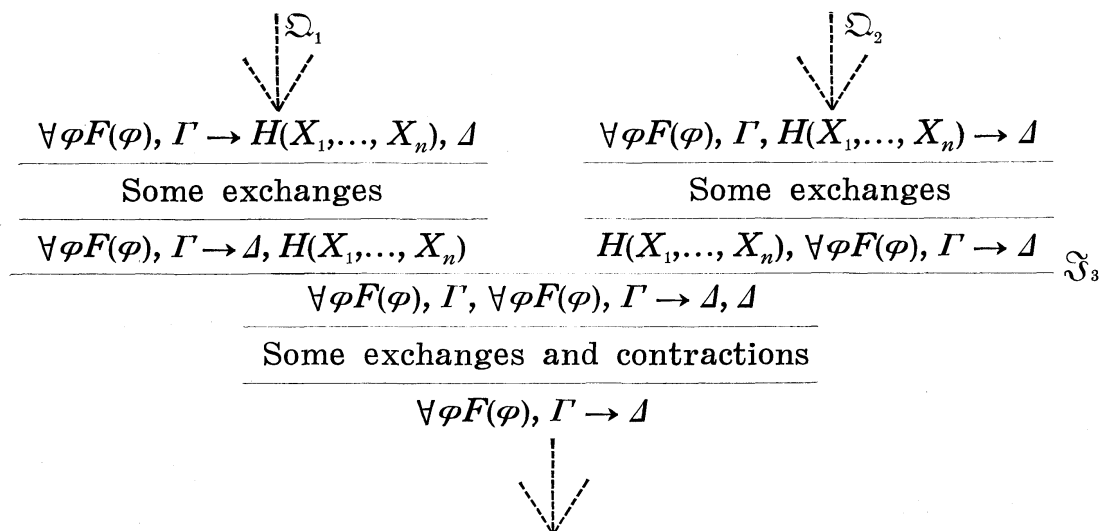
2.4. In a similar way, we have the following proof-figure  $\mathfrak{Q}_2$ :

$$\begin{array}{c}
 \downarrow \\
 \frac{F(H), \Gamma, H(X_1, \dots, X_n) \rightarrow \Delta}{\forall \varphi F(\varphi), \Gamma, H(X_1, \dots, X_n) \rightarrow \Delta} \mathfrak{S}_2
 \end{array}$$

Here the inseparative number of  $\mathfrak{S}_2$  is  $k-1$ , and therefore, the ordinal number of  $\mathfrak{Q}_2$  is  $3^{k-1}\alpha$ .

2.5. Now we reduce the proof-figure  $\mathfrak{P}$  to a proof-figure  $\mathfrak{P}'$  of the

following form :



Clearly the ordinal number of the lower sequence of  $\mathfrak{S}_3$  is  $2 \cdot 3^{k-1} \alpha$  and is smaller than the ordinal number  $3^k \alpha$  of  $\mathfrak{Q}$ , hence the ordinal number of  $\mathfrak{P}'$  is smaller than the ordinal number of  $\mathfrak{P}$ .

Therefore by finite reductions every proof-figure is reduced to a separative proof-figure.

Since every provable sequence is an end-sequence of a proof-figure, whose beginning sequences have no logical symbols, we attain our aim, if we prove the following theorem.

The end-sequence of a separative proof-figure is provable without cut.

### § 3. Proof-figure with potential.

In this section we assign a potential to each sequence of a separative proof-figure and using this notion we define the ordinal number of such proof-figure.

#### 3.1. Zero-part.

A formula  $A$  of a separative proof-figure is called a formula in the zero-part of this proof-figure, if and only if every leading formula of each formula related to  $A$  is a weakening formula or a beginning formula. (The case, where no formulas are related to  $A$ , is contained in this case.)

Clearly, if a formula belongs to the zero-part, then its ancestors belong also to the zero-part.

### 3.2. Degree.

We define the degree of a formula  $A$  in a separative proof-figure recursively as follows:

3.2.1. If  $A$  is a beginning or weakening formula or formula in the zero-part, then the degree of  $A$  is one.

3.2.2. If  $A$  is the successor of  $B$  and is not the chief formula of a logical inference nor contraction, then the degree of  $A$  is equal to the degree of  $B$ .

3.2.3. If  $A$  is the chief formula of a contraction, then the degree of  $A$  is equal to the maximal number of the degrees of two predecessors of  $A$ .

3.2.4. If  $A$  is not in the zero-part and is the successor of  $B$  and the chief formula of a logical inference except  $\wedge$  right, then the degree of  $A$  is equal to  $d+1$ , where  $d$  is the degree of  $B$ .

3.2.5. If  $A$  is not in the zero-part and is the chief formula of a inference  $\wedge$  right, then the degree of  $A$  is  $d+1$ , where  $d$  is the maximal number of the degrees of the two predecessors of  $A$ .

The degree of a cut is the maximal number of the degrees of the two cut-formulas of the cut.

### 3.3. Potential.

A separative proof-figure is called a proof-figure with potential, if to each sequence of this proof-figure corresponds a natural number called its potential satisfying the following conditions:

3.3.1. If a sequence  $\mathcal{S}_1$  is above a sequence  $\mathcal{S}_2$ , then the potential of  $\mathcal{S}_1$  is not less than the potential of  $\mathcal{S}_2$ .

3.3.2. If a sequence  $\mathcal{S}_1$  is an upper sequence of an inference except cut and a sequence  $\mathcal{S}_2$  is the lower sequence of the inference, then the potential of  $\mathcal{S}_1$  is equal to the potential of  $\mathcal{S}_2$ .

3.3.3. If  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are two upper sequences of a cut, then the potential of  $\mathcal{S}_1$  is equal to the potential of  $\mathcal{S}_2$ .

3.3.4. If a sequence  $\mathcal{S}$  is an upper sequence of a cut, then the potential of  $\mathcal{S}$  is not less than the degree of the cut.

3.3.5. The potential of the end-sequence is zero.

We see easily that every separative proof-figure may be considered as a proof-figure with potential by choosing a suitable potential. Therefore to prove *FC* of *QL* we have only to prove that the end-sequence of a proof-figure with potential is provable without cut.

3.4. Ordinal number in the second reduction.

We assign to each sequence of a proof-figure with potential a natural number called the ordinal number of this sequence recursively as follows:

- 3.4.1. The ordinal number of a beginning sequence is one.
- 3.4.2. If  $\mathfrak{S}_1$  is the upper sequence of an inference  $\mathfrak{J}$  on structure and  $\mathfrak{S}_2$  is the lower sequence of  $\mathfrak{J}$ , then the ordinal number of  $\mathfrak{S}_2$  is equal to the ordinal number of  $\mathfrak{S}_1$ .
- 3.4.3. If  $\mathfrak{S}_1$  is the upper sequence of an inference on logical symbol except  $\wedge$  right and  $\mathfrak{S}_2$  is the lower sequence, then the ordinal number of  $\mathfrak{S}_2$  is  $\alpha+1$ , where  $\alpha$  is the ordinal number of  $\mathfrak{S}_1$ .
- 3.4.4. If  $\mathfrak{S}_1$  and  $\mathfrak{S}_2$  are two upper sequences and  $\mathfrak{S}$  is the lower sequence of an inference  $\wedge$  right, then the ordinal number of  $\mathfrak{S}$  is  $\alpha+\beta$ , where  $\alpha$  or  $\beta$  is the ordinal number of  $\mathfrak{S}_1$  or  $\mathfrak{S}_2$  respectively.
- 3.4.5. If  $\mathfrak{S}_1$  and  $\mathfrak{S}_2$  are two upper sequences and  $\mathfrak{S}$  is the lower sequence of a cut, then the ordinal number of  $\mathfrak{S}$  is

$$\left. \begin{array}{c} \alpha + \beta \\ 3 \\ \dots \\ 3 \end{array} \right\} \sigma - \tau$$

where  $\alpha$  or  $\beta$  is the ordinal number of  $\mathfrak{S}_1$  or  $\mathfrak{S}_2$  respectively and  $\tau$  or  $\sigma$  is the potential of  $\mathfrak{S}$  or  $\mathfrak{S}_1$  (and  $\mathfrak{S}_2$ ) respectively.

We define the ordinal number of a proof-figure with potential as the ordinal number of its end-sequence.

§ 4. The second reduction.

In this section, we reduce a proof-figure with potential to a proof-figure with potential with a smaller ordinal number, by the method of § 4, § 5 and § 6 in [2].



We treat separately the following several cases.

4.1. The case, where the end-place contains an explicit logical inference.

This case can be treated in the same way as in § 4, [2]

4.2. The case, where the end-place contains an implicit beginning sequence and no logical inferences.

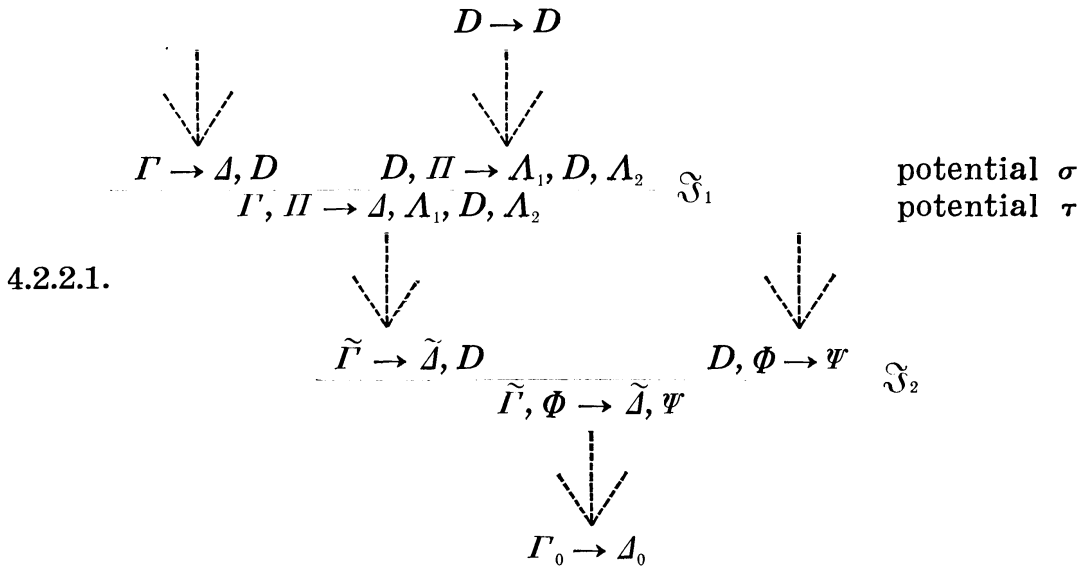
Let  $D \rightarrow D$  be one of the implicit beginning sequences in the end-place. We shall consider separately the following two cases.

4.2.1. The case, where one and only one of the two  $D$ 's is explicit.

This case is treated in the same way as in 5.1, [2].

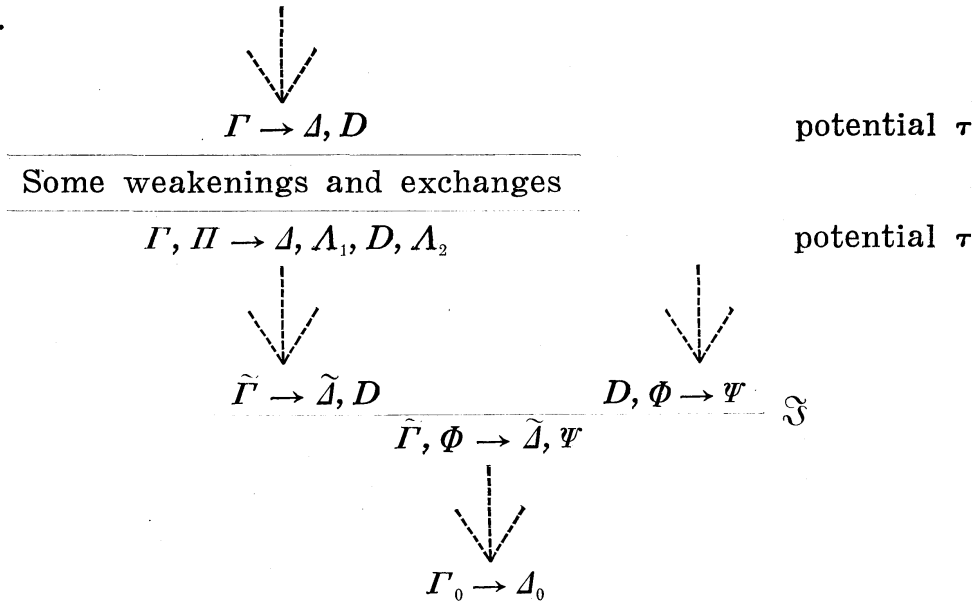
4.2.2. The case, where two  $D$ 's are implicit.

Let the proof-figure be of the following form :



We reduce 4.2.2.1 to the following 4.2.2.2.

4.2.2.2.



where every sequence, corresponding to a sequence in 4.2.2.1 loaded on  $\mathfrak{S}_1$ , has the potential  $\tau$ , and every other sequence has the same potential as one of the corresponding sequences in 4.2.2.1, and moreover every beginning formula with logical symbols belongs to the first or the second class according as the corresponding one in 4.2.2.1. belongs to the first or the second class.

Obviously, the ordinal number of the proof-figure 4.2.2.2 is less than the ordinal number of 4.2.2.1. (See 5.1 in [2].) We verify now the separativity and the conditions on the potentials of 4.2.2.2.

4.2.2.3. In case, where  $D$  has no logical symbols, the separativity of 4.2.2.2 follows from that of 4.2.2.1. And clearly, the cut formulas of  $\mathfrak{S}$  belong to the zero-part, so the degree of  $\mathfrak{S}$  is one; therefore it is, of course, not greater than the degree of  $\mathfrak{S}_2$ . Hence, the conditions on the potentials are fulfilled.

4.2.2.4. The case, where  $D$  has a logical symbol and two  $D$ 's in the beginning sequence  $D \rightarrow D$  in 4.2.2.1 belong to the first class. The right cut-formula  $D$  of the cut  $\mathfrak{S}_2$  is related to the right  $D$  in the beginning sequence  $D \rightarrow D$ , so each leading formula of the right cut-formula of  $\mathfrak{S}_2$  is a weakening formula or a beginning formula belonging to the first class. That is, each leading formula of the right cut-formula of  $\mathfrak{S}$  is a weakening formula or a beginning formula belonging to the first class. Hence, the separativity of

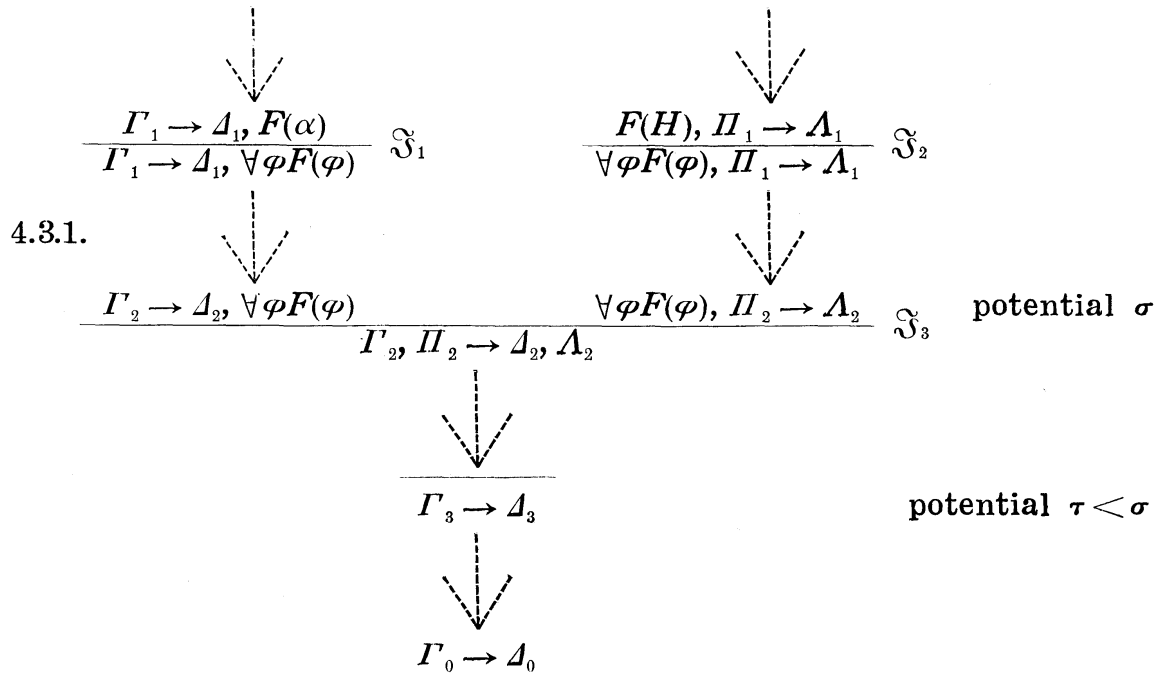
4.2.2.2 holds. And since both cut-formulas of  $\mathfrak{S}$  belong to the zero-part, the degree of  $\mathfrak{S}$  is one, therefore the conditions on the potentials are fulfilled.

4.2.2.5. The case, where  $D$  has a logical symbol and two  $D$ 's in  $D \rightarrow D$  belong to the second class. By a similar reason as in 4.2.2.4, each leading formula of  $D$  in the sequence  $I' \rightarrow \Delta, D$  in 4.2.2.2 is a weakening formula or a beginning formula belonging to the second class. Therefore we see easily that the proof-figure 4.2.2.2 is separative. And the degree of the right or left cut-formula  $\mathfrak{S}$  is not greater than the degree of the right or left cut-formula of  $\mathfrak{S}_2$  respectively. Hence the degree of  $\mathfrak{S}$ , which is not greater than the degree of  $\mathfrak{S}_2$ , is not greater than the potential of the upper sequences of  $\mathfrak{S}$ .

4.3. The case, where the end-place does not contain an implicit beginning sequence nor a logical inference.

In this case, we may assume, in the same way as in §6 [2], that there exists a suitable cut  $\mathfrak{S}$ . We have to consider separately several cases according to the form of the outermost logical symbol of the cut-formula  $D$  of  $\mathfrak{S}$ . But, since other cases are to be treated similarly, we treat only the case, where the outermost logical symbol of  $D$  is  $\forall$  on  $f$ -variable.

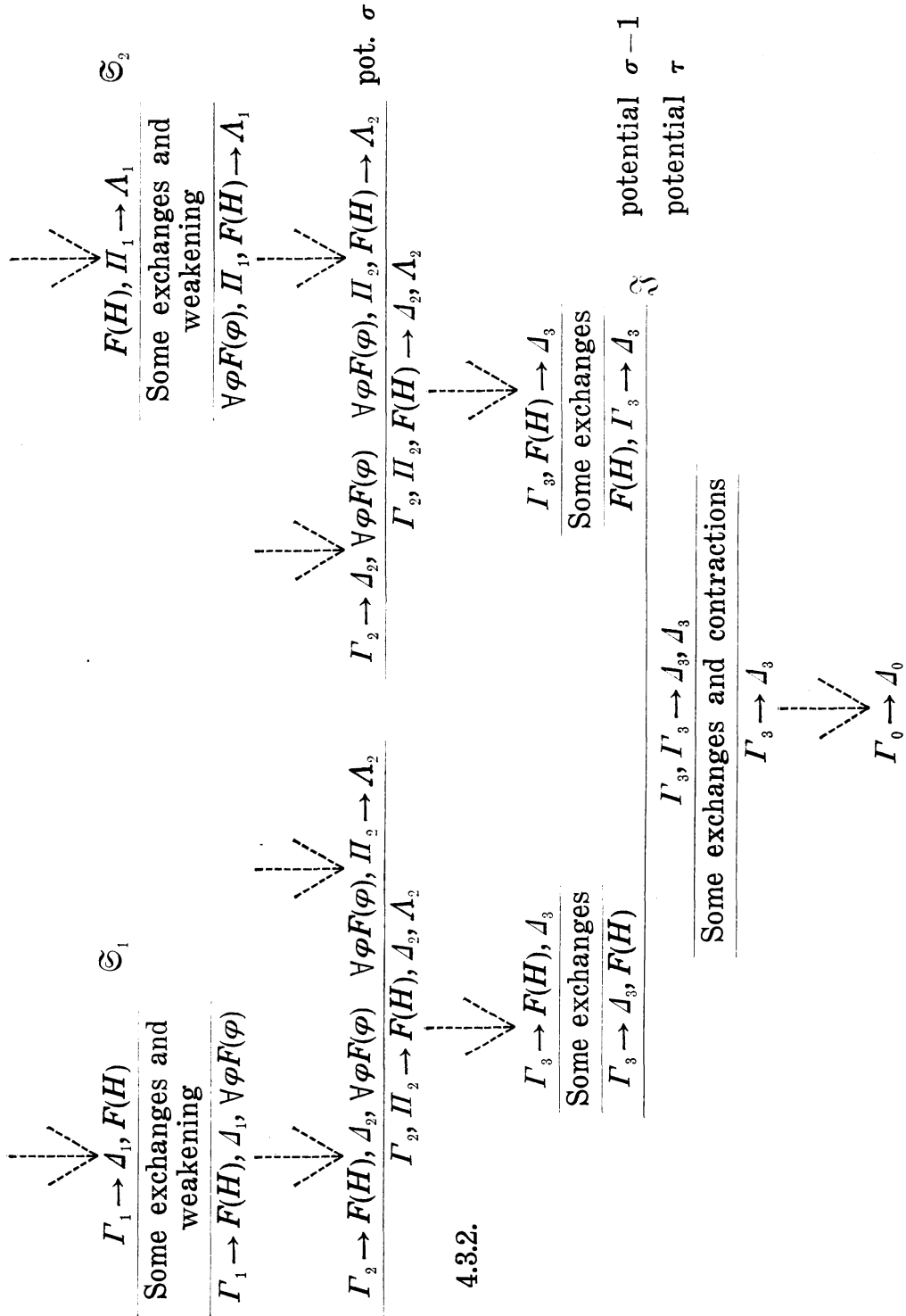
We assume that the proof-figure is of the following form:



where  $\Gamma_3 \rightarrow \Delta_3$  is the uppermost sequence under  $\mathfrak{S}_3$  whose potential is less than  $\sigma$ .

After a suitable change of free  $f$ -variables, we can assume that every inference-figure  $\forall$  right on  $f$ -variable over  $\mathfrak{S}_1$  has no  $\alpha$  as its eigen-variable.

We reduce this proof-figure to the following 4.3.2.



4.3.2.

where the potentials of sequences loaded on  $\mathfrak{S}$  are  $\sigma - 1$  and the potentials of the other sequences are equal to those of the corresponding sequences in 4.3.1 respectively, and a beginning formula with logical symbols belongs to the first or to the second class according as the corresponding one in 4.3.1 belongs to the first or to the second class. But if  $H(\alpha_1, \dots, \alpha_n)$  has logical symbols, the corresponding formula of a beginning formula  $H(X_1, \dots, X_n)$  over the sequence  $\mathfrak{S}_1$  may possibly be a formula  $\alpha[X_1, \dots, X_n]$  over  $\mathfrak{S}_1$ , which has no logical symbols. In such a case we determine as follows the class to which  $H(X_1, \dots, X_n)$  belongs.

By the separativity of 4.3.1, the inference  $\mathfrak{S}_2$  is separative, so the range of  $n$  terms  $(X_1, \dots, X_n)$  is separative. Hence, either there exists an original formula  $H(X_1, \dots, X_n)$  in the right side of a sequence which is not a weakening formula while there exists no such formula in the left side of a sequence, or all original formulas  $H(X_1, \dots, X_n)$  in the right side of a sequence are weakening formulas. In the first case the initial beginning formula  $H(X_1, \dots, X_n)$  over  $\mathfrak{S}_1$  will be classed to the first class, and in the second case to the second class.

We see easily that the ordinal number of 4.3.2 is less than the ordinal number of 4.3.1. The degree of the cut  $\mathfrak{S}$  or the potential of the upper sequences of  $\mathfrak{S}$  is by one less than the degree of the cut  $\mathfrak{S}_2$  or the potential of the upper sequences of  $\mathfrak{S}_2$  respectively, therefore the conditions on the potentials are fulfilled. And the above definition of the first or the second class of the beginning formulas with logical symbol in 4.3.2, and one of the following lemmas imply the separativity of 4.3.1.

4.3.3. Lemma 1. In the logical system *PL*, let  $A$  be an original formula of  $F(H)$  in  $\mathfrak{S}_1$  in 4.3.2 and be related to a formula  $B$ , then  $B$  is an original formula of  $F(H)$  in  $\mathfrak{S}_2$  and is homologous to  $A$ .

4.3.4. Lemma 2. In the logical system *QL*, let  $A$  be an original formula of  $F(H)$  in  $\mathfrak{S}_1$  and be related to  $B$ , then  $B$  is an original formula of  $F(H)$  in  $\mathfrak{S}_2$  and is homologous to  $A$ .

The lemma 1 is obvious, because  $n$  is zero and an original formula of  $F(H)$  is  $H$ . And since *QL* contains no inference  $\forall$  on variable of type (0), the lemma 2 is also obvious.

Therefore the *FC* on *PL* (or on *QL*) is proved.

REMARK. Clearly the following fact is hereby proved:

The provable sequence in *PL* (or in *QL*) is an end-sequence of a suitable separative proof-figure without cut in *PL* (or in *QL*).

### References

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Errata to 'On a Generalized Logic Calculus' Jap. J. Math., 24 (1954) pp. 149-156.
- [2] —: On the fundamental conjecture of *GLC* I, J. Math. Soc. Japan, 7 (1955) pp. 249-275.
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### ERRATA

Construction of the set theory from the theory  
of ordinal numbers  
(This Journal Vol. 6, No. 2, pp. 196-219)

by Gaisi TAKEUTI

p.	l.	Errata	Correction
206	30	$\vdash g_2(x)$	$\vdash x > g_2(x)$
208	10	$\alpha[y]$	$fn(y)$
209	9	$\leq$	$\leq$
216	7	$\varphi_2[v, u]$	$\varphi_2[u, v]$
	9	$\varphi_2[u, v]$	$\varphi_2[v, u]$
217	20	from 37	from 39