

THE IRREDUCIBLE DECOMPOSITION OF AN AFFINE HOMOGENEOUS CONVEX DOMAIN

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1. Introduction. A convex domain not containing any affine line in a finite dimensional real vector space is said to be *affine homogeneous* if the group of all affine automorphisms of it acts transitively. In the present note, an affine homogeneous convex domain is simply called a homogeneous convex domain. A homogeneous convex domain is said to be *reducible* if it is affinely equivalent to the direct product of homogeneous convex domains of lower dimension; otherwise it is said to be *irreducible*. It is known that every homogeneous convex cone is uniquely decomposed into the direct product of irreducible homogeneous convex cones ([1], [7]). The main purpose of the present note is to prove that this fact holds for an arbitrary homogeneous convex domain. The proof is given in §4 by means of the notion of clans due to E. B. Vinberg ([7]). Using this, we can generalize results in [5], [6], [3] to the case of reducible homogeneous convex domains, as we see in §5. The reader is referred to [7] for details concerning the results on clans used in the present note.

2. Homogeneous convex domains and clans. We first recall the notion of clans. A finite dimensional algebra \mathfrak{S} over the real number field is called a *clan* if the following conditions are satisfied:

- (1) $a(bc) - (ab)c = b(ac) - (ba)c$ for all $a, b, c \in \mathfrak{S}$;
- (2) There exists a linear form α on \mathfrak{S} such that $\alpha(ab) = \alpha(ba)$ for all $a, b \in \mathfrak{S}$ and $\alpha(aa) > 0$ for all $a \neq 0 \in \mathfrak{S}$;
- (3) For every $a \in \mathfrak{S}$, the eigenvalues of the left multiplication $x \in \mathfrak{S} \rightarrow ax \in \mathfrak{S}$ are real.

Let Ω be a homogeneous convex domain in the real number space \mathbf{R}^n . Then Ω admits a simply transitive triangular affine Lie group T . Let \mathfrak{t} be the Lie algebra of T and p any point of Ω . Then the mapping $D \in \mathfrak{t} \rightarrow D(p) \in \mathbf{R}^n$ is a linear isomorphism. Therefore, for every $a \in \mathbf{R}^n$, we have a unique element $D_a \in \mathfrak{t}$ satisfying $D_a(p) = a$. We denote by L_a the linear part of the operator D_a . We now define a multiplication $a\Delta b$ in \mathbf{R}^n by $a\Delta b = L_a(b)$ and a linear form α on \mathbf{R}^n by $\alpha(a) =$ the trace of L_a . Then the algebra \mathbf{R}^n provided with the linear form α is a clan,

which is denoted by $\mathfrak{S}(\Omega)$. Moreover, it was proved by Vinberg ([7, Theorem 2, p. 368]) that the set of all homogeneous convex domains and the set of all clans are in one-to-one correspondence (up to isomorphism); the correspondence assigns to a homogeneous convex domain Ω the clan $\mathfrak{S}(\Omega)$ of Ω .

In the correspondence stated above, we can see that a homogeneous convex domain Ω is affinely equivalent to the direct product of homogeneous convex domains Ω_1 and Ω_2 if and only if the clan $\mathfrak{S}(\Omega)$ is isomorphic to the direct sum $\mathfrak{S}(\Omega_1) + \mathfrak{S}(\Omega_2)$ of clans.

Let \mathfrak{S} be a clan. By an ideal of \mathfrak{S} we mean a two-sided ideal of \mathfrak{S} . A clan is said to be *reducible* if it is the direct sum of non-trivial ideals; otherwise it is said to be *irreducible*. Then, it follows that a homogeneous convex domain is irreducible if and only if its clan is irreducible. Hence for our purpose, it suffices to prove that an arbitrary clan admits a unique decomposition into the direct sum of irreducible ideals.

We next recall a normal decomposition of a clan from [7, p. 374]. Let \mathfrak{S} be a clan provided with a multiplication Δ and let L_a (resp. R_a) be the left (resp. right) multiplication by an element a of \mathfrak{S} . We denote by $(,)$ the canonical inner product in \mathfrak{S} defined by $(a, b) = \text{trace of } L_a b b$. Then \mathfrak{S} admits a direct sum decomposition (called a *normal decomposition*) by subspaces

$$\mathfrak{S} = \sum_{1 \leq i \leq j \leq r-1} \mathfrak{S}_{ij} + \sum_{1 \leq i \leq r-1} \mathfrak{S}_{ir} \quad (r \geq 2)$$

satisfying the following conditions:

(2.1) The direct summands \mathfrak{S}_{ij} are mutually orthogonal with respect to the inner product $(,)$;

(2.2) $\mathfrak{S}_{ii} = R e_i$ ($1 \leq i \leq r-1$), where e_i is an idempotent;

(2.3) The linear operators L_{e_i} and R_{e_i} are scalar operators on every direct summand and they are given by

$$e_i \Delta x_{ij} = \frac{1}{2} x_{ij}, \quad x_{ij} \Delta e_i = 0 \quad \text{for all } x_{ij} \in \mathfrak{S}_{ij} \quad (1 \leq i < j \leq r);$$

$$e_j \Delta x_{ij} = \frac{1}{2} x_{ij}, \quad x_{ij} \Delta e_j = x_{ij} \quad \text{for all } x_{ij} \in \mathfrak{S}_{ij} \quad (1 \leq i < j \leq r-1);$$

$$e_i \Delta x_{jk} = x_{jk} \Delta e_i = 0 \quad \text{for all } x_{jk} \in \mathfrak{S}_{jk} \quad (i \neq j, k).$$

Let \mathfrak{S} be the clan of a homogeneous convex domain Ω and $\mathfrak{S} = \sum_{1 \leq i \leq j \leq r-1} \mathfrak{S}_{ij} + \sum_{1 \leq i \leq r-1} \mathfrak{S}_{ir}$ a normal decomposition of \mathfrak{S} . Then it is known by [7] that Ω is affinely equivalent to a convex cone if and only if \mathfrak{S} admits a unit element. It is easy to see that such a clan is characterized by the

condition $\mathfrak{S}_{ir} = (0)$ ($1 \leq i \leq r - 1$). Moreover, let V be the cone fitted onto Ω (cf. [7]). Then a normal decomposition of the clan $\mathfrak{S}(V)$ is given by

$$\mathfrak{S}(V) = \sum_{1 \leq i \leq j \leq r} \mathfrak{S}_{ij} + \sum_{1 \leq i \leq r} \mathfrak{S}_{i,r+1},$$

where $\mathfrak{S}_{i,r+1} = (0)$ ($1 \leq i \leq r$).

3. Some Lemmas. In this section, we prove some lemmas needed later. Let \mathfrak{S} be a clan. Then we take and fix a normal decomposition of \mathfrak{S} by subspaces \mathfrak{S}_{ij} ($1 \leq i \leq j \leq r - 1$) and \mathfrak{S}_{ir} ($1 \leq i \leq r - 1$). A general element x of \mathfrak{S} is written as

$$x = \sum_{1 \leq i \leq j \leq r-1} x_{ij} + \sum_{1 \leq i \leq r-1} x_{ir}$$

according to the normal decomposition of \mathfrak{S} . By using (2.2) and (2.3), we have the following identities:

$$e_i \Delta(x \Delta e_j) = \frac{1}{2} x_{ij} \quad \text{for all } x \in \mathfrak{S} \quad (1 \leq i < j \leq r - 1);$$

$$y \Delta e_j = y_{jj} \quad \text{for all } y = \sum_{1 \leq i \leq r-1} (y_{ii} + y_{ir}) \in \mathfrak{S} \quad (1 \leq j \leq r - 1);$$

$$e_j \Delta z = \frac{1}{2} z_{jr} \quad \text{for all } z = \sum_{1 \leq i \leq r-1} z_{ir} \in \mathfrak{S} \quad (1 \leq j \leq r - 1).$$

Let \mathfrak{M} be an ideal of \mathfrak{S} and define subspaces \mathfrak{M}_{ij} of \mathfrak{M} by $\mathfrak{M}_{ij} = \mathfrak{M} \cap \mathfrak{S}_{ij}$. Then, by using the above identities, we have $\mathfrak{M} = \sum_{1 \leq i \leq j \leq r-1} \mathfrak{M}_{ij} + \sum_{1 \leq i \leq r-1} \mathfrak{M}_{ir}$ (direct sum). Since $\dim \mathfrak{S}_{ii} = 1$, the subspace \mathfrak{M}_{ii} is equal either to (0) or to \mathfrak{S}_{ii} .

We first prove the following:

- LEMMA 1.** (1) If $\mathfrak{M}_{jj} = \mathfrak{S}_{jj}$, then $\mathfrak{M}_{ij} = \mathfrak{S}_{ij}$ and $\mathfrak{M}_{jk} = \mathfrak{S}_{jk}$ for $j \neq i, k$.
 (2) If $\mathfrak{M}_{jj} = (0)$, then $\mathfrak{M}_{jk} = (0)$ for $j < k$ and \mathfrak{M}_{ij} is equal either to (0) or to \mathfrak{S}_{ij} for $i < j$.

PROOF. (1) For any $x_{ij} \in \mathfrak{S}_{ij}$ and $x_{jk} \in \mathfrak{S}_{jk}$ ($i < j < k$), the identities $x_{ij} = x_{ij} \Delta e_j$ and $x_{jk} = 2e_j \Delta x_{jk}$ hold by (2.3), which imply the assertion. (2) For every $x_{jk} \in \mathfrak{S}_{jk}$ ($j < k$), the formula $x_{jk} \Delta x_{jk} = ((x_{jk}, x_{jk}) / (e_j, e_j)) e_j$ holds (cf. [7, p. 376]). From this and the condition $\mathfrak{M}_{jj} = (0)$, we have $\mathfrak{M}_{jk} = (0)$. If $\mathfrak{M}_{ii} = (0)$, then $\mathfrak{M}_{ij} = (0)$. If $\mathfrak{M}_{ii} \neq (0)$, then we have $\mathfrak{M}_{ij} = \mathfrak{S}_{ij}$ by (1). q.e.d.

Let us denote by I the index set $\{1, 2, \dots, r - 1\}$ of \mathfrak{S} . Then a subset I_1 of I is said to be *admissible* if the condition $\mathfrak{S}_{\varepsilon(i)\varepsilon(j)} = (0)$ holds for every pair (i, j) of indices i and j satisfying $i \in I_1$ and $j \notin I_1$, where $(\varepsilon(i), \varepsilon(j))$ means (i, j) or (j, i) in accordance with $i < j$ or $j < i$, respectively.

It follows from definition that the complementary subset of an admissible subset of I is also admissible. A subset of I is said to be *irreducible* if it does not contain any proper admissible subset.

LEMMA 2. *The index set I is represented as $I = I_1 \cup \dots \cup I_p$, the disjoint union of irreducible and admissible subsets I_α ($1 \leq \alpha \leq p$). This decomposition is unique up to order.*

PROOF. For a pair (i, j) of indices $i, j \in I$, we define a relation $i \sim j$ if there exists a sequence $\{i_1, i_2, \dots, i_m\}$ of indices in I such that $i_1 = i$, $i_m = j$ and $\mathfrak{L}_{\varepsilon(i_{k-1})\varepsilon(i_k)} \neq (0)$ ($2 \leq k \leq m$). Then it is easy to see that the relation \sim is an equivalence relation in I , and that for every index $i \in I$, the equivalence class $\{j \in I; j \sim i\}$ is irreducible and admissible. Conversely, every irreducible and admissible subset is nothing but an equivalence class. q.e.d.

For an ideal \mathfrak{M} of a clan \mathfrak{L} , we define a subset $I(\mathfrak{M})$ of the index set I by $I(\mathfrak{M}) = \{i \in I; \mathfrak{M}_{ii} = \mathfrak{L}_{ii}\}$. Then we have the following:

LEMMA 3. *If a clan \mathfrak{L} is the direct sum of non-trivial ideals \mathfrak{M}_α ($1 \leq \alpha \leq p$), then the subsets $I(\mathfrak{M}_\alpha)$ of I are admissible and satisfy the condition $I = I(\mathfrak{M}_1) \cup \dots \cup I(\mathfrak{M}_p)$ (disjoint).*

PROOF. We first remark that $\mathfrak{L} = \mathfrak{M}_1 + \dots + \mathfrak{M}_p$ is the orthogonal direct sum with respect to the canonical inner product $(,)$. Therefore, the assertion follows from the condition (2.1) and Lemma 1. q.e.d.

Conversely we have the following:

LEMMA 4. *Let I_α ($1 \leq \alpha \leq p$) be admissible subsets of I such that $I = I_1 \cup \dots \cup I_p$ (disjoint). Then there exist unique ideals \mathfrak{L}_α of \mathfrak{L} satisfying the conditions $I(\mathfrak{L}_\alpha) = I_\alpha$ ($1 \leq \alpha \leq p$) and $\mathfrak{L} = \mathfrak{L}_1 + \dots + \mathfrak{L}_p$ (direct sum).*

PROOF. The multiplication Δ in \mathfrak{L} satisfies the following relations (cf. [7, the formulas (34)–(36), p. 376]):

$$\begin{aligned} \mathfrak{L}_{ij}\Delta\mathfrak{L}_{kl} &= (0) \quad (j \neq k, l); \quad \mathfrak{L}_{ij}\Delta\mathfrak{L}_{jk} \subset \mathfrak{L}_{ik} \quad (i < j < k); \\ \mathfrak{L}_{ij}\Delta\mathfrak{L}_{kj} &\subset \mathfrak{L}_{\varepsilon(i)\varepsilon(k)} \quad (i \leq j, k \leq j). \end{aligned}$$

We now define subspaces \mathfrak{L}_α of \mathfrak{L} by

$$(3.1) \quad \mathfrak{L}_\alpha = \sum_{s \leq t, s, t \in I_\alpha} \mathfrak{L}_{st} + \sum_{s \in I_\alpha} \mathfrak{L}_{sr} \quad (1 \leq \alpha \leq p).$$

Then, by using these relations and the conditions (2.2), (2.3), we can see that \mathfrak{L}_α is an ideal of the clan \mathfrak{L} for every α ($1 \leq \alpha \leq p$). Since $\mathfrak{L}_{\varepsilon(i)\varepsilon(j)} = (0)$ for $i \in I_\alpha$ and $j \in I_\beta$ ($\alpha \neq \beta$), we have $\mathfrak{L} = \mathfrak{L}_1 + \dots + \mathfrak{L}_p$ (direct sum).

Clearly the condition $I(\mathfrak{I}_\alpha) = I_\alpha$ holds for every α ($1 \leq \alpha \leq p$). Moreover, the uniqueness of \mathfrak{I}_α follows from Lemma 1. q.e.d

Note that, for the ideal \mathfrak{I}_α in the above lemma, the decomposition (3.1) is a normal decomposition of the clan \mathfrak{I}_α if we rename the indices in $I_\alpha = \{p_1, \dots, p_{s-1}\}$ ($p_1 < \dots < p_{s-1}$) by $p_k \rightarrow k$ ($1 \leq k \leq s - 1$), $r \rightarrow s$. It follows from Lemmas 3 and 4 that a clan \mathfrak{I} is irreducible if and only if the index set I of \mathfrak{I} is irreducible (cf. [1], [5], [6]).

4. Irreducible decomposition. By making use of the lemmas obtained in the preceding section, we can prove the following:

THEOREM 1. *Every homogeneous convex domain is affinely decomposed into the direct product of irreducible homogeneous convex domains. This decomposition is unique up to order.*

As was noted in §2, it suffices for the proof of the above theorem to prove the following:

THEOREM 2. *Every clan is decomposed, uniquely up to order, into the direct sum of irreducible ideals.*

PROOF. Let \mathfrak{I} be a reducible clan. Then, by Lemma 2, the index set I of \mathfrak{I} is uniquely represented as $I = I_1 \cup \dots \cup I_p$, the disjoint union of irreducible and admissible subsets I_α ($1 \leq \alpha \leq p$). For the index subsets I_α , we have the ideals \mathfrak{I}_α of \mathfrak{I} defined by (3.1) and they satisfy $\mathfrak{I} = \mathfrak{I}_1 + \dots + \mathfrak{I}_p$ (direct sum) by Lemma 4. Moreover the ideal \mathfrak{I}_α is irreducible, since the index set I_α is irreducible. Now, let $\mathfrak{I} = \mathfrak{M}_1 + \dots + \mathfrak{M}_q$ be another decomposition of \mathfrak{I} by irreducible ideals \mathfrak{M}_β ($1 \leq \beta \leq q$). Then the sets $I(\mathfrak{M}_\beta)$ are irreducible and admissible, and they satisfy the condition $I = I(\mathfrak{M}_1) \cup \dots \cup I(\mathfrak{M}_q)$ (disjoint union) by Lemma 3. Thus, by Lemma 2, the conditions $p = q$ and $\{I_1, \dots, I_p\} = \{I(\mathfrak{M}_1), \dots, I(\mathfrak{M}_p)\}$ hold. Hence, by Lemma 4, the decomposition is unique up to order. q.e.d

5. Some results on reducible domains. Throughout this section, we study reducible homogeneous convex domains exclusively. We first fix notations. For a homogeneous convex domain Ω , we denote by $G(\Omega)$ the group of all affine automorphisms of Ω . It is known that Ω admits a $G(\Omega)$ -invariant Riemannian metric which is called the canonical metric (cf. e.g., [7], [4]). We denote by $I(\Omega)$ the group of all isometries of Ω with respect to the canonical metric. For a topological group G , we denote by G° the connected component of G containing the identity element.

We have the following by [4, Proposition 1.1] and [6, Theorem 3.1].

THEOREM 3. *Let Ω be a homogeneous convex domain. If the irreducible decomposition of Ω contains no homogeneous convex cones as its components, then it coincides with the de Rham decomposition of Ω with respect to the canonical metric.*

The irreducible homogeneous convex domain $\{(x_1, x_2, \dots, x_n) \in \mathbf{R}^n; x_1 > x_2^2 + \dots + x_n^2\}$ ($n \geq 2$) is called an *elementary domain*.

THEOREM 4. *Let Ω be a homogeneous convex domain having neither a homogeneous convex cone nor an elementary domain as its irreducible components. Then $I(\Omega)^\circ$ coincides with $G(\Omega)^\circ$.*

PROOF. Let $\Omega = \Omega_1 \times \dots \times \Omega_p$ be the irreducible decomposition of Ω . Then, by Theorem 3 and [2, Theorem 3.5, p. 240], we have $I(\Omega)^\circ = I(\Omega_1)^\circ \times \dots \times I(\Omega_p)^\circ$. By [5, Theorem 6.1], we see that the equalities $I(\Omega_i)^\circ = G(\Omega_i)^\circ$ ($1 \leq i \leq p$) hold. This implies $I(\Omega)^\circ = G(\Omega)^\circ$, since $G(\Omega)^\circ = G(\Omega_1)^\circ \times \dots \times G(\Omega_p)^\circ$. q.e.d.

In the rest of this section, we show that the theorem and propositions in [3] remain valid for a homogeneous convex domain having no homogeneous convex cones as its irreducible components. In the following, we use the same notation as in [3].

PROPOSITION 1. *Let Ω be a homogeneous convex domain and V the cone fitted onto Ω . If the irreducible decomposition of Ω contains no homogeneous convex cones as its components, then V is irreducible.*

PROOF. Let $\Omega = \Omega_1 \times \dots \times \Omega_p$ be the irreducible decomposition of Ω . For simplicity, we assume that $p = 2$. The proof in the case $p \geq 3$ is similar to that in the case $p = 2$. For $s = 1, 2$, let $\mathfrak{X}(\Omega_s) = \sum_{1 \leq i \leq j \leq r_s - 1} \mathfrak{X}_{ij}^{(s)} + \sum_{1 \leq i \leq r_s - 1} \mathfrak{X}_{ir_s}^{(s)}$ be a normal decomposition of the clan $\mathfrak{X}(\Omega_s)$ corresponding to the homogeneous convex domain Ω_s . In view of the proof of Lemma 4 in §3 and the remark after it, we can see that a normal decomposition of the clan $\mathfrak{X}(\Omega) = \mathfrak{X}(\Omega_1) + \mathfrak{X}(\Omega_2)$ (direct sum) is given by $\mathfrak{X}(\Omega) = \sum_{1 \leq i \leq j \leq r-1} \mathfrak{X}_{ij} + \sum_{1 \leq i \leq r-1} \mathfrak{X}_{ir}$ ($r = r_1 + r_2 - 1$), where

$$\mathfrak{X}_{ij} = \begin{cases} \mathfrak{X}_{ij}^{(1)} & (1 \leq i \leq j \leq r_1 - 1) \\ (0) & (1 \leq i \leq r_1 - 1, r_1 \leq j \leq r - 1) \\ \mathfrak{X}_{ir_1}^{(1)} & (1 \leq i \leq r_1 - 1, j = r) \\ \mathfrak{X}_{kl}^{(2)} & (r_1 \leq i \leq j \leq r - 1, k = i - r_1 + 1, l = j - r_1 + 1) \\ \mathfrak{X}_{kr_2}^{(2)} & (r_1 \leq i \leq r - 1, j = r, k = i - r_1 + 1) . \end{cases}$$

As was stated in §2, a normal decomposition of the clan $\mathfrak{X}(V)$ is given by $\mathfrak{X}(V) = \sum_{1 \leq i \leq j \leq r} \mathfrak{X}_{ij} + \sum_{1 \leq i \leq r} \mathfrak{X}_{i,r+1}$, where $\mathfrak{X}_{i,r+1} = (0)$ ($1 \leq i \leq r$). We now

put $n_{ij} = n_{ji} = \dim \mathfrak{R}_{ij}$. By assumption, Ω_1 and Ω_2 are not affinely equivalent to a homogeneous convex cone. Hence, there exist indices k and l such that $1 \leq k \leq r_1 - 1$, $r_1 \leq l \leq r - 1$, $n_{kr} \neq 0$ and $n_{lr} \neq 0$. Since Ω_1 and Ω_2 are irreducible, for every pair (i, j) of indices with $1 \leq i \leq j \leq r$, there exists a sequence $\{i_1, i_2, \dots, i_q\}$ of indices satisfying the conditions $1 \leq i_k \leq r$ ($1 \leq k \leq q$), $i_1 = i$, $i_q = j$, and $n_{i_{k-1}i_k} \neq 0$ ($2 \leq k \leq q$). Therefore, the index set of $\mathfrak{R}(V)$ is irreducible, and hence, V is irreducible (see also [1]). q.e.d.

By using Proposition 1 stated above and the same argument as in the proof of [3, Proposition 2], we have the following:

PROPOSITION 2. *Let Ω be a homogeneous convex domain in \mathbf{R}^n having no homogeneous convex cones as its irreducible components. If a subgroup G of $G(\Omega)$ acts on Ω transitively, then the centralizer of G in the group of all affine transformations of \mathbf{R}^n is trivial. In particular, the center of G is trivial.*

We next prove the following:

LEMMA 5. *Let Ω be a homogeneous convex domain having no homogeneous convex cones as its irreducible components. If a connected Lie subgroup G of $I(\Omega)$ acts on Ω transitively, then the center of G is trivial.*

PROOF. Let $\Omega = \Omega_1 \times \dots \times \Omega_p$ be the irreducible decomposition of Ω . Then, by Theorem 3, we have $I(\Omega)^\circ = I(\Omega_1)^\circ \times \dots \times I(\Omega_p)^\circ$. Since G is connected, we see that $G \subset I(\Omega)^\circ$. Let π_i be the natural projection of $I(\Omega)^\circ$ onto $I(\Omega_i)^\circ$ and put $\pi_i(G) = G_i$. Then the center of G is a subgroup of $C(G_1) \times \dots \times C(G_p)$, where $C(G_i)$ is the center of G_i . Hence, we have only to show that $C(G_i)$ is trivial. Since G acts on Ω transitively, we see that G_i is a Lie subgroup of $I(\Omega_i)$ acting on Ω_i transitively. Therefore, our assertion follows from [3, Corollary 2]. q.e.d.

In view of the proof of [3, Theorem], we have the following by Lemma 5.

THEOREM 5. *Let M be a homogeneous Riemannian manifold whose universal covering is isometric to a homogeneous convex domain Ω endowed with the canonical metric. If the irreducible decomposition of Ω contains no homogeneous convex cones as its components, then M is simply connected, that is, M itself is isometric to Ω .*

REMARK. In Lemma 5, if the irreducible decomposition of Ω contains neither a homogeneous convex cone nor an elementary domain as its components, then the assertion of the lemma follows immediately from Proposition 2 and Theorem 4.

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