Tôhoku Math. Journ.
Vol. 18, No. 4, 1966

ON GENERALIZED CESÀRO MEANS OF INTEGRAL ORDER — CORRIGENDA

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(Received December 7, 1966)

In a recent paper [2] with the above title, I discussed inclusion relations between Riesz summability (R, λ, p) and generalized Cesàro summability (C, λ, p), where p is a non-negative integer (for an account of such relations, including those involving summability of non-integral order, and latest results, see [1]); one of the theorems in [2] was as follows:

THEOREM 4. $(C, \lambda, p) \subseteq (R, \lambda, p) \quad (p = 0, 1, 2, \cdots)$.

It has been pointed out to me by Dr. A. Meir that the three lines of argument preceding (38) on p. 426 of [2] are fallacious. However, (38) is in fact still true, though with the restriction $0 \leqq r \leqq n-\nu$ — and this is enough, since we need only the case $r = n-\nu$. More precisely, we are given that

$$h(x) = \frac{(\omega - x)^p}{(x-\lambda_{n+1}) \cdots (x-\lambda_{\nu+p+1})}$$

where $0 \leqq n-p \leqq \nu \leqq n$, $x < \omega \leqq \lambda_{n+1}$, and we show that

$$H \equiv \frac{(-1)^{p+1} h^{(n-\nu)}(\xi)}{(n-\nu)!} \geqq 0 \quad \text{for any} \quad \xi < \omega .$$

(If required, the more general case of (38), with $0 \leqq r \leqq n-\nu$, would follow from this on noting that $h^{(r)}(\omega -) = 0$). Though the ideas are elementary, I have been unable to find a less tedious method than the following, to replace the paragraph between (37) and (38) of [2]:

Write $h(\xi)$ in the form $(\omega - \xi)^{n-\nu} g(\xi)$ and express $g(\xi)$ as a sum of partial fractions; differentiate each partial fraction of $h(\xi)$ thus obtained $n-\nu$ times by Leibniz' formula, and use the binomial theorem to reduce the result to the form

$$H=\sum_{i=n+1}^{\nu+p+1}\frac{(\lambda_i-\omega)^p}{(\lambda_i-\xi)^{n-\nu+1}\rho_i},\qquad \rho_i=\prod_{j=n+1}^{\nu+p+1}{}'(\lambda_i-\lambda_j)$$

(my colleague Dr. R. A. Schaufele has obtained the same result somewhat more briefly by the use of Laplace transforms).

But if we now denote

$$k(x)=\frac{(x-\omega)^p}{(x-\xi)^{n-\nu+1}}\quad(x\geqq\omega>\xi)$$

then it follows from the expansion formula and mean-value theorem for divided differences that

$$H=k[\lambda_{n+1},\cdots,\lambda_{\nu+p-n}]=\frac{1}{(\nu+p-n)!}k^{(\nu+p-n)}(\eta)$$

for some η in $\lambda_{n+1}\leqq\eta\leqq\lambda_{\nu+p+1}$; and we can evaluate $k^{(\nu+p-n)}(\eta)$ by Leibniz' formula (and the binomial theorem) to give

$$H=\binom{p}{n-\nu}\frac{(\eta-\omega)^{n-\nu}(\omega-\xi)^{\nu+p-n}}{(\eta-\xi)^{p+1}}\geqq 0\,.$$

Thus (38) follows (with $r=n-\nu$) and the proof of the theorem is concluded as in [2].

Finally, the following misprint should be corrected in [2], p. 435, statement of Theorem 6: for $o(\lambda_n^\mu)$ read $O(\lambda_n^\mu)$.

REFERENCES

[1] D. BORWEIN AND D. C. RUSSELL, On Riesz and generalised Cesàro summability of arbitrary positive order. (Submitted to Math. Zeit.)

[2] D. C. RUSSELL, On generalized Cesàro means of integral order, Tôhoku Math. Journ., 17(1965), 410-442.

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