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On some properties of covering groups of a topological group.

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Recently C. Chevalley has developed in his book "Theory of Lie groups" (cited with L.G.) the theory of covering groups of a connected, locally connected and locally simply connected topological group with new definitions of a covering space and of the simply connectedness of a space.

The purpose of this paper is to investigate some properties of covering groups of a topological group with these new conceptions. In § 1 we shall give an algebraic characterization of the simply-connectedness of a topological group and give another proof of the existence theorem of a simply-connected covering group under usual conditions. In § 2, § 3 we shall consider the generalized universal covering group under weaker conditions, i.e. for a connected, locally connected topological group with the first countability axiom.

§ 1. *Simply connected topological groups.*

We use here the following definitions from L.G., the definition that a set $E \subseteqq X$ is evenly covered by X^* with respect to a continuous mapping f of X^* into X (Chap. II, § VI, Def. 2); the definition of a covering space (X^*, f) of X, where X and X^* are connected (conn.) and locally connected (l.c.) space with a continuous mapping f of X^* onto X (§ VI, Def. 3) the definition of the simply connectedness of a conn. and l.c. space (§ IX, Def. 2); and the definition of a covering group of a conn. and l.c. topological group (§ VIII, Def. 2).

Definition. 1. Let G_1, G_2 be two topological groups and U_1 (U_2) be a neighbourhood of the unit element of G_1 (G_2) respectively. We mean by a *U_1-U_2-local isomorphism* of G_1 and G_2 a homeomorphism f of U_1 onto U_2 which has the following properties:

(i) the conditions $a,b,ab \in U_1$ imply $f(ab)=f(a)\, f(b)$ in U_2.

(ii) the conditions $a,b \in U_1$, $f(ab) \in U_2$ imply $ab \in U_1$.

Now we construct a topolngical group $Gr(U)$ from a neighbourhood U of the unit element e in a topological group G with the property $U=U^{-1}$ as follows. To each element a $(a \neq e)$ in U take an abstract element A. $Gr(U)$ has these $\{A\}$ as generators. If to a, a^{-1} $(a \neq e)$ in U correspond

A, A^*, then we take as the defining relations of $Gr(U)$ $A\cdot A^*=1$ $(e \neq a \in U)$ and $ABC^*=1$ for every relation $ab=c$ in U. As the system of neighbourhoods of the unit element of $Gr(U)$ we take $V^*=\{A\,;\ a \in V \subseteq U\}$ for every neighbourhood V *in* U of e of G. Then we have

Lemma 1.[1] *The topological group $G_r(U)$ constructed above has the following properties:*

(i) *If U is connected, then $Gr\ (U)$ is also connected.*

(ii) *Let $U^*=\{A\,;\ a \in U\}$, then $Gr(U)$ and G are U^*-U-local isomorphic.*

(iii) *For any topological group G' which is U'-U-local isomorphic with G*

$$G \cong Gr(U)/N^{2)} \tag{1}$$

holds for a discrete normal subgroup of $Gr(U)$.

Lemma 2.[3] *Let G^* be a conn. and l.c. topological group and N^* be its discrete normal subgroup. If $G=G^*/N^*$ is also conn. and l.c., then for the natural mapping f of G^* onto G (G^*, f) is a covering group of G.*

Using these Lemmas we have the following characterization of the simply connectedness of a topological group:

Theorem 1. *A necessary and sufficient condition that a conn. and l.c. topological group G be simply-connected is that*

$$G \cong Gr.(U) \tag{2}$$

holds for every connected neighbourhood $U=U^{-1}$ of the unit element of G.

Proof. (i) The necessity of the condition. From Lemma 1 follows $G \cong Gr(U)/N^*$ for a discrete normal subgroup N^* of $Gr(U)$. Then follows from Lemma 2 that $Gr(U)$ is a covering group of G if U is connected. By the assumption that G is simply connected, we conclude taht $N^*=1$ and $G \cong Gr(U)$.

(ii) The sufficiency of the condition. Let U be a neighbourhood of the unit element e with $U=U^{-1}$ of a topological group G. We call a set of finite elements of G

$$W=\{p,a_1,a_2,\dots,a_r,q\} \qquad (p=a_0,\ q=a_{r+1})$$

a *U-chain* if $a_i^{-1}a_{i+1} \in U$ $(i=\dots,r)$ hold. We identify two U-chains W and $W_0=\{p,a_1,\dots,a_k,\ a_k,\ a_{k+1},\dots,a_r,\ q\}$. Let $w'=\{p,\ b_1,\dots,b_r,\ q\}$ be another U-chain. We denote $W \approx W'$ (U) if $a_i b_i^{-1} \in U$ $(i=1,\dots r)$ hold. We call two U-chains W and W' *U-homotopic* and denote $W \sim W'$ (U) if $W \approx W_1 \approx W_2 \approx \dots \approx W_s \approx W'$ (U) holds. Now we have

Lemma. 3.[4] *If $A_1^{\varepsilon(1)}\dots A_r^{\varepsilon(r)}=B_1^{\delta(1)}\dots B_s^{\delta(s)}$ $(\varepsilon(i)=\pm 1,\ \delta(j)=\pm 1)$ holds in $Gr(U)$, then U-chains $W=\{1,\ A_1^{\varepsilon(1)},\ A_1^{\varepsilon(1)}A_2^{\varepsilon(2)},\dots,A_1^{\varepsilon(1)}\dots A_r^{\varepsilon(r)}\}$* and

$W'=\{1, B_1^{\delta(1)}, B^{\delta(1)}B^{\delta(2)},\dots,B_1^{\delta(1)}\cdots B_s^{\delta(s)}\}$ *are U-homotopic in* $Gr(U)$.

Now let G be a conn. and l.c. topological group satisfying the condition of Theorem 1. Let (X^*, f) be any covering space of G. For any $x_0 \in G$ take x_1^*, x_2^* from $f^{-1}(x_0)\subseteqq X^*$. If we can show $x_1^*=x_2^*$, then f is univalent and G must be simply connected.

For this purpose take $e^* \in f^{-1}(e)\subseteqq X^*$. For any element $x \in G$ we can take an open and conn. neighbourhood $V(e,x)$ of e in G, so that $x\cdot V(e,x)$ is evenly covered by X^*. Then for an element $x^* \in f^{-1}(x)\subseteqq X^*$ there is a compoent $V^*(x^*)=f_\alpha^{-1}(x\cdot V(e,x))$ of $f^{-1}(x\cdot V(e,x))$ which contains x^* so that $V^*(x^*)$ is open, conn. and homeomorphic to $x\cdot V(e,x)$ by the mapping f. If we take an open and conn. neighbourhood $U(e,x)$ of e in G so that $U(e,x)^3\subseteqq V(e,x)$, then $U^*(x)=f^{-1}(x\cdot U(e,x))\cap V^*(x^*)$ $=f_\alpha^{-1}(x\cdot U(e,x))$ is also open, conn. and homeomorphic to $x\cdot U(e,x)$ by the mapping f.

Since X^* is conn. we can choose $y_1^*,\dots y^*_{r-1}$ and $U^*(y_1^*),\dots,U^*(y^*_{r-1})$ from X^* so that the finite set $C_1^*=\{e^*,y_1^*,\dots, y^*_{r-1}, x_1^*\}$ satisfies $y^*_{i+1} \in U^*(y_i^*)$ $(i=0,1,\dots, r-1\,;\ y_0^*=e^*, y_r^*=x_1^*)$. Analogously we can choose $C_2^*=\{e^*,z_1^*,\dots, z^*_{s-1}, x_2^*\}$ so that $z_{j+1}^* \in U^*(z_j^*)$ $(j=0,1,\dots,s-1\,;\ z_0^*=e^*, z^*_s=x_2^*)$. From C_1^*, C_2^* we have by the mapping f two finite subsets of G $C_1=\{e=y_0, y_1,\dots, y_{r+1}, x_0,=y_r\}$, $C_2=\{e=z_0, z_1,\dots,z_{s+1}, x_0=z_s\}$ with $y_{i+1} \in U(y_i)$ $(i=0,1,\dots,r-1)$, $z_{j+1} \in U(z_j)$ $(j=0,1,\dots,s-1)$.

Now take an open and conn. neighbourhood U_0 of e in G such that

$$U_0=U_0^{-1}\subseteqq\{\cap_i U(e, y_i)\}\cap\{\cap_j U(e, z_j)\}.$$

Since $U(e, y_i)$, $U(e, z_j)$ are conn. we can take U_0-chains

$$U_i=\{y= y_{i0}, y_{i1}\dots, y_{ir(i)}=y_{i+1}\},\ y_{ik+1} \in y_{ik}\, U_0$$
$$U_j'=\{z_j=z_{j0}, z_{j1},\dots, z_{js(j)}=z_{j+1}\},\ z_{jk+1} \in z_{jk}\, U_0$$
$$(i=0,1,\dots;\ r-1\,;\ j=0,1,\dots, s-1)$$

Since $y_{ik} \in y_iU(e, y_i)$ and $y_{ik}\cdot U(e, y_i)^2\subseteqq y_iU(e, y_i)^3\subseteqq y_iV(e, y_i)$, $y_{ik}U(e, y_i)^2$ is evenly covered by X^*. Then put $U(e, y_{ik})=U(e, y_i)$, $V(e, y_{ik})=U(e, y_i)^2$.

If we interpolate these U_0-chains U_i (U_j') between y_i and y_{i+1} (z_j and z_{j+1}) in C_1 (C_2), we have two U_0-chains with the following properties. (We denote these new U_0-chains and their elements with the same letters as the old ones.)

(i) $$C_1=\{e=y_0, y_1,\dots, y_{r-1}, x_0=y_r\},\qquad y_{i+1} \in y_i\cdot U_0$$

$$C_2=\{e=z_0, z_1,\ldots,z_{s-1}, x_0=z_s\}, \qquad z_{j+1} \in z_j U_0$$

(ii) $U_0=U_0^{-1}\subseteqq U(e, y_i), \ U_0\subseteqq U(e, z_j).$

(iii) $U(e, y_i)^2\subseteqq V(e, y_i), \quad U(e, z_j)^2\subseteqq V(e, z_j),$

$V(e, y_i)$ and $V(e, z_j)$ are evenly covered by X^*.

(iv) Let $U^*(y_i^*)$ $(U^*(z_j^*))$ be the component of $f^{-1}(y_i U(e, y_i))$ $(f^{-1}(z_j\cdot U(e, z_j))$ which contains y_i^* (z_j^*), then we have

$$C_1^*=\{e^*, y_1^*,\ldots, y_{r-1}^*, x_1^*\}, \qquad y_{i+1}^* \in U^*(y_i^*)$$

$$C_2^*=\{e^*, z^*,\ldots, z_{s-1}^*, x_2^*\}, \qquad z_{j+1}^* \in U^*(z_j^*)$$

on X^*.

Now we use the condition that $G\cong Gr(U_0)$ and apply Lemma 3 for C_1 and C_2. Then C_1 and C_2 are U_0-homotopic. Hence it is sufficient to see that $x_1^*=x_2^*$ holds under the condition $C_1\approx C_2(U_0)$. Therefore, let $r=s$ and $y_i^{-1}z_i \in U_0$ $(i=1,\ldots,r)$. Under these assumptions we shall prove

$$z_i^* \in V^*(y_i^*) \qquad (i=0, 1,\ldots,r) \tag{3}$$

by the mathematical induction on i. For $i=0$ $y_0^*=z_0^*=e^*$. Hence we assume that the relation (3) holds for $i-1$. From $y_{i+1} \in y_iU_0$, $z_{i+1} \in y_{i+1}U_0 \subseteqq y_i\cdot U_0^2$, $z_i \in y_iU_0$ follows that y_{i-1}, z_{i-1}, z_i are contained in $y_i\cdot V(e, y_i)$. Since y_{i-1}^* and y_i^*, y_{i-1}^* and z_{i-1}, z_{i-1}^* and z_i^* belong in the same component of $f^{-1}(y_i\cdot V(e,y_i))$, we can conclude that z_i^* and y_i^* are in the same component $V^*(y_i^*)$ of $f^{-1}(y_i\cdot V(e,y_i))$. Thus (3) is proved. Hence $x_1^*=y_r^*$ and $x_2^*=z_r^*$ belong to the same component and so $x_1^*=x_2^*$ holds, Q.E.D.

Theorem. 2. *Let G be a conn., l.c. and locally simply connected topological group. Let a neighbourheed $U_0=U_0^{-1}$ of the unit element of G be simply connected. Then $Gr(U_0)$ is a simply connected topological group.*

Proof. For any covering space (X^*, f) of G simply connected sets xU_0 are evenly covered by X^*. Hence in the proof of Theorem 1 we can choose as $V(e,x)$ for any $x \in G$ the simply connected neighbourhood U_0. Then we can prove Theorem 2 just in the same way as in the proof of Theorem 1, Q.E.D.

From Lemma 1,2 follows that $Gr(U_0)$ is connected. Hence we have the following existence theorem of simply connected covering space for topological groups (c.f. L.G. Cap. II, § IX, Theorem 4) :

Corollary. *Let G be conn., l.c. and locally simply connected topological group. Then G has a simply connected covering group.*

§**2.** *Generalized universal covering group.*

We shall consider here conn. and l.c. topological groups with the 1-st countability axiom. Since we can not always expect the existence of a simply connected covering group for such a group, we shall define the following generalized covering group.

Definition 2. *Let G be a conn. and l.c. topological group with the* 1-*st countability axiom. If there exists a topological group G_0^* satisfying the following conditions we call G_0^* a generalized universal covering group of G*:

P. I. *G_0^* is conn., l.c. and simply connected.*

P. II. *There exists a normal subgroup F_0^* in the center of G_0^* with*

$$G_0^*/F_0^* \cong G \tag{4}$$

P. III. *F_0^* is totally disconnected, and, moreover, every neibourhood V^* of the unit element e^* contains an open subgroup H^* of F_0^*.*

Now we shall define a group G_0^{**} for a conn. and l.c. topological group G as follows. Let $\{U_n; n=1,2,3,\ldots\}$ $(U_1 \supset U_2 \supset U_3 \supset \ldots\ldots)$ be a basis of neighbourhoods of the unit element e in G. We can assume that U_n are open, conn. and $U_n^{-1}=U_n$ hold. Let $Gr(U_n)$ be the topological group defined in § 1. From Lemma 1 follows that

$$Gr(U_m)/N_n^m \cong Gr(U_n) \qquad (n<m) \tag{5}$$

for a discrete normal subgroup N_n^m in the center of $Gr(U_m)$. The natural mapping of this homomorphism shall be denoted by φ_n^m. Then the relation

$$\varphi_n^m \, \varphi_m^l = \varphi_n^l \qquad (n<m<l) \tag{6}$$

holds. Hence we have an inverse system of topological groups

$$\{Gr(U_n),\ \varphi_n^m\} \tag{7}$$

Let its limit group be G_0^{**}. Now let

$$Gr(U_n)/F_n \cong G \tag{8}$$

for a discrete subgroup F_n in the center of $Gr(U_n)$. Then $\varphi_n^m(F_m)=F_n$ and $\{F_n,\ \varphi_n^m\}$ forms also an inverse system of topological groups. Let the limit group of this system be F_0^*. Then we have the relation $G_0^{**}/F_0^* \cong G$. Since F_n lies in the center of $Gr(U_n)$, F_0^* lies also in the center of G_0^{**}. Since F_n is discrete, it is easy to see that F_0^* satisfies the property **P.** III.

Now we assume that the group G_0^{**} is conn. and l.c. Then we can prove that

$$G_0^{**} = Gr(U^*) \tag{9}$$

holds for any conn. open neighbourfood U^* of G_0^{**} with $U^*=U^{*-1}$.

For let the natural mapping of G_0^{**} onto $Gr(U_n)$ by the homomorphism

$$G_0^{**}/N_n^* \cong Gr(U_n)$$

be ψ_n. Then the basis of neibourhoods of the unit element e^* in G_0^{**} is $\{\psi_n^{-1}(U_{nk})\}$ where $\{U_{nk}\}$ are neighbourhoods of the unit element in $Gr(U_n)$. Hence we can choose a group $Gr(U_n)$ such that $\psi_n(U^*)=U_{nk}\supset U_n$ in $Gr(U_n)$. Since U_{nk} is open, conn., $U_{nk}=U_{nk}^{-1}$ the relation $Gr(U_n)=Gr(U_{nk})$ hlods. Now it is also easy to see that we have $G_0^{**}=Gr(U^*)$ from this relation. From Theorem 1 follows then that G_0^{**} is simply connected. Therefore, we have the following theorem:

Theorem 3. *If the group G_0^{**} constructed above is conn. and l.c., then G_0^{**} is a generalized universal covering grovp of G.*

Now we shall consider the converse problem, *i. e.* " if G has a generalized universal covering group G_0^*, is G_0^{**} then conn. and l.c. and is G_0^* isomorphic with G_0^{**}?" For this purpose we shall prove the following Lemmas:

Lemma 4. *Let G^* be a topological group satisfying* **P.** I, **P.** II. *Then for any conn. and l.c. topological group G_1 which is locally isomorphic with G there exists a normal subgroup H^* of G^* such that*

$$G_1 \cong G^*/H^* \tag{10}$$

holds.

Proof. Let G_1 be U_1-U-locally isomorphic with G. We can assume that $U_1=U_1^{-1}$ and open. Let f be the natural mapping of G^* onto G by (4). Let U^* be the component of $f^{-1}(U)$ which oontains the unit element e^* in G^*. Put $U_0=f(U^*)\subseteq U$. $U_0=U_0^{-1}$ is open and conn. From Lemma 1 follows then

$$G_1 \cong Gr(U_0)/H_0^* \tag{11}$$

Now we shall apply the following Lemma:

Lemma 5. *Let $G_1=Gr(U_1)$, $G_2=Gr(U_2)$ and let f be a continuous open mapping of U_1 onto U_2 with*

$$f(ab)=f(a)f(b) \qquad (a,b,\ ab\in U_1).$$

Then we can always extend the mapping f to a continuous open hommorphism of G_1 onto G_2.

Hence the mapping f of U^* onto U_0 can be made to a continuous open homomorphism of $Gr(U^*)=G^*$ onto $Gr(U_0)$. Therefore, we have

$$G^*/H_1^* \cong Gr(U_0) \tag{12}$$

Combining (11) and (12) we have (10), Q. E. D.

Lemma 6. *Let G_0^{**} be the limit grovp defined by* (7). *Let G^* be any topological group with the properties* **P.** I, **P.** II. *Then for a suitable normal, subgroup H^* of G^* the relation*

$$G_0^{**} \cong G^*/H^* \tag{13}$$

holds.

Proof. From Lemma 4 follows $Gr(U_n) \cong G^*/N_n^*$. Let f_n be the natural mapping for this homomorphism. It is easy to see that $f_n = \psi_n^m f_m$ $(n<m)$ holds, where ψ_n^m is the natural mapping defined in (5). Hence by a well knewn theorem we have a continuous open homorphism f of G^* onto the limit gooup G_0^{**} of the inverse system of groups $\{Gr(U_n), \psi_n^m\}$, Q. E. D.

Theorem 4. *Let G be a conn. and l.c. topological group with the* 1-*st countability axiom. If G has a generalized universal covering group G_0^* then the group G_0^{**} defined by* (7) *is conn. and l.c., and hence G_0^{**} is also a generalized universal covering grcup of G. Moreover, every generalized universal covering graup of G is topologically isomorphic with G_0^{**}.*

Proof. Let G_0^* be a generalized universal covering group of G. Then we have $G_0^{**} \cong G_0^*/H^*$ by Lemma 6. Since G_0^* is conn. and l.c., G_0^{**} is also conn. and l.c. Hence we have the first half of Theorem 4. Now let F_0^* be the normal subgroup defined by (4). Clearly H^* is a subgroup of F_0^*. Hence it is easy to see that H^* has also the property **P.** III. Let N^* be any open subgroup of H^*. Then H^*/N^* is discrete. Since $G_0^{**} \cong (G_0^*/N^*)/(H^*/N^*)$ holds, G_0^*/N^* is a covering group of G_0^{**}. Since G_0^{**} is simply connected, we can easily conclude that $H^* = \{e^*\}$, that is, $G_0^* \cong G_0^{**}$, Q. E. D.

§ 3. *Some properties of the generalized universal covering group.*

Definition 3. *Let G be a conn. and l.c. topological group with the* 1-*st countability axiom. We assume further G has the generalized uuiversal covering group G_0^*. Let f be the natural mapping by $G \cong G_0^*/F_0^*$. The totality of all the homeomorphisms φ of G_0^* onto itself which satisfies*

$$f \cdot \varphi = f \tag{14}$$

is called the Poincaré group of G.

Theorem 5. *Let G_0^* be the generalized universal covering group of a conn. and l.c. topological group G with the* 1-*st countability axiom. Let $G \cong G_0^*/F_0^*$. Then the Poincaré group P of G is algebraically iscmorphic to F_0^*.*

Proof. Let $y \in F_0^*$. Then the homeomorphic mapping of G_0^* onto itself $\varphi_y(x)=x\cdot y$ $(x \in G_0^*)$ belongs clearly to P. Conversely we shall prove that any $\varphi \in P$ is a φ_y $(y \in F_0^*)$. From $f\cdot\varphi(x)=f(x)$ follows $\varphi(x)=x\cdot y(x)$, $y(x)\in F_0^*$. Then $y(x)=\varphi(x)\cdot(x)^{-1}$ is a continuous mapping of G_0^* into F_0^*. Since G_0^* is conn., so is its image in F_0^*. On the other hand F_0^* is totally disconnected. Hence $y(x)=y$ is independent of $x \in G_0^*$, that is $\varphi(x)=x\cdot y=\varphi_y(x)$, Q.E.D.

The structure of the Poincaré group of an infinite product group is very simple.

Theorem 6. *Let G_n $(n=1,2,\ldots)$ and the infinite direct product group $G=\mathbf{P}_{n=1}^{\infty} G_n$ have the genealized universal covering goups G_n^* and G^* respectively. Let $G_n\cong G_n^*/F_n^*$, and $G\cong G^*/F^*$. Then*

$$G^*\cong\mathbf{P}_{n=1}^{\infty} G_n^*, \qquad F^*\cong\mathbf{P}_{n=1}^{\infty} F_n^*(\textit{algebraically}). \tag{15}$$

Proof. It is easy to see that $G_0^*=\mathbf{P}_{n=1}^{\infty} G_n^*$ satisfies the condition of Theorem 1, that is, G_0^* is simply connected. On the other hand it is also easy to see that $F_0^*=\mathbf{P}_{n=1}^{\infty} F_n^*$ is contained in the center of G_0^*, $G \cong G_0^*/F_0^*$, and F_0^* satisfies the property **P.** III. Hence we have (15) by Theorem 4, Q. E. D.

Example. Let G_n be isomorphic to the additive group of real numbers mod. 1. and let $G=\mathbf{P}_{n=1}^{\infty} G_n$. Then the generalized universal covering group G_n^* of G_n is isomorphic to the additive group of all the real numbers. Then the generalized universal covering group of G is given by $G^*=\mathbf{P}_{n=1}^{\infty} G_n^*$.

Theorem 7. *Let G_1 and G_2 have the generalized universal covering groups G_1^* and G_2^* respectively. Let $G_1\cong G_1^*/F_1^*$, and $G_2\cong G_2^*/F_2^*$. A necessary and sufficient condition for the local-isomorphism of G_1 and G_1 is that*

(i) $\qquad G_1^*\cong G_2^*$

(ii) *There are mutually isomorphic open subgroups H_1^* and H_2^* of F_1^* and F_2^* raspectively:*

$$H_1^*\cong H_2^*, \quad H_1^*\subseteq F_1^*, \quad H_2^*\subseteq F_2^*. \tag{16}$$

Proof. Let G_1 and G_2 be U_1-U_2-locally isomorphic. We can assume that $U_1=U_1^{-1}, U_2=U_2^{-1}$ and are open, conn. Then $Gr(U_1)\cong Gr(U_2)$ holds. In general $Gr(U_{1n})\cong Gr(U_{2n})$ if $U_{1n}\subseteq U_1$ and $U_{2n}\subseteq U_2$ are locolly isomorphic. Hence the limit groups G_1^* and G_2^* of $\{Gr(U_{1n})\}$ and $\{Gr(U_{2n})\}$ respectively are topologically isomorphic. Moreover, let $G_1^*/H_1^* \cong Gr(U_1)$ and $G_2^* H_2^* \cong Gr(U_2)$. Then H_1^* and H_2^* are open subgroups of F_1^* and F_2^* respectively and $H_1^* \cong H_2^*$. The sufficiency of the conditions can be proved quite analogously, Q. E. D.

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References

1) For the proof of Lemma 1, c.f. O. Schreier, [1] Abstrakte kontinuierliche Gruupe, Hamb. Abh., **4** (1925), 15–32, [2] Die Verwandtschaft stetiger Gruppen im Grossen, ibid. **5** (1926) 233–244.
2) $\cong$ means topological isomorphism.
3) For the proof of Lemma 2, c.f. L.G. Chap. II, § X, Prop. 4, Cor. 2.
4) For the proof of Lemma 3 c.f. Y. Kawada, Ueber die Ueberlagerungsgruppe und die stetige projektive Darstellung topologischer Gruppen, Jap. Jour. Math., **17** (1940), 139–164, § 1, 3 Hilfssatz.
5) C. f. S. Lefschetz, Algebraic topology, (1942), p. 54.
6) C. f. O. Schreier, [1] and [2].