Journal of the Mathematical Society of Japan Vol. 4, No. 1, July, 1952.

Correction.

On the group of automorphisms of a function field.

This Journal Vol. 3, No. 1.

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A correction should be made for the proof of Lemma 1, as the one given in our paper is not valid, when the ramification order of some prime divisor of K over K' is divisible by the characteristic p of K. We shall show, however, that the conclusion of this lemma is also true in this exceptional case, although the estimation of the order of automorphisms given in our paper fails then to hold. First we shall prove the following local theorem.

THEOREM. Let k be an algebraically closed field with characteristic $p \neq 0$, $\Omega = k((t))$ a field of formal power series of one variable t, and Ω' a finite separable extension of Ω whose degree n is divisible by p. If a continuous automorphism σ of Ω' over k leaves invariant Ω, then the first coefficient α_1 of the expansion

$$t^{\sigma} = \alpha_1 t + \cdots$$

is a root of unity.

PROOF. We first show that we have only to prove our assertion in this special case: Ω' is a cyclic extension of Ω of degree p. Let Ω^* be a finite Galois extension of Ω containing Ω', and L the intermediate field between Ω^* and Ω corresponding to the p-Sylow group of the Galois group of Ω^*/Ω. As the degree e of L over Ω is prime to p, there is no other extension of Ω of degree e in an algebraic closure $\bar{\Omega}$ of Ω^*. Hence, if $\bar{\sigma}$ is any extension of σ to $\bar{\Omega}$, we have $(\Omega' L)^{\bar{\sigma}} = \Omega' L$. Let H be an intermediate field between $\Omega' L$ and L with degree p over L, then H is cyclic over L, and some power $\bar{\sigma}^h$ of σ leaves invariant H. Thus we have reduced our theorem to the special case. So we shall assume in the following that Ω'/Ω is cyclic of degree p. Then we can choose an Artin-Schreier's generater π of Ω' over Ω satisfying the following equation:

$$\wp(\pi)=\pi^{p}-\pi=\frac{\beta}{t^{\lambda}}+\cdots \qquad (\lambda, p)=1\,.$$

As π^{σ} is then also a generator of the same nature, we have

$$\wp(\pi^{\sigma})-\wp(\pi)\in\wp(\Omega)\,, \quad \text{and} \quad \alpha_1^{\lambda}=1\,.$$

THE PROOF OF LEMMA 1 IN THE EXCEPTIONAL CASE.

Let P be a prime divisor of K, whose ramification order is divisible by p, and Q the prime divisor of K' such that $P\mid Q$. We may assume without loss of generality that $\nu_Q(x)>0$. Then some power σ^h of σ leaves invariant P, and we have

$$x^{\sigma^h}=\frac{\alpha x}{x+\beta} \qquad (\alpha\neq 0)\,.$$

From the above theorem, we have $\alpha^{\lambda}=1$. Hence the order of σ is $\leqq h\cdot\lambda\,[K:K']$.