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A NOTE ON CONFORMAL MARTINGALES

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1. In a forthcoming paper, P. A. Meyer establishes by a very nice method that $(H^1)^* = BMO$ for right continuous martingales, but he does not deal with conformal martingales. The purpose of this note is to extend the fundamental results given by R. K. Getoor and M. J. Sharpe [1] on conformal martingales to locally square integrable martingales under the assumption such that (F_t) has no times of discontinuity. Our proof is an adaptation of the proof due to Getoor and Sharpe.

2. The reader is assumed to be familiar with the basic notions of the theory of stochastic integrals relative to martingales as given in [2].

By a system (Ω, F, F_t, P) is meant a complete probability space (Ω, F, P) with an increasing right continuous family $(F_t)_{t\geqq 0}$ of sub σ-fields of F. We assume as usual that F_0 contains all P-null sets. Let $\mathscr{M} = \mathscr{M}(F_t)$ (resp. $\mathscr{M}_c(F_t)$) be the class of all right continuous (resp. continuous) L^2-bounded martingales X over (F_t) such that $X_0 = 0$. Denote by $\mathscr{M}^{loc}(F_t)$ the class of all locally square integrable martingales X over (F_t) such that $X_0 = 0$.

For each $X \in \mathscr{M}^{loc}(F_t)$, we define:

$$\begin{aligned} ||X||_H &= E[\langle X, X\rangle_\infty^{1/2}] \\ ||X||_B^2 &= \sup_t \operatorname*{ess.sup}_\omega E[\langle X, X\rangle_\infty - \langle X, X\rangle_t | F_t] \\ H^1 &= \{X \in \mathscr{M}^{loc} \mid ||X||_H < +\infty\} \\ BMO &= \{X \in \mathscr{M}^{loc} \mid ||X||_B < +\infty\}\,. \end{aligned}$$

Clearly $BMO \subset \mathscr{M}(F_t) \subset H^1$. BMO is a normed linear space with the norm $||\cdot||_B$. H^1 is also a normed linear space with the norm $||\cdot||_H$, but it should be noted that this is not the same H^1-space introduced by P. A. Meyer for right continuous martingales. Probably, our H^1-space is not complete.

The next inequality is proved in [1] only for continuous H^1-martingales.

THEOREM 1. *For every* $X \in \mathscr{M}^{loc}$

$$||X||_H \leqq \sup\{E[\langle X, Y\rangle_\infty];\, 0 \leqq \langle X, Y\rangle_\infty \quad and \quad ||Y||_B \leqq 1\}\,.$$

PROOF. Fix $X \in \mathcal{M}^{loc}$ and let $(T_n)_{n=1,2,\dots}$ be an increasing sequence of F_t-stopping times reducing X to $\mathcal{M}$; that is, $X^{T_n} = (X_{t\wedge T_n}) \in \mathcal{M}$ for all $n \geqq 1$. For each $c > 0$, denote by (H_t^c) (resp. (M_t^c)) a right continuous modification of $E[(c+\langle X, X\rangle_\infty)^{-1/2}|F_t]$ (resp. $E[(c+\langle X, X\rangle_\infty)^{-1}|F_t]$). Clearly $0 \leqq H_t^c \leqq c^{-1/2}$ and $Y = H_-^c \cdot X \in \mathcal{M}^{loc}$, where $H_-^c = (H_{t-}^c, F_t)$. The definition of the stochastic integral $H_-^c \cdot X$ are taken from [2]. By using Jensen's inequality, $(H_-^c)^2 \leqq M_-^c$ and so from Fatou's lemma

$$\begin{aligned}
E[\langle Y, Y\rangle_\infty - \langle Y, Y\rangle_t|F_t] &= E\left[\int_t^{+\infty}(H_{s-}^c)^2 d\langle X, X\rangle_s|F_t\right] \\
&\leqq \varliminf_n E\left[\int_t^{+\infty} M_{s-}^c d\langle X^{T_n}, X^{T_n}\rangle_s|F_t\right] \\
&\leqq \varliminf_n E[(c + \langle X, X\rangle_\infty)^{-1}(\langle X^{T_n}, X^{T_n}\rangle_\infty - \langle X^{T_n}, X^{T_n}\rangle_t)|F_t] \\
&\leqq E[(c + \langle X, X\rangle_\infty)^{-1}\langle X, X\rangle_\infty|F_t] \\
&\leqq 1
\end{aligned}$$

from which $\|Y\|_B \leqq 1$ for every $c > 0$. Clearly $\langle X, Y\rangle_t = \int_0^t H_{s-}^c d\langle X, X\rangle_s \geqq 0$. If $E[\langle X, Y\rangle_\infty] = +\infty$ for some $c > 0$, then the theorem is evident, and so suppose that for every $c > 0$, $E[\langle X, Y\rangle_\infty] < +\infty$. Then, by remarking the fact that $\langle X, Y\rangle_t$ is positive and increasing in t, we get from the dominated convergence theorem

$$\begin{aligned}
E[\langle X, Y\rangle_\infty] &= \lim_n E\left[\int_0^{+\infty} H_{s-}^c d\langle X^{T_n}, X^{T_n}\rangle_s\right] \\
&= \lim_n E[H_\infty^c\langle X^{T_n}, X^{T_n}\rangle_\infty] \\
&= \lim_n E[(c + \langle X, X\rangle_\infty)^{-1/2}\langle X^{T_n}, X^{T_n}\rangle_\infty] .
\end{aligned}$$

On the other hand, from the monotone convergence theorem

$$\begin{aligned}
\|X\|_H = E[\langle X, X\rangle_\infty^{1/2}] &= E[\lim_{c\downarrow 0}\lim_n (c + \langle X, X\rangle_\infty)^{-1/2}\langle X^{T_n}, X^{T_n}\rangle_\infty] \\
&= \lim_{c\downarrow 0}\lim_n E[(c + \langle X, X\rangle_\infty)^{-1/2}\langle X^{T_n}, X^{T_n}\rangle_\infty] .
\end{aligned}$$

Thus $\|X\|_H \leqq \lim_{c\downarrow 0} E[\langle X, Y\rangle_\infty]$, which completes the proof.

3. We deal entirely with locally square integrable martingales.

DEFINITION 1. Let X and Y belong to $\mathcal{M}^{loc}(F_t)$. Then a complex-valued martingale $X + iY$ is called conformal if $\langle X, Y\rangle = 0$ and $\langle X, X\rangle = \langle Y, Y\rangle$.

Similarly, we can define such a complex-valued martingale by using another increasing process $[X, X]$ instead of $\langle X, X\rangle$. We call it a [,]-conformal martingale for the sake of convenience. Originally the concept of a conformal martingale was introduced by Getoor and Sharpe. They proved that if X is a continuous local martingale, then there exists a "conjugate" Y such that $X+iY$ is conformal. We shall give an extension of this fundamental result to our case, following the idea of them. But it seems to be more difficult to establish the existence of the [,]-conformal martingales, because $\langle X, X\rangle = \langle Y, Y\rangle$ does not always imply $[X, X] = [Y, Y]$ (the converse is clear).

DEFINITION 2. A system $(\tilde{\Omega}, \tilde{F}, \tilde{F}_t, \tilde{P})$ is said to be a lifting of (Ω, F, F_t, P) under the surjection $\pi: \tilde{\Omega} \to \Omega$ if

1°. $\pi^{-1}(F_t) \subset \tilde{F}_t$ for each t and $\pi^{-1}(F) \subset \tilde{F}$

2°. $P = \tilde{P} \circ \pi^{-1}$ on F

3°. if $X \in \mathscr{M}(F_t)$, then $X \circ \pi$ is a martingale over $(\tilde{F}_t)$.

Notice that this is not quite the same definition as that given in [1] where for every $X \in \mathscr{M}_c$ $X \circ \pi$ is a martingale over $(\tilde{F}_t)$.

Now we assume that (F_t) has no times of discontinuity; thus for every $X \in \mathscr{M}^{loc}$, $\langle X, X\rangle$ is continuous.

THEOREM 2. *Assume that (Ω, F, P) is separable. Then there exists a lifting $(\tilde{\Omega}, \tilde{F}, \tilde{F}_t, \tilde{P})$ of (Ω, F, F_t, P) under $\pi: \tilde{\Omega} \to \Omega$ which satisfies the following conditions*:

1°. *There exists a linear mapping $\alpha: \mathscr{M}(F_t) \mapsto \mathscr{M}_c(\tilde{F}_t)$ such that*

(1) *for every $X \in \mathscr{M}(F_t)$, $X \circ \pi + i\alpha(X)$ is conformal*

(2) *for every $X \in \mathscr{M}(F_t)$ and $C \in L^2(X)$, $\alpha(C \cdot X) = (C \circ \pi) \cdot \alpha(X)$*

2°. *There exists a linear mapping $\bar{\alpha}: \mathscr{M}(\tilde{F}_t) \mapsto \mathscr{M}(F_t)$ such that*

(1) *$\bar{\alpha} \circ \alpha$ is the identity on $\mathscr{M}(F_t)$*

(2) *if $X \in \mathscr{M}(F_t)$ and $\tilde{X} \in \mathscr{M}(\tilde{F}_t)$, then*

$$\tilde{E}[\alpha(X)_\infty \tilde{X}_\infty] = E[X_\infty \bar{\alpha}(\tilde{X})_\infty]$$

(3) *for every $\tilde{X} \in \mathscr{M}(\tilde{F}_t)$, $\|\bar{\alpha}(\tilde{X})\|_B \leqq \|\tilde{X}\|_B$.*

Let $X^* \in \mathscr{M}(F_t)$ be fundamental for $\mathscr{M}(F_t)$; the existence of such an element X^* is guaranteed by the separability of (Ω, F, P). Put now: $A_t = \langle X^*, X^*\rangle_t$ and $b_t = \inf\{s > 0; A_s > t\}$. Denote by (G_t) the right continuous family (F_{b_t}). Each A_t is a G_t-stopping time. Let (K_t) be the right continuous family (G_{A_t}). As $b_{A_t} \geqq t$, $F_t \subset K_t$.

LEMMA 1. *If $X \in \mathscr{M}(F_t)$, then for any fixed $r < s$, a.s.*

$$\{X \text{ constant on } [r, s[\} \supset \{\langle X, X\rangle \text{ constant on } [r, s[\}\ .$$

(See the proof of Lemma (4, 1) in [1], p. 284.)

REMARK. The reverse inclusion is not necessarily true. Now we state such an example communicated by R. K. Getoor. Let (Ω, F, F_t, P) be a system such that (F_t) has no times of discontinuity and such that there exists a totally inaccessible stopping time T with $F_{T-} \neq F_T$. Let now $Y \neq 0$ be a bounded F_T-measurable random variable with $E[Y|F_{T-}] = 0$. Denote by $X = (X_t)$ a right continuous modification of $E[Y|F_t]$. It is easy to see that $X = 0$ on $[0, T[$ and $X_T = Y$. On the other hand, as $E[\langle X, X\rangle_T] = E[X_T^2] > 0$, the sample continuity of $\langle X, X\rangle$ implies that $\langle X, X\rangle$ can not be constant on $[0, T[$.

LEMMA 2. *$\mathscr{M}(F_t)$ is a stable subspace of $\mathscr{M}(K_t)$.*

PROOF. If $X \in \mathscr{M}(F_t)$, X is uniformly integrable. By Doob's optional sampling theorem, $(X_{b_{A_t}}, K_t)$ belongs to $\mathscr{M}(K_t)$. On the other hand $\langle X, X\rangle_{b_{A_t}} = \langle X, X\rangle_t$, because the process A is constant on $[t, b_{A_t}]$. Therefore, as $t \leqq b_{A_t}$, we get

$$\begin{aligned} E[(X_{b_{A_t}} - X_t)^2] &= E[X_{b_{A_t}}^2 - 2E(X_{b_{A_t}}|F_t)X_t + X_t^2] \\ &= E[X_{b_{A_t}}^2 - X_t^2] \\ &= E[\langle X, X\rangle_{b_{A_t}} - \langle X, X\rangle_t] = 0 \end{aligned}$$

from which for every $t \geqq 0$ $X_{b_{A_t}} = X_t$. Consequently $\mathscr{M}(F_t) \subset \mathscr{M}(K_t)$.

Next, fix $X \in \mathscr{M}(F_t)$ and suppose that C is a bounded previsible process over (K_t) having the form: $C_t = C_{t_0} I_{]t_0,+\infty[}(t)$, where C_{t_0} is K_{t_0}-measurable. Then we have

$$\begin{aligned} (C \cdot X)_t &= C_{t_0}(X_t - X_{t_0}) I_{[t_0,+\infty[}(t) \\ &= C_{t_0}(X_t - X_{t_0})(I_{[t_0,b_{A_{t_0}}[}(t) + I_{[b_{A_{t_0}},+\infty[}(t)) \end{aligned}$$

From Lemma 1, X is a.s. constant on $[t_0, b_{A_{t_0}}[$, and so

$$(C \cdot X)_t = C_{t_0} I_{\{b_{A_{t_0}} \leqq t\}}(X_t - X_{t_0}) .$$

Thus $(C \cdot X)_t$ is F_t-measurable for each t. The stability of $\mathscr{M}(F_t)$ in $\mathscr{M}(K_t)$ now follows by the Monotone Class Theorem.

Except these two lemmas, there is no necessity to change the proof due to Getoor and Sharpe. That is, the other portions of our proof are mere translations to our case of the proof given by them for continuous martingales. However, we shall recall briefly the proof, modified to fit the present case, for the reader's convenience.

Now let (Ω', F', F'_t, P') be a separable system which carries a sequence $(B^n)_{n\geqq 1}$ of independent real Brownian motions with $B_0^n = 0$ and $\langle B^n, B^n\rangle_t = t$ for all n. Denote by $(\tilde{\Omega}, \tilde{F}, \tilde{G}_t, \tilde{P})$ the product of the systems (Ω, F, G_t, P) and (Ω', F', F'_t, P') with π the projection of $\tilde{\Omega}$ onto Ω. Put $\tilde{F}_t =$

$\tilde{G}_{A_t\circ\pi}$. It follows from Lemma 2 that $X_t \circ \pi = X_{b_{A_t}} \circ \pi$ is a martingale over $(\tilde{F}_t)$ for every $X \in \mathscr{M}(F_t)$. Consequently, $(\tilde{\Omega}, \tilde{F}, \tilde{F}_t, \tilde{P})$ is a lifting of (Ω, F, F_t, P) under π. Now we shall define the mapping α. By the continuity of $\langle X, X\rangle$ we have $\langle X\circ\pi, X\circ\pi\rangle = \langle X, X\rangle\circ\pi$ for every $X\in \mathscr{M}(F_t)$. Put now: $\tilde{N}_t^k(\omega, \omega') = B^k_{A_t(\omega)}(\omega')$. As $E[A_\infty] < +\infty$, each $\tilde{N}^k$ is an $\tilde{F}_t$-martingale which is clearly continuous. Obviously, $\langle \tilde{N}^j, \tilde{N}^k\rangle = 0$ if $j \neq k$, $\langle \tilde{N}^k, \tilde{N}^k\rangle_t = A_t\circ\pi$ and if $X\in\mathscr{M}(F_t)$, $\langle \tilde{N}^k, X\circ\pi\rangle = 0$ for all k. Let $(X^n)_{n\geqq 1}$ be an integral basis for $\mathscr{M}(F_t)$ whose existence is guaranteed by the separability of the space (Ω, F, P). Denote by D^n a previsible version of $d\langle X^n, X^n\rangle/dA$ and put: $C^n = (D^n)^{1/2}$, $\tilde{M}^n = (C^n\circ\pi)\cdot\tilde{N}^n$. Then the process $\tilde{M}^n$ belongs to $\mathscr{M}_c(\tilde{F}_t)$ and it does not depend on the choice of D^n. It is clear that $\langle \tilde{M}^n, X\circ\pi\rangle = 0$ for all n and $X\in\mathscr{M}(F_t)$, and that $\langle \tilde{M}^j, \tilde{M}^k\rangle = \delta_{jk}\langle X^j, X^k\rangle\circ\pi$.

On the other hand, for each $X\in\mathscr{M}(F_t)$, $X = \sum_n h^n\cdot X^n$, convergent in $\mathscr{M}(F_t)$ with $h^n = d\langle X, X^n\rangle/d\langle X^n, X^n\rangle$. Then the sum $\sum_n (h^n\circ\pi)\cdot\tilde{M}^n$ converges in $\mathscr{M}_c(\tilde{F}_t)$ because $\langle h^n\cdot X^n, h^n\cdot X^n\rangle\circ\pi = \langle (h^n\circ\pi)\cdot\tilde{M}^n, (h^n\circ\pi)\cdot\tilde{M}^n\rangle$ for each n; $(h^n\circ\pi)\cdot\tilde{M}^n$ does not depend on the choice of h^n.

The mapping $\alpha\colon \mathscr{M}(F_t)\to\mathscr{M}_c(\tilde{F}_t)$ given by

$$\alpha(X) = \alpha(\sum_n h^n\cdot X^n) = \sum_n (h^n\circ\pi)\cdot\tilde{M}^n$$

is well defined and linear. It should be noted that for every $X\in\mathscr{M}(F_t)$ $\alpha(X)$ is always continuous because of the continuity of A. From the above relation, for every $X\in\mathscr{M}(F_t)$, $\langle\alpha(X), \alpha(X)\rangle = \langle X\circ\pi, X\circ\pi\rangle$ and $\alpha(X)$ is orthogonal to $\mathscr{M}(F_t)\circ\pi$. Consequently for all $X\in\mathscr{M}(F_t)$ $X\circ\pi + i\alpha(X)$ is a conformal martingale over $(\tilde{F}_t)$.

Next, we shall explain briefly the definition of the adjoint mapping $\bar{\alpha}$. Denote by $\tilde{\mathscr{N}}$ the stable subspace of $\mathscr{M}(\tilde{F}_t)$ generated by the $X^n\circ\pi$ and $\tilde{M}^n$, by L_1 the projection of $\mathscr{M}(\tilde{F}_t)$ onto $\tilde{\mathscr{N}}$, and let $L_2\colon\tilde{\mathscr{N}}\to\tilde{\mathscr{N}}$ be defined as follows: if $\tilde{X}\in\tilde{\mathscr{N}}$ has an expansion of the form $\sum_n C^n\cdot(X^n\circ\pi) + \sum_n D^n\cdot\tilde{M}^n$, then $L_2(X) = \sum_n D^n\cdot(X^n\circ\pi) + \sum_n C^n\cdot\tilde{M}^n$; $L_2(X)$ does not depend on the versions of C^n and D^n. It is clear that for every $X\in\mathscr{M}(F_t)$ $L_2(\alpha(X)) = X\circ\pi$. Define $L_3\colon\tilde{\mathscr{N}}\to\mathscr{M}(K_t)$ by letting $L_3\tilde{X}$ for $\tilde{X}\in\tilde{\mathscr{N}}$ be the unique right continuous martingale over (K_t) such that $(L_3\tilde{X})_t\circ\pi = E[\tilde{X}_\infty|\pi^{-1}(K_t)]$. Finally, denote by L_4 the projection of $\mathscr{M}(K_t)$ onto $\mathscr{M}(F_t)$ which is well defined by Lemma 2. Then the mapping $\bar{\alpha} = L_4L_3L_2L_1\colon\mathscr{M}(\tilde{F}_t)\to\mathscr{M}(F_t)$ satisfies all the properties necessarily for the theorem.

If $X\in\mathscr{M}^{loc}(F_t)$ and (T_n) reduces X to $\mathscr{M}(F_t)$, then for every n

$$\alpha(X^{T_{n+1}}) = \alpha(X^{T_n}) \quad\text{on}\quad [0, T_n\circ\pi]$$

and so $\alpha(X)$ can be defined in $\mathscr{M}^{loc}(F_t)$. It is clear that $X\circ\pi + i\alpha(X)$ is conformal for each $X \in \mathscr{M}^{loc}(F_t)$.

4. By using the fact that $\alpha(X)$ is continuous for every $X \in \mathscr{M}^{loc}(F_t)$, we can prove that $(H^1)^* = BMO$. The next lemma implies that for each $Y \in BMO$ $f(X) = E[\langle X, Y\rangle_\infty]$ defines a bounded linear functional on H^1.

LEMMA 3. *If $X \in H^1$ and $Y \in BMO$, then*

$$E\left[\int_0^{+\infty} |d\langle X, Y\rangle_s|\right] \leqq \sqrt{2}\, \|X\|_H \|Y\|_B .$$

(For the proof, see Theorem (3.5) in [1], p. 282.)

To show $(H^1)^* \subset BMO$, it suffices to prove the next theorem, which is the exact extension of Theorem (8.1) in [1]. Of course, we assume that the space (Ω, F, P) is separable, and that (F_t) has no times of discontinuity.

THEOREM 3. *For any $f \in (H^1)^*$, there exists a unique $Y \in BMO$ such that $f(X) = E[\langle X, Y\rangle_\infty]$ and $2^{-5}\|Y\|_B \leqq \|f\| \leqq \sqrt{2}\, \|Y\|_B$.*

PROOF. Denote by $\tilde{H}_c^1$ the space of all continuous H^1-martingales over $(\tilde{F}_t)$. Then $\alpha(H^1)$ is a linear subspace of $\tilde{H}_c^1$, and it is easily checked that $(\tilde{\Omega}, \tilde{F}, \tilde{P})$ is separable.

Given a bounded linear functional f on H^1, there is induced a bounded linear functional g on $\alpha(H^1)$ by the relation $g(\alpha(X)) = f(X)$ (note that $\alpha(X) = 0$ if and only if $X = 0$). Then we get $\|f\| = \|g\|$ from $\|\alpha(X)\|_H = \|X\|_H$. From the Hahn-Banach theorem, there exists a functional g' in $(\tilde{H}_c^1)^*$ such that $g = g'$ on $\alpha(H^1)$ and $\|g\| = \|g'\|$. Hence, by Theorem (8.1) in [1], there exists a unique continuous BMO-martingale $\tilde{Y}$ over $(\tilde{F}_t)$ such that for every $\tilde{X} \in \tilde{H}_c^1$, $g'(\tilde{X}) = \tilde{E}[\langle \tilde{X}, \tilde{Y}\rangle_\infty]$ and $2^{-5}\|\tilde{Y}\|_B \leqq \|g'\|$. As $\|\bar{\alpha}(\tilde{Y})\|_B \leqq \|\tilde{Y}\|_B$, $\bar{\alpha}(\tilde{Y})$ is a BMO-martingale over (F_t). In particular, for every $X \in H^1$, we get

$$f(X) = g(\alpha(X)) = g'(\alpha(X)) = \tilde{E}[\langle \alpha(X), \tilde{Y}\rangle_\infty] .$$

Fix now $X \in H^1$, and let (T_n) be an increasing sequence of F_t-stopping times reducing X to $\mathscr{M}(F_t)$. Then each $T_n \circ \pi$ is also an $\tilde{F}_t$-stopping time and $\alpha(X^{T_n}) = \alpha(X)^{T_n \circ \pi}$. Since $\bar{\alpha}(\tilde{Y}) \in BMO$, $\alpha(X) \in \tilde{H}_c^1$ and $\tilde{Y}$ is a BMO-martingale over $(\tilde{F}_t)$, we find

$$\begin{aligned} f(X) &= \lim_n \tilde{E}[\langle \alpha(X), \tilde{Y}\rangle_{T_n \circ \pi}] \\ &= \lim_n \tilde{E}[\alpha(X^{T_n})_\infty \tilde{Y}_\infty] \\ &= \lim_n E[X_{T_n} \bar{\alpha}(\tilde{Y})_\infty] \\ &= \lim_n E[\langle X, \bar{\alpha}(\tilde{Y})\rangle_{T_n}] \\ &= E[\langle X, \bar{\alpha}(\tilde{Y})\rangle_\infty] \end{aligned}$$

by using Lemma 3, the property of α and the dominated convergence theorem. Thus $f(X) = E[\langle X, Y\rangle_\infty]$, where $Y = \bar{\alpha}(\tilde{Y}) \in BMO$. The uniqueness of Y follows easily from the fact $BMO \subset H^1$.

Finally, by Lemma 3, $||f|| \leqq \sqrt{2}||Y||_B$ and so we get

$$2^{-5}||Y||_B \leqq 2^{-5}||\tilde{Y}||_B \leqq ||g'|| = ||f|| \leqq \sqrt{2}\,||Y||_B \,.$$

This completes the proof.

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