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CORRECTION AND SUPPLEMENT TO "ON CLOSED GEODESICS OF LENS SPACES"*)

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Statement of Theorem 3 contains some errors. Sentence "Another geodesic is closed geodesic of length 2π and of multiplicity 1." in the lines 23-24 and 28-29 on page 410 should be replaced by "All closed geodesics of length $2\pi/q$ are obtained as above. Other geodesics through these points are closed geodesics of length 2π (resp. π) in case of (i) (resp. (ii)) and of multiplicity 1."

Nevertheless arguments in the lines 4-14 on page 410 are valid when q is a prime or $\cos(2\pi p_1/q) = \cdots = \cos(2\pi p_n/q)$ holds. So Corollaries 1, 2 to Theorem 3 remain true. Thus Theorem 3 determines closed geodesics of length $2\pi/q$.

Now we shall determine all closed geodesics of lens spaces. Generally closed geodesics of a lens space of constant curvature 1 are of length $2\pi k/q (k = 1, 2, \cdots, q$: if q is odd and $k = 1, 2, \cdots, q/2$: if q is even). First let us introduce the equivalence relation $\overset{(k)}{\sim}$ in $\{p_1, \cdots, p_n\}$. Let s_i $(i = 1, \cdots, n)$ be an integer such that $s_i p_i \equiv k \pmod q$. Note that s_i is determined modulo q. Then we define $p_i \overset{(k)}{\sim} p_j$ if and only if $\cos(2\pi p_i s_j/q) = \cos(2\pi k/q)$. As is easily seen, this is an equivalence relation. Let $\{p_1 = p_{j_1}, \cdots, p_{j_{m_1}}; \cdots; p_{j_{m_{b+1}+1}}, \cdots, p_{j_{m_b}} = p_{j_n}\}$ be a partition of $\{p_1, \cdots, p_n\}$ with respect to this equivalence relation. Now a point $p = \varphi(x_1, \cdots, x_n)$ is said to be $\overset{(k)}{\sim}$-adapted if there exists an $s \in \{m_1, \cdots, m_b\}$ and $x_{2j-1} = x_{2j} = 0$ holds for $p_j \in \{p_1, \cdots, p_n\} - \{p_{j_{m_{s-1}+1}}, \cdots, p_{j_{m_s}}\}$. Now we have

THEOREM 3′. *Let $k = 1, \cdots, q-1$ if q is odd and $k = 1, \cdots, q/2 - 1$ if q is even. Then through every $\overset{(k)}{\sim}$-adapted point there exists a unique closed geodesic of length $2\pi k/q$ with initial direction $\varphi_*(x_2, -x_1, \cdots, x_{2n}, -x_{2n-1})$. This closed geodesic is of multiplicity 1 if and only if the base point is never $\overset{(k_i)}{\sim}$-adapted for all divisors $k_i(<k)$ of k. Every closed geodesic of length $2\pi k/q$ may be obtained as above. Closed geodesics other than stated above are of length 2π (resp. π) if q is odd (resp. even) and*

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of multiplicity 1.

Finally we shall add one more corollary to this theorem.

COROLLARY 3. *A lens space of constant curvature* 1 *with fundamental group of order q is homogeneous if and only if through every point there exists a closed geodesic of length $2\pi/q$.*

PROOF. "only if" part is trivial. We consider the "if" part. From the assumption, equivalence class of $\overset{(1)}{\sim}$ is $\{p_1, \cdots, p_n\}$. Since p_1 and consequently s_1 is equal to 1, we have $\cos(2\pi p_i/q) = \cos(2\pi/q)$ $(i = 1, \cdots, n)$. This means that the lens space has the homogeneous riemannian metric of constant curvature.

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