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STABILITY OF HARMONIC MAPS AND STANDARD MINIMAL IMMERSIONS

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1. Introduction. Let f be a smooth map of a compact Riemannian manifold M into another Riemannian manifold N. The *energy functional* $E(f)$ for f is defined by

$$E(f) = (1/2)\int_M \|df\|^2 dv_M .$$

A smooth map f of M into N is called a *harmonic map* if f is a critical point of the energy functional E. A harmonic map f is called *stable* if every second variation of E at f is nonnegative. Let S^n be an n-dimensional Euclidean sphere. Then the following remarkable theorems are known.

THEOREM (Xin [22]). *For $n \geqq 3$ there exists no nonconstant stable harmonic map from S^n to any Riemannian manifold.*

THEOREM (Leung [5]). *For $n \geqq 3$ there exists no nonconstant stable harmonic map from any compact Riemannian manifold to S^n.*

It is natural to ask what kind of a compact Riemannian manifold M has the property that there exists no nonconstant stable harmonic map from M to any Riemannian manifold nor from any compact Riemannian manifold to M. We call such an M *harmonically unstable*. We know topological restrictions on harmonically unstable Riemannian manifolds; if M is harmonically unstable, then by a classical result on closed geodesics we have $\pi_1(M) = \{1\}$ and by the theorem of Sacks and Uhlenbeck [15] $\pi_2(M) = \{1\}$.

The purpose of this note is to classify harmonically unstable compact symmetric spaces.

THEOREM 1. *A compact symmetric space M is harmonically unstable, if and only if M is a product of simply connected compact irreducible symmetric spaces belonging to the following list;*

(i) *simple Lie groups of type A_n $(n \geqq 2)$, B_2 and C_n $(n \geqq 3)$,*

(ii) *$SU(2n)/Sp(n)$ $(n \geqq 3)$,*

(iii) *spheres S^n $(n \geqq 3)$,*

(iv) *quaternionic Grassmann manifolds* $Sp(p+q)/Sp(p)\times Sp(q)$ $(p \geqq q \geqq 1)$,

(v) E_6/F_4,

(vi) *Cayley projective plane* $F_4/Spin(9)$.

The method of Xin and Leung was to deform a harmonic map along conformal vector fields of S^n and take the average of the second variations. Our method generalized theirs. We deform a harmonic map along gradient vector fields of the first eigenfunctions for the Laplacian of a compact symmetric space, and use the standard immersion of the compact symmetric space into the first eigenspace in order to compute the average of the second variations. In [11], using the same method, we investigated the stability of minimal submanifolds in compact symmetric spaces.

Theorem 1 is proved in Section 3. In Section 4 we study the harmonic instability of convex hypersurfaces of a Euclidean space and isoparametric minimal hypersurfaces in a unit sphere.

2. Trace formulas and standard minimal immersions. Let M be an n-dimensional compact Riemannian manifold and Φ an isometric immersion of M into the Euclidean space $\boldsymbol{E}^N$ with the inner product $\langle\ ,\ \rangle$. We denote by R and B the curvature tensor of M and the second fundamental form of Φ, respectively. The equation of Gauss is given by

$$\langle R(X, Y)Z, W\rangle = \langle B(X, W), B(Y, Z)\rangle - \langle B(X, Z), B(Y, W)\rangle$$

for $X, Y, Z, W \in T_x(M)$. Let $\{v_1, \cdots, v_n\}$ be an orthonormal basis of $T_x(M)$. The Ricci tensor of M is defined by

$$\rho(X, Y) = \sum_{i=1}^{n} \langle R(X, v_i)v_i, Y\rangle\ , \quad \text{for} \quad X, Y \in T_x(M)\ ,$$

and the mean curvature vector of Φ is defined by

$$\eta = (1/n)\sum_{i=1}^{n} B(v_i, v_i)\ .$$

For a vector v in $\boldsymbol{E}^N$ we consider a vector field $V = \operatorname{grad} f_v$ on M, where $f_v(x) = \langle \Phi(x), v\rangle$ for $x \in M$. We denote by ψ_t the flow generated by V.

Let $f\colon N \to M$ be a harmonic map, where N is an m-dimensional compact Riemannian manifold. We now define a quadratic form Q_f on $\boldsymbol{E}^N$ by

$$Q_f(v) = (d^2/dt^2)E(\psi_t \circ f)|_{t=0}\ .$$

Then the trace of Q_f on $\boldsymbol{E}^N$ is given as follows (cf. Leung [6]):

$$\text{(1)} \qquad \operatorname{Tr} Q_f = \int_N \sum_{\alpha=1}^{m} \sum_{i=1}^{n} (2\| B(df(e_\alpha), v_i)\|^2 - \langle B(df(e_\alpha), df(e_\alpha)), B(v_i, v_i)\rangle) dv_N ,$$

where $\{e_1, \cdots, e_m\}$ is an orthonormal basis at $x \in N$ and $\{v_1, \cdots, v_n\}$ is an orthonormal basis at $f(x) \in M$.

On the other hand, let $h: M \to N'$ be a harmonic map, where N' is a Riemannian manifold with the metric $\langle \, , \rangle'$. We define a quadratic form Q_h on $\boldsymbol{E}^N$ by

$$Q_h(v) = (d^2/dt^2)E(h \circ \psi_t)|_{t=0} .$$

Then the trace of Q_h on $\boldsymbol{E}^N$ is given as follows (cf. Pan [14]):

$$\text{(2)} \qquad \operatorname{Tr} Q_h = \int_M \sum_{i,j=1}^{n} \sum_{k=1}^{n} (2\langle B(v_i, v_k), B(v_k, v_j)\rangle - \langle B(v_i, v_j), B(v_k, v_k)\rangle)\langle dh(v_i), dh(v_j)\rangle' dv_M ,$$

where $\{v_1, \cdots, v_n\}$ is an orthonormal basis at $x \in M$.

Next we consider the case where M is a product submanifold of $\boldsymbol{E}^N$. Suppose that M is a product manifold $M_1 \times \cdots \times M_r$ and Φ is a product isometric immersion $\Phi_1 \times \cdots \times \Phi_r$, where, for each s with $1 \leqq s \leqq r$, M_s is an $n(s)$-dimensional compact Riemannian manifold and Φ_s is an isometric immersion of M_s into $\boldsymbol{E}^{N(s)}$. We denote by B_s the second fundamental form of Φ_s. Then the formulas (1) and (2) are written as follows:

$$\text{(1)}' \qquad \operatorname{Tr} Q_f = \sum_{s=1}^{r} \int_N \sum_{\alpha=1}^{m} \sum_{k(s)} (2\| B_s((df)_s(e_\alpha), v_{k(s)})\|^2 - \langle B_s((df)_s(e_\alpha), (df)_s(e_\alpha)), B_s(v_{k(s)}, v_{k(s)})\rangle) dv_N ,$$

and

$$\text{(2)}' \qquad \operatorname{Tr} Q_h = \sum_{s=1}^{r} \int_M \sum_{i(s), j(s)} \sum_{k(s)} (2\langle B_s(v_{i(s)}, v_{k(s)}), B_s(v_{k(s)}, v_{j(s)})\rangle - \langle B_s(v_{i(s)}, v_{j(s)}), B_s(v_{k(s)}, v_{k(s)})\rangle)\langle dh(v_{i(s)}), dh(v_{j(s)})\rangle' dv_M ,$$

where $(df)_s$ denotes the $T(M_s)$-component of df, $\{v_{k(s)}; k(s) = n(1) + \cdots + n(s-1) + 1, \cdots, n(1) + \cdots + n(s-1) + n(s)\}$ is an orthonormal basis of $T_x(M_s)$, and each of the indices $i(s)$, $j(s)$ and $k(s)$ runs from $n(1) + \cdots + n(s-1) + 1$ to $n(1) + \cdots + n(s-1) + n(s)$.

We shall review quickly the definition of the standard minimal immersions of compact irreducible symmetric spaces.

Let $M = G/K$ be an n-dimensional compact irreducible symmetric space and g_0 a G-invariant Riemannian metric on M induced by the Killing form of the Lie algebra of G. We should note that the scalar curvature of (M, g_0) is equal to $n/2$. Let Δ be the Laplacian of (M, g_0) acting on functions. For the k-th eigenvalue λ_k of Δ, we choose an orthonormal basis $\{f_0, \cdots, f_{m(k)}\}$ of the k-th eigenspace with respect to the L^2-inner product defined by g_0. We consider the mapping Φ_k of M into $E^{m(k)+1}$ defined by

$$\Phi_k\colon M \ni x \mapsto C\cdot(f_0(x), \cdots, f_{m(k)}(x)) \in E^{m(k)+1} ,$$

where $C = (\mathrm{Vol}(M, g_0)/(m(k)+1))^{1/2}$. Then Φ_k is a full G-equivariant minimal isometric immersion of $(M, (\lambda_k/n)g_0)$ into the unit sphere $S^{m(k)}(1)$ (cf. Takahashi [20], Wallach [21]). This Φ_k is called the *k-th standard minimal immersion* of M. In [10] we studied some properties of the first standard immersions of compact irreducible symmetric spaces.

3. Main Results. First we shall show the following.

THEOREM 2. *Let M be an n-dimensional compact minimal submanifold immersed in a unit sphere $S^{N-1}(1)$. If the Ricci curvature ρ of M satisfies $\rho > n/2$, then M is harmonically unstable.*

PROOF. We use the same notation as in Section 2 and denote by $\underline{\rho}$ the minimum value of the Ricci curvature of M. Let $f\colon N \to M$ be a nonconstant harmonic map. By the equation of Gauss, (1) becomes

$$\begin{aligned}\mathrm{Tr}\, Q_f &= \int_N \sum_{\alpha=1}^{m} \sum_{i=1}^{n} (\langle B(df(e_\alpha), df(e_\alpha)), B(v_i, v_i)\rangle - 2\langle R(df(e_\alpha), v_i)v_i, df(e_\alpha)\rangle) dv_N \\ &= \int_N \sum_{\alpha=1}^{m} (n\langle B(df(e_\alpha), df(e_\alpha)), \eta\rangle - 2\rho(df(e_\alpha), df(e_\alpha))) dv_N .\end{aligned}$$

Since the minimality of M in $S^{N-1}(1)$ implies $\langle B(X, Y), \eta\rangle = \langle X, Y\rangle$, we have

$$\mathrm{Tr}\, Q_f \leqq 2(n - 2\underline{\rho})E(f) .$$

Since $\underline{\rho} > n/2$ and $E(f) > 0$, we get $\mathrm{Tr}\, Q_f < 0$. Thus f is unstable. Next let $h\colon M \to N'$ be a nonconstant harmonic map. In the formula (2) we choose an orthonormal basis $\{v_1, \cdots, v_n\}$ at $x \in M$ such that $\langle dh(v_i), dh(v_j)\rangle' = a_i\delta_{ij}$, where each a_i is nonnegative. By the equation of Gauss and the minimality of M in $S^{N-1}(1)$, (2) becomes

$$\begin{aligned}\mathrm{Tr}\, Q_h &= \int_M \sum_{i,k=1}^{n} (2\langle B(v_i, v_k), B(v_k, v_i)\rangle - \langle B(v_i, v_i), B(v_k, v_k)\rangle) a_i dv_M \\ &= \int_M \sum_{i=1}^{n} (n\langle B(v_i, v_i), \eta\rangle - 2\rho(v_i, v_i)) a_i dv_M \\ &\leqq 2(n - 2\underline{\rho})E(h) .\end{aligned}$$

Thus h is unstable. q.e.d.

COROLLARY 3. *Let M be an n-dimensional compact minimal submanifold immersed in a unit sphere $S^{N-1}(1)$. If the Ricci curvature ρ*

of M satisfies $\rho > n/2$, then $\pi_1(M) = \{1\}$ and $\pi_2(M) = \{1\}$.

REMARK. The result is sharp; M satisfies $\rho = n/2$ and $\pi_2(M) \neq \{1\}$, if M is an n-dimensional compact irreducible Hermitian symmetric space isometrically imbedded in a unit sphere by its first standard minimal immersion or if M is the Clifford minimal hypersurface $S^2(\sqrt{1/2}) \times S^2(\sqrt{1/2})$ of the unit sphere $S^5(1)$.

Applying Theorem 2 to the first standard minimal immersions of compact irreducible symmetric spaces, we obtain the following theorem.

THEOREM 4. *Let M be an n-dimensional compact irreducible symmetric space and denote by λ_1 and c the first eigenvalue of the Laplacian on functions and the scalar curvature of M respectively. Then the following four conditions are equivalent:*

(a) $\lambda_1 < 2c/n$.

(b) *There exists no nonconstant stable harmonic map from M to any Riemannian manifold.*

(c) *There exists no nonconstant stable harmonic map from any compact Riemannian manifold to M.*

(d) *The identity map of M is unstable as a harmonic map.*

PROOF. (b) $\Rightarrow$ (d) and (c) $\Rightarrow$ (d) are trivial. Since the first eigenvalue for the Jacobi operator of the identity map as a harmonic map is $\mathrm{Min}\{0, \lambda_1 - 2c/n\}$, we have (a) $\Leftrightarrow$ (d) (cf. Smith [16]). We have only to show (a) $\Rightarrow$ (b) and (a) $\Rightarrow$ (c). We may assume that the Riemannian metric on M is the metric g_0 defined in Section 2. Since the scalar curvature of (M, g_0) is $n/2$, we suppose that $\lambda_1 < 1$. Let $\Phi_1: (M, (\lambda_1/n)g_0) \to S^{m(1)}(1)$ be the first standard minimal isometric immersion of M. Since M is an Einstein manifold and the scalar curvature of $(M, (\lambda_1/n)g_0)$ is equal to $n^2/2\lambda_1$, we have $\rho = \underline{\rho} = n/2\lambda_1$, where $\underline{\rho}$ is the minimum of the Ricci curvature ρ of $(M, (\lambda_1/n)g_0)$. Hence we get $n - 2\underline{\rho} = n - n/\lambda_1 = n(1 - 1/\lambda_1) < 0$. By Theorem 2 we obtain the conditions (b) and (c). q.e.d.

From Theorem 4 we see that a compact irreducible symmetric space is simply connected if its identity map is unstable. The eigenvalues of the Laplacian on a simply connected compact symmetric space can be computed by using the formula of Freudenthal and the theorems of Sugiura [17], [18]. Simply connected compact irreducible symmetric spaces with the unstable identity map were determined by Smith [16] and Nagano [9] as follows. (There seem to be inaccuracies in [9].)

PROPOSITION 5. *Let M be a simply connected compact irreducible* *symmetric space. Its identity map is unstable if and only if M belongs to the following list;*

(i) *simple Lie groups of type A_n $(n \geqq 2)$, B_2 and C_n $(n \geqq 3)$,*
(ii) $SU(2n)/Sp(n)$ $(n \geqq 3)$,
(iii) S^n $(n \geqq 3)$,
(iv) $Sp(p+q)/Sp(p)\times Sp(q)$ $(p \geqq q \geqq 1)$,
(v) E_6/F_4,
(vi) $F_4/Spin(9)$.

PROOF OF THEOREM 1. Suppose that $M = M_1\times\cdots\times M_r$ is a product of compact irreducible symmetric spaces belonging to the list of Theorem 1. We isometrically imbed each M_i into some Euclidean space $\boldsymbol{E}^{N(i)}$ by using the first standard minimal immersion of M_i. Thus we get a product isometric imbedding of $M = M_1\times\cdots\times M_r$ into $\boldsymbol{E}^N = \boldsymbol{E}^{N(1)}\times\cdots\times\boldsymbol{E}^{N(r)}$. Applying (1)′ and (2)′ to this product isometric imbedding, we obtain the harmonic instability of M.

Conversely, suppose that a compact symmetric space M is harmonically unstable. Since M is simply connected, M is a product $M_1\times\cdots\times M_r$, where each M_i is a simply connected compact irreducible symmetric space. Without loss of generality, we assume that the identity map of some M_1 is stable as a harmonic map. Fix a point $(y_1, \cdots, y_r)$ in M. We define a mapping $f\colon M \to M$ by

$$f\colon M \ni (x_1, x_2, \cdots, x_r) \mapsto (x_1, y_2, \cdots, y_r) \in M\,.$$

Then f is a nonconstant stable harmonic map, a contradiction. Hence each M_i has the unstable identity map. By Proposition 5 M is a product of compact irreducible symmetric spaces belonging to the list of Theorem 1. q.e.d.

4. **Other Examples.** By virtue of the formulas (1) and (2) we can find many examples of harmonically unstable Riemannian manifolds. We look at convex hypersurfaces of a Euclidean space and isoparametric minimal hypersurfaces of a unit sphere.

PROPOSITION 6. *Let M be an n-dimensional compact convex hypersurface in the Euclidean space $\boldsymbol{E}^{n+1}$. If the principal curvatures $\kappa_i > 0$ $(i = 1, \cdots, n)$ of M satisfies*

$$\kappa_i < \kappa_1 + \cdots + \kappa_{i-1} + \kappa_{i+1} + \cdots + \kappa_n\,,$$

for each i with $1 \leqq i \leqq n$, the M is harmonically unstable.

PROOF. Let $f\colon N \to M$ and $h\colon M \to N'$ be nonconstant stable harmonic maps, where N is a compact Riemannian manifold. By simple computations, (1) and (2) give

$$\operatorname{Tr} Q_f = \int_N \sum_{\alpha=1}^{m} \sum_{i=1}^{n} \kappa_i(\kappa_i - \sum_{j=1,\, j\neq i}^{n} \kappa_j)\langle df(e_\alpha),\, v_i\rangle^2 dv_N$$

and

$$\operatorname{Tr} Q_h = \int_M \sum_{i=1}^{n} \kappa_i\Big(\kappa_i - \sum_{j=1,\, j\neq i}^{n} \kappa_j\Big)\langle dh(v_i),\, dh(v_i)\rangle' dv_M \,. \qquad \text{q.e.d.}$$

EXAMPLE. (1) Let M be a compact convex hypersurface in E^{n+1}, $n \geqq 3$, such that its principal curvatures $\kappa_i > 0$ $(i = 1, \cdots, n)$ satisfy $1/(n-1) < \kappa_i \leqq 1$. Then M is harmonically unstable.

(2) Let M be an ellipsoid in E^{n+1} defined by an equation $ax_0^2 + x_1^2 + \cdots + x_n^2 = 1$, $a > 0$. If $n \geqq 3$ and $0 < a < n-1$, then M is harmonically unstable.

PROOF of (2). It is easy to check that M has at most two distinct principal curvatures $\kappa_1 = \{a(a-1)x_0^2 + 1\}^{-1/2}$ and $\kappa_2 = a\{a(a-1)x_0^2 + 1\}^{-3/2}$ with multiplicities $n-1$ and 1, respectively. Hence we have $\kappa_1 \leqq \kappa_2 \leqq a\kappa_1$ for $a \geqq 1$, and $a\kappa_1 \leqq \kappa_2 \leqq \kappa_1$ for $a \leqq 1$. The rest of the proof is immediate. q.e.d.

The above example (2) is a slight extension of the result of Leung [6].

Next let M be an isoparametric hypersurface in an $(n+1)$-dimensional unit sphere with g distinct principal curvatures. Let $\kappa_0 > \kappa_1 > \cdots > \kappa_{g-1}$ be the distinct principal curvatures of M and denote by m_i the multiplicity for κ_i. Münzner [7], [8] showed the following;

(i) $m_\alpha = m_{\alpha+2}$ (indices modulo g) for any $\alpha \in \{0, \cdots, g-1\}$,

(ii) $g = 1, 2, 3, 4$ or 6,

(iii) if $g = 3$ or 6, then $m_0 = m_1$.

Recently Abresch [1] showed that if $g = 6$, then $m_0 = m_1 = 1$ or 2.

Assume that M is a minimal submanifold in the unit sphere. Then, using the result of [7], we can show that in each case $\kappa_0, \cdots, \kappa_{g-1}$ are given as follows:

PROPOSITION 7.

(1) *If* $g = 1$, *then* $\kappa_0 = 0$.

(2) *If* $g = 2$, *then* $\kappa_0 = \sqrt{m_1/m_0}$ *and* $\kappa_1 = -\sqrt{m_0/m_1}$.

(3) *If* $g = 3$, *then* $\kappa_0 = \sqrt{3}$, $\kappa_1 = 0$ *and* $\kappa_2 = -\sqrt{3}$.

(4) *If* $g = 4$, *then* $\kappa_0 = (\sqrt{m_0 + m_1} + \sqrt{m_1})/\sqrt{m_0}$, $\kappa_1 = (\sqrt{m_0 + m_1} - \sqrt{m_0})/\sqrt{m_1}$, $\kappa_2 = (\sqrt{m_1} - \sqrt{m_0 + m_1})/\sqrt{m_0}$ *and* $\kappa_3 = -(\sqrt{m_0 + m_1} + \sqrt{m_0})/\sqrt{m_1}$.

(5) *If* $g = 6$, *then* $\kappa_0 = 2 + \sqrt{3}$, $\kappa_1 = 1$, $\kappa_2 = 2 - \sqrt{3}$, $\kappa_3 = -(2 - \sqrt{3})$, $\kappa_4 = -1$ *and* $\kappa_5 = -(2 + \sqrt{3})$.

By Proposition 7 and the equation of Gauss we can compute the Ricci curvature ρ of M. The following isoparametric minimal hypersurfaces satisfy $\rho > n/2$;

(1) $g = 1$, $m_0 = n \geqq 3$,

(2) $g = 2$, $m_0 \geqq 3$ and $m_1 \geqq 3$,

(3) $g = 3$, $m_0 = m_1 \geqq 3$,

(4) $g = 4$, m_0, $m_1 \geqq 4$,

(5) $g = 6$, $m_0 = m_1 \geqq 5$.

By Theorem 2 we obtain the following:

PROPOSITION 8. *If M is an isoparametric compact minimal hypersurface in a unit sphere belonging to the above list, then M is harmonically unstable.*

We consider each of the five cases. Let M be an isoparametric compact minimal hypersurface in a unit sphere with g distinct principal curvatures.

(1) $g = 1$: M is a great sphere S^n.

(2) $g = 2$: M is a Clifford minimal hypersurface $S^p(\sqrt{p/n}) \times S^q(\sqrt{q/n})$ $(n = p + q,\ 1 \leqq p, q \leqq n,\ m_0 = p,\ m_1 = q)$ (cf. Cartan [2]).

(3) $g = 3$: According to Cartan [3], all the isoparametric compact minimal hypersurfaces with $g = 3$ and $m_0 = m_1 \geqq 3$ are homogeneous hypersurfaces $Sp(3)/Sp(1)^3$ $((m_0, m_1) = (4, 4))$ and $F_4/Spin(8)$ $((m_0, m_1) = (8, 8))$, which appear as principal orbits of the isotropy representations of the symmetric spaces $SU(6)/Sp(3)$ and E_6/F_4, respectively (cf. [19]).

(4) $g = 4$: According to Takagi and Takahashi [19], all the homogeneous compact minimal hypersurfaces with $g = 4$ and m_0, $m_1 \geqq 4$ are $(Sp(2) \times Sp(p))/(Sp(1)^2 \times Sp(p-1))$ $(p \geqq 3,\ (m_0, m_1) = (4, 4p-5))$, $U(5)/(SU(2) \times SU(2) \times T^1)$ $((m_0, m_1) = (4, 5))$ and $(U(1) \times Spin(10))/(T^1 \times SU(4))$ $((m_0, m_1) = (6, 9))$, which appear as principal orbits of the isotropy representations of the symmetric spaces $Sp(p + 2)/Sp(2) \times Sp(p)$, $SO(10)/U(5)$ and $E_6/U(1) \times Spin(10)$, respectively. There are many inhomogeneous isoparametric compact minimal hypersurfaces satisfying $g = 4$ and m_0, $m_1 \geqq 4$ (cf. Ozeki and Takeuchi [12], [13], Ferus, Karcher and Münzner [4]).

(5) $g = 6$: By the result of Abresch there exists no isoparametric hypersurface in a unit sphere with $g = 6$ and $m_0 = m_1 \geqq 5$.

REMARK. I take this occasion to correct errors in my paper [10]:

(i) P38 ↓ 12, "except $SO(9)/SO(6) \times SO(3)$ and" should be replaced by "except $Spin(5)$, $Spin(7)$, $Spin(8)$, $SO(9)/SO(6) \times SO(3)$ and".

(ii) P38 ↓ 13, "non simply connected spaces $SO(p + q)/S(O(p) \times O(q))$ $(p \geqq q \geqq 1)$" should be replaced by "non simply connected spaces $SO(n)$ $(n \geqq 5)$, $SO(p + q)/S(O(p) \times O(q))$ $(p \geqq q \geqq 1)$".

(iii) P38 $\downarrow$ 18, "one of $SO(9)/SO(6)\times SO(3)$ and" should be replaced by "one of $Spin(8)$, $SO(9)/SO(6)\times SO(3)$ and".

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