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A REMARK ON THE HAUSDORFF MOMENT PROBLEM

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In his publication "Density Properties of Hausdorff Moment Sequences" [2] Trautner gives a rather complicated proof of the following theorem:

Let $d_n = \int_0^1 t^n d\chi(t)$, $\chi \in V[0, 1]$. *If* $d_{n_k} = O(c^{n_k})$, $(0 < c < 1$, n_k *natural numbers) and* $\sum 1/n_k = \infty$, *then* $d_n = O(c^n)$ *for all* n, *and hence* $d_n = \int_0^c t^n d\chi(t)$.

This theorem is a direct consequence of the following well-known theorem of Boas [1]:

Let f *be integrable on* (a, b), $0 \leqq a < b$, $\{\lambda_n\}$ *a sequence of complex numbers with* $\operatorname{Re} \lambda_n \to \infty$, $\arg \lambda_n \to 0$, $\sum 1/|\lambda_n| = \infty$, $|\lambda_m - \lambda_n| \geqq |m - n| h$, $h > 0$. *If* $\int_a^b t^{\lambda_n} f(t) dt = O((a + \varepsilon)^{\lambda_n})$ *for all* $\varepsilon > 0$, *then* $f(t) = 0$ *almost everywhere on* (a, b).

PROOF. Without loss of generality let $\chi(1) = 0$ (otherwise we consider $\int_0^1 t^n d\tilde{\chi}(t)$, where $\tilde{\chi}(t) = \chi(t) - \chi(1)$). Then $d_{n_k} = \int_0^1 t^{n_k} d\chi(t) = -n_k \int_0^1 t^{n_k - 1} \chi(t) dt$. If $d_{n_k} = O(c^{n_k})$, we get $\int_0^1 t^{n_k - 1} \chi(t) dt = O(c^{n_k - 1})$. If now $\sum 1/n_k = \infty$, then $\sum 1/(n_k - 1) = \infty$. From the above theorem it follows $\chi(t) = 0$ a.e. on $(c, 1)$, hence $d_n = O(c^n)$ for all n, i.e. $d_n = \int_0^c t^n d\chi(t)$.

REFERENCES

[1] R. P. BOAS, Remarks on a moment problem, Studia Math. 13, 59–61.
[2] R. TRAUTNER, Density properties of Hausdorff moment sequences, Tôhoku Math. J., 24 (1972), 347–352.

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