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INFINITESIMAL TORELLI THEOREM FOR COMPLETE INTERSECTIONS IN CERTAIN HOMOGENEOUS KÄHLER MANIFOLDS

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Introduction. After Griffiths [7] formulated a problem which is nowadays called *the infinitesimal Torelli problem*, several authors tried to solve it and obtained an affirmative answer in many cases. One of the important works is due to Kiĭ [10] which we state here only in the form suitable for our purpose. (See [8, (4.6.1), p. 159] and [10, Theorem 1, p. 54] for more general statement.)

(0.1) KIĬ'S THEOREM. *Let X be an n-dimensional compact Kähler manifold. Assume that the canonical bundle K_X of X is written as $K_X = L^{\otimes m}$, where L is a line bundle on X and m is a positive integer. If the base locus of $|L|$ has codimension $\geqq 2$ and if* $\dim H^0(\Omega_X^{n-1} \otimes L) \leqq \dim H^0(L) - 2$, *then the derivative*

$$P_*: H^1(T_X) \to \mathrm{Hom}(H^0(\Omega_X^n), H^1(\Omega_X^{n-1})) ,$$

of the period map is injective so that the infinitesimal Torelli theorem holds.

An easy but important consequence of (0.1) is the infinitesimal Torelli theorem for any non-singular complete intersection with ample canonical bundle in a projective space, a result originally obtained by Usui [16] and Peters [15] independently.

In this article we try to solve the problem for non-singular complete intersections in a Kähler C-space (i.e., compact simply connected homogeneous Kähler manifold) whose second Betti number is one. Though our result is far from being complete, it covers an important subclass of Kähler C-spaces, namely, the class of irreducible Hermitian symmetric spaces of compact type which contains, for instance, all Grassmannians. Therefore, ours may be regarded as an extension of the case of projective spaces.

Now we state our result. For more precise statements, see (3.8), (3.10), (3.11) and (4.5).

(0.2) THEOREM. *Let $Y = G/U$ be a Kähler C-space with $b_2(Y) = 1$,* *where G is a simply connected complex simple Lie group and U is a parabolic subgroup of G. Let X be a non-singular complete intersection in Y with the ample canonical bundle. If either* $\dim X$ *is sufficiently big* (*cf.* (3.8) *for more precise statement*) *or one of the following conditions is satisfied, then the infinitesimal Torelli theorem holds for X*:

(0.2.1) *Y is an irreducible Hermitian symmetric space of compact type.*

(0.2.2) $\mathrm{Lie}\, G = C_l, E_6, E_4$ *or* G_2.

(0.2.3) $\dim X = 2$ *and Y is not of type* (E_8, α_4)
(*cf.* §1 *for the notation* (E_8, α_4)).

The plan of this article is as follows: In §1, we recall known results on Kähler C-spaces with $b_2 = 1$ and give Table 1. In §2, we reduce the problem to the estimate of $h^0(\Omega_Y^p(1))$ by means of (0.1) (cf. Proposition (2.4)), where $\mathcal{O}_Y(1)$ is the ample generator of $\mathrm{Pic}(Y)$. Though we have the generalized Borel-Weil Theorem ([6] and [14]) for the cohomology groups of homogeneous vector bundles, it works only for ones induced by an irreducible representation of U. Since Ω_Y^p is not induced even by a completely reducible U-module in general, we cannot apply this theorem directly. In §3, we define a filtration on Ω_Y^p whose successive quotients are induced by the completely reducible U-modules $G^i(\Lambda^p\mathfrak{n}^+)$. Then we can compute $h^0(\Omega_Y^p(1))$ by using the induced spectral sequence. The difficulty is in determining the irreducible decomposition of $G^i(\Lambda^p\mathfrak{n}^+)$. For $p = 1$, we list in Table 3 the lowest weights which are determined by (3.9). On the other hand, we give two criteria, Lemmas (3.5) and (3.6), on the vanishing of $h^0(\Omega_Y^p(1))$. These, together with Tables 1 and 3, imply most of (0.2). In §4, we restrict ourselves to special cases and give a rough estimate for $h^0(\Omega_Y^p(1))$ by a rather concrete calculation.

Since the restriction on $\dim X$ in (0.2) is caused merely by our technical weakness and should be removed, we hope that our result can be extended to all Kähler C-spaces. Moreover, since (0.2) covers the case of hypersurfaces, it would be interesting to investigate the generic Torelli problem for them, which we discuss in a forthcoming paper [13].

1. Kähler C-spaces and the generalized Borel-Weil theorem. A simply connected compact homogeneous Kähler manifold is called a *Kähler C-space*. Let Y be a Kähler C-space with the second Betti number $b_2(Y) = 1$. We first recall how Y can be constructed.

Let $\mathfrak{g}$ be a complex simple Lie algebra. If $\mathfrak{h}$ is a Cartan subalgebra of $\mathfrak{g}$ and

$$\Delta = (\alpha_1, \cdots, \alpha_l)\ , \quad l = \mathrm{rank}\, \mathfrak{g}\ ,$$

is a base of the root system Φ of $\mathfrak{g}$ with respect to $\mathfrak{h}$, we denote by Φ^+ (resp. Φ^-) the subset of all positive (resp. negative) roots. Then we have a Cartan decomposition

$$\mathfrak{g} = \mathfrak{h} + \sum_{\alpha \in \Phi^-} \mathfrak{g}_\alpha + \sum_{\alpha \in \Phi^+} \mathfrak{g}_\alpha .$$

Choose a simple root α_r, $1 \leqq r \leqq l$, and put

$$\Phi(\alpha_r) = \left\{\alpha \in \Phi\colon \alpha = \sum_{i=1}^{l} n_i\alpha_i,\ n_r = 0\right\} ,$$

$$\Phi(\mathfrak{n}^+) = \left\{\alpha \in \Phi^+\colon \alpha = \sum_{i=1}^{l} n_i\alpha_i,\ n_r > 0\right\} ,$$

$$\Phi(\mathfrak{u}) = \Phi(\alpha_r) \cup \Phi(\mathfrak{n}^+) .$$

Using these, we define Lie subalgebras of $\mathfrak{g}$ as follows:

$$\mathfrak{g}_1 = \mathfrak{h} + \sum_{\alpha \in \Phi(\alpha_r)} \mathfrak{g}_\alpha , \quad \mathfrak{n}^+ = \sum_{\alpha \in \Phi(\mathfrak{n}^+)} \mathfrak{g}_\alpha , \quad \mathfrak{u} = \mathfrak{h} + \sum_{\alpha \in \Phi(\mathfrak{u})} \mathfrak{g}_\alpha .$$

If we take a simply connected complex simple Lie group G and a connected Lie subgroup U of G in such a way that $\operatorname{Lie} G = \mathfrak{g}$ and $\operatorname{Lie} U = \mathfrak{u}$, then the factor space $Y = G/U$ is a Kähler C-space with $b_2(Y) = 1$. Conversely, every Kähler C-space with $b_2 = 1$ can be constructed in this way. For this reason, we denote the manifold thus constructed by $Y = (\mathfrak{g}, \alpha_r)$ in what follows.

Many properties are known about a Kähler C-space Y with $b_2 = 1$. We collect here some of them.

(1.1) FACT. (1.1.1) Y is a rational manifold admitting an "algebraic cell-decomposition". Thus $H^q(\Omega_Y^p) = 0$ if $p \neq q$. (See [2, Theorems 2 and 3] and [4, Satz I].)

(1.1.2) The Picard group $\operatorname{Pic}(Y)$ is isomorphic to $\boldsymbol{Z}$ and one of its generators is very ample. (See [5, n°3 and n°4].)

We denote the very ample generator of $\operatorname{Pic}(Y)$ by $\mathcal{O}_Y(1)$ and $\mathcal{O}_Y(1)^{\otimes a}$ by $\mathcal{O}_Y(a)$. Let $k = k(Y)$ be the positive integer defined by $K_Y = \mathcal{O}_Y(-k)$. Then k is given by the following formula. (See [3, p. 521].)

(1.2) $$k(Y) = 2 \textstyle\sum_{\alpha \in \Phi(\mathfrak{n}^+)} (\alpha, \alpha_r)/(\alpha_r, \alpha_r),$$

where (,) denotes the Euclidean scalar product induced by the Killing form on the real vector space spanned by Φ in $\mathfrak{h}^*$.

Now let $\lambda_1, \cdots, \lambda_l$ be the fundamental weights of $\mathfrak{g}$, i.e., $\langle \lambda_i, \alpha_j \rangle := 2(\lambda_i, \alpha_j)/(\alpha_j, \alpha_j) = \delta_{ij}$. We set

$$\delta = \sum_{i=1}^{l} \lambda_i = 1/2 \sum_{\alpha \in \Phi^+} \alpha .$$

(1.3) DEFINITION. A weight λ is called

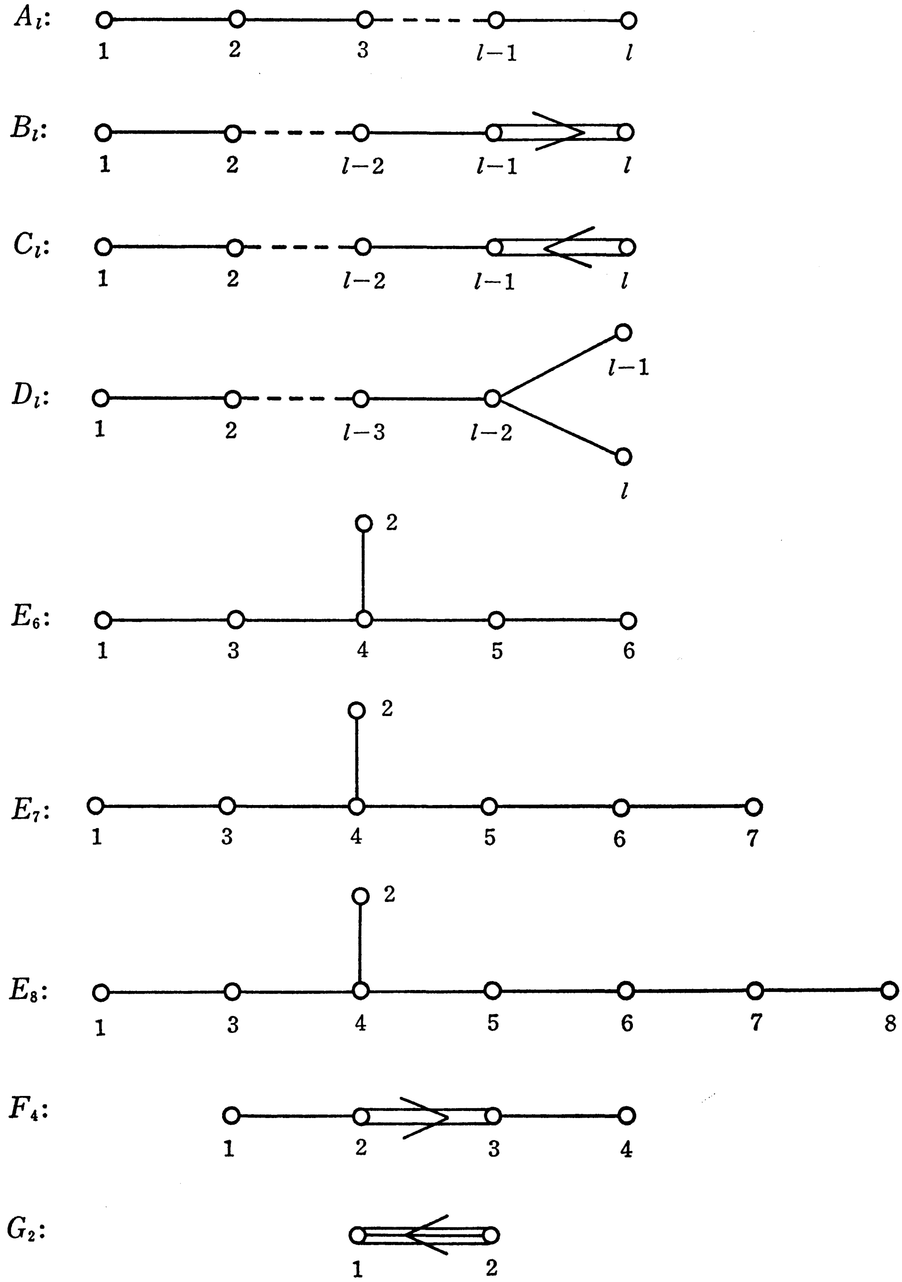
(1.3.1) singular if $(\lambda, \alpha) = 0$ for at least one $\alpha \in \Phi$.

(1.3.2) regular with index p if it is not singular and there exist exactly p roots $\alpha \in \Phi^+$ with $(\lambda, \alpha) < 0$.

TABLE 1.

$\mathfrak{g}$	r	$\dim Y$	k	$h^0(\mathcal{O}_Y(1))$	q_r
A_l	$1 \leqq r \leqq l$	$r(l+1-r)$	$l+1$	$\binom{l+1}{r}$	$r(1-r/(l+1))$
B_l	$1 \leqq r \leqq l-1$	$2r(l-r)+r(r+1)/2$	$2l-r$	$\binom{2l+1}{r}$	r
C_l	$2 \leqq r \leqq l$	$2r(l-r)+r(r+1)/2$	$2l+1-r$	$\binom{2l}{r}-\binom{2l}{r-2}$	r $(r<l)$ $l/2$ $(r=l)$
D_l	$1 \leqq r \leqq l-2$	$2r(l-r)+r(r-1)/2$	$2l-1-r$	$\binom{2l}{r}$	r
	$l-1$	$l(l-1)/2$	$2l-2$	2^{l-1}	$l/4$
E_6	1	16	12	27	4/3
	2	21	11	78	2
	3	25	9	351	10/3
	4	29	7	2925	6
E_7	1	33	17	133	2
	2	42	14	912	7/2
	3	47	11	8645	6
	4	53	8	365750	12
	5	50	10	27664	15/2
	6	42	13	1539	4
	7	27	18	56	3/2
E_8	1	78	23	3875	4
	2	92	17	147250	8
	3	98	13	6696000	14
	4	106	9	6899079264	30
	5	104	11	146325270	20
	6	97	14	2450240	12
	7	83	19	30380	6
	8	57	29	248	2
F_4	1	15	8	52	2
	2	20	5	1274	6
	3	20	7	273	6
	4	15	11	26	2
G_2	2	5	3	14	2

TABLE 2.



(1.4) GENERALIZED BOREL-WEIL THEOREM. ([6, p. 228] and [14, p. 371]) *Let* $E_{-\lambda}$ *be a homogeneous vector bundle on* $Y = G/U$ *induced by an ir-*

reducible representation of U with the lowest weight $-\lambda$.

(1.4.1) *If $\lambda+\delta$ is singular, then*

$$H^i(Y, E_{-\lambda})=0 \quad \text{for all} \quad i\,.$$

(1.4.2) *If $\lambda+\delta$ is regular with index p, then*

$$H^i(Y, E_{-\lambda})=0 \quad \text{for all} \quad i\neq p$$

and $H^p(Y, E_{-\lambda})$ is an irreducible G-module with the lowest weight $-\mu$. Here μ is the uniquely determined element of the closure of the fundamental Weyl chamber of Lie G *such that $\mu+\delta$ is congruent to $\lambda+\delta$ under the Weyl group.*

Since $\mathcal{O}_Y(1)$ is induced by the irreducible representation of U with the lowest weight $-\lambda_r$ (cf. [5], n°3 and n°4), we can compute $h^0(\mathcal{O}_Y(1))$ by using Weyl's formula (cf. [9], p. 139) by virtue of (1.4):

(1.5) $h^0(\mathcal{O}_Y(1))=\deg(\lambda_r)=\prod_{\alpha\in\Phi^+}(\lambda_r+\delta, \alpha)/\prod_{\alpha\in\Phi^+}(\delta, \alpha)$.

We close this section by giving in Table 1 the list of Kähler C-spaces with $b_2=1$. In Table 1, q_r is the coefficient of α_r in $\lambda_r=\sum_{i=1}^l q_i\alpha_i$, which we shall need later. We follow [9, p. 58] in numbering the simple roots in the Dynkin diagrm for $\mathfrak{g}$ as in Table 2.

(1.6) Remark. It is easy to see the following: $(B_l, \alpha_l)\cong(D_{l+1}, \alpha_l)$, $(C_l, \alpha_1)\cong(A_{2l-1}, \alpha_1)$, $(D_l, \alpha_l)\cong(D_l, \alpha_{l-1})$, $(E_6, \alpha_5)\cong(E_6, \alpha_3)$, $(E_6, \alpha_6)\cong(E_6, \alpha_1)$, $(G_2, \alpha_1)\cong(B_3, \alpha_1)$. Note also that (A_l, α_r), (B_l, α_1), (C_l, α_l), (D_l, α_1), (D_l, α_{l-1}), (E_6, α_1) and (E_7, α_7) are the irreducible Hermitian symmetric spaces of compact type. They fall into the following six classes, see, e.g., [3, p. 521]:

I. (A_l, α_r) (Grassmannians): $Y=SU(l+1)/S(U(r)\times U(l+1-r))$.

II. (B_l, α_1) and (D_l, α_1) (Quadrics Q^N): $Y=SO(N+2)/SO(2)\times SO(N)$, $N=2l-1,\ 2l-2$.

III. (C_l, α_l): $Y=Sp(l)/U(l)$.

IV. (D_l, α_{l-1}): $Y=SO(2l)/U(l)$.

V. (E_6, α_1): $Y=E_6/\mathrm{Spin}(10)\times T^1$.

VI. (E_7, α_7): $Y=E_7/E_6\times T^1$.

2. Reduction of the problem.

Let Y be a Kähler C-space with $b_2(Y)=1$ and $\dim Y=N\geqq 3$. For positive integers $d_1, \cdots d_{N-n}$, define

$$E=\mathcal{O}_Y(d_1)\oplus\cdots\oplus\mathcal{O}_Y(d_{N-n})\,, \qquad 1\leqq n\leqq N-1\,.$$

A global section x of E transversal to the zero-section determines an n-dimensional submanifold X of Y called a non-singular complete intersection of type $(d_1, \cdots, d_{N-n})$. We always assume that K_X is ample, i.e., $\sum_{i=1}^{N-n} d_i>k(Y)$ in the following.

If $\dim X = 1$, then the infinitesimal Torelli theorem is clearly true for X, since $|K_X|$ gives a projective embedding. So we may assume $\dim X > 1$. Put $s = \#\{d_i; d_i = 1\}$.

(2.1) LEMMA. (2.1.1) *If $s = 0$ or $n \geqq 4$, then*

$$H^0(\Omega_X^{n-1}(1)) \cong H^0(\Omega_Y^{n-1}(1) \otimes \mathcal{O}_X) .$$

(2.1.2) *If $n = 3$, then there is an exact sequence*

$$0 \to H^0(\Omega_Y^2(1) \otimes \mathcal{O}_X) \to H^0(\Omega_X^2(1)) \to H^1(\Omega_X^1)^{\oplus s} .$$

(2.1.3) *If $n = 2$, then there is an exact sequence*

$$0 \to H^0(\mathcal{O}_X)^{\oplus s} \to H^0(\Omega_Y^1(1) \otimes \mathcal{O}_X) \to H^0(\Omega_X^1(1)) \to 0 .$$

PROOF. First we note that $H^q(\Omega_X^p) \cong H^q(\Omega_Y^p)$ holds for $p + q < n$. The standard exact sequence

$$0 \to N_{X/Y}^* \to \Omega_Y^1 \otimes \mathcal{O}_X \to \Omega_X^1 \to 0$$

induces a filtration F for $\Omega_Y^{n-1} \otimes \mathcal{O}_X$,

$$\Omega_Y^{n-1} \otimes \mathcal{O}_X = F^0 \supset F^1 \supset \cdots \supset F^{n-1} \supset F^n = 0 ,$$

whose successive quotients are

$$\mathrm{Gr}_F^p(\Omega_Y^{n-1} \otimes \mathcal{O}_X) = F^p/F^{p+1} \cong (\Lambda^p N_{X/Y}^*) \otimes \Omega_X^{n-1-p} .$$

Tensoring $\mathcal{O}_X(1)$ with these, we get a spectral sequence

$$E_1^{p,q-p} = H^q((\Lambda^p N_{X/Y}^*) \otimes \Omega_X^{n-1-p}(1)) \Rightarrow H^q(\Omega_Y^{n-1}(1) \otimes \mathcal{O}_X) .$$

Note that

$$(\Lambda^p N_{X/Y}^*) \otimes \mathcal{O}_X(1) = \sum_{1 \leq i_1 < \cdots < i_p \leq N-n} \mathcal{O}_X\Big(-\sum_{j=1}^{p} d_{i_j} + 1\Big)$$

is a direct sum of negative line bundles for $p \geqq 1$ if $s = 0$ and for $p \geqq 2$ if $s > 0$. Now the assertion follows from the vanishing theorem of Kodaira-Nakano, the remark at the beginning of the proof, (1.1.1) and the above spectral sequence. q.e.d.

(2.2) LEMMA. *If $n \geqq 3$, then*

$$H^0(\Omega_Y^{n-1}(1) \otimes \mathcal{O}_X) \cong H^0(\Omega_Y^{n-1}(1)) .$$

If $n = 2$, then there is an exact sequence

$$0 \to H^0(\Omega_Y^1(1)) \to H^0(\Omega_Y^1(1) \otimes \mathcal{O}_X) \to H^1(\Omega_Y^1)^{\oplus s} \to 0 .$$

PROOF. The section $x \in H^0(Y, E)$ determines the Koszul resolution of $\mathcal{O}_X$ which in turn defines a spectral sequence

$$E_1^{-p,q} = H^q(Y, (\Lambda^p E^*) \otimes \Omega_Y^{n-1}(1)) \Rightarrow H^{q-p}(\Omega_Y^{n-1}(1) \otimes \mathcal{O}_X) .$$

We get the assertion for $n \geqq 3$ by the same reasoning as in the proof of (2.1). If $n = 2$, then we get the following exact sequence

$$0 \to H^0(\Omega_Y^1(1)) \to H^0(\Omega_Y^1(1) \otimes \mathcal{O}_X) \to \operatorname{Ker}\{H^1(\Omega_Y^1)^{\oplus s} \xrightarrow{d_1} H^1(\Omega_Y^1(1))\} \to 0 .$$

But the vanishing of $H^1(\Omega_Y^1(1)) = H^{N-1}(T_Y(-k-1))^*$ is shown in [1, (5), p. 66]. q.e.d.

By the same argument as in the proof of (2.2), we get:

(2.3) LEMMA. $h^0(\mathcal{O}_X(1)) = h^0(\mathcal{O}_Y(1)) - s$.

Summing up, we have by (0.1):

(2.4) PROPOSITION. *Let X be a non-singular complete intersection of type $(d_1, \cdots, d_{N-n})$ in a Kähler C-space Y with $b_2(Y) = 1$. If K_X is ample and* $\dim X = n \geqq 2$, *then the infinitesimal Torelli theorem holds for X in the following cases:*

(2.4.1) $h^0(\Omega_Y^{n-1}(1)) \leqq h^0(\mathcal{O}_Y(1)) - s - 2$ *if* $n \neq 3$.

(2.4.2) $h^0(\Omega_Y^2(1)) \leqq h^0(\mathcal{O}_Y(1)) - 2s - 2$ *if* $n = 3$.

Here s stands for the cardinality of $\{d_i : d_i = 1\}$.

(2.5) REMARK. It can be shown that the Kuranishi space of deformations of X is smooth since the natural family of displacements of X in Y gives a complete family (see [1, p. 65]).

3. Proof of the theorem (general case). We keep the notation in §1. It is known that $\mathfrak{n}^+$ is invariant with respect to the adjoint representation of U on $\mathfrak{g}$. The homogeneous vector bundle on Y induced by this U-module $\mathfrak{n}^+$ is Ω_Y^1. Thus in applying the generalized Borel-Weil theorem (1.4) to compute $h^0(\Omega_Y^p(1))$, it would be convenient if $\Lambda^p\mathfrak{n}^+$ is a completely reducible U-module. If Y is an irreducible Hermitian symmetric space, then this is the case and its irreducible decomposition is given by Kostant [14, p. 379]. But it is not so in general.

Let $F^i(\Lambda^p\mathfrak{n}^+)$ be the linear subspace of $\Lambda^p\mathfrak{n}^+$ spanned by vectors whose weights are of the form:

$$\lambda = \sum_{j=1}^{l} n_j\alpha_j , \quad n_r \geqq i .$$

It is obvious that $F^i(\Lambda^p\mathfrak{n}^+)$ is also invariant with respect to $\mathrm{Ad}(U)$ and we have a descending filtration of $\Lambda^p\mathfrak{n}^+$:

$$0 \subset \cdots \subset F^{i+1}(\Lambda^p\mathfrak{n}^+) \subset F^i(\Lambda^p\mathfrak{n}^+) \subset \cdots \subset F^p(\Lambda^p\mathfrak{n}^+) = \Lambda^p\mathfrak{n}^+ .$$

We set $G^i(\Lambda^p\mathfrak{n}^+) := F^i(\Lambda^p\mathfrak{n}^+)/F^{i+1}(\Lambda^p\mathfrak{n}^+)$. Then we have:

(3.1) LEMMA. *$G^i(\Lambda^p\mathfrak{n}^+)$ is a completely reducible U-module and $\Lambda^p\mathfrak{n}^+ \cong \bigoplus_i G^i(\Lambda^p\mathfrak{n}^+)$ as $\mathfrak{g}_1$-modules.*

PROOF. Since $\mathfrak{u} = \mathfrak{g}_1 + \mathfrak{n}^+$, we only have to check that the adjoint representation is trivial on $\mathfrak{n}^+$, which is easy. q.e.d.

We denote the homogeneous vector bundle corresponding to $G^i(\Lambda^p\mathfrak{n}^+)$ by $G^i\Omega_Y^p$.

(3.2) PROPOSITION. *There is a spectral sequence:*

$$E_1^{i,q-i} = H^q(Y, G^i\Omega_Y^p(m)) \Rightarrow H^q(Y, \Omega_Y^p(m)) .$$

(3.3) COROLLARY. $h^0(\Omega_Y^p(1)) \leqq \sum_i h^0(G^i\Omega_Y^p(1))$.

(3.4) REMARK. The lowest weight $-\lambda$ of any irreducible component of $G^i(\Lambda^p\mathfrak{n}^+)$ satisfies the following properties.

(3.4.1) $-\lambda$ is a sum of p distinct roots of $\mathfrak{n}^+$.

(3.4.2) $n_r = i$ in the expression

$$-\lambda = \sum_{j=1}^{l} n_j\alpha_j , \quad n_j \in N \cup \{0\} .$$

(3.4.3) $(-\lambda, \alpha_j) \leqq 0$ for $j \neq r$.

We give here two lemmas on the vanishing of $H^0(\Omega_Y^p(1))$.

(3.5) LEMMA. *If $H^0(\Omega_Y^p(1)) = 0$, then $H^0(\Omega_Y^{p+1}(1)) = 0$.*

PROOF. For any point $y \in Y$, we can choose exactly N elements $v_1, \cdots, v_N \in H^0(T_Y)$, depending on y, which span the tangent space to Y at y. Consider the pairing

$$\langle\ ,\ \rangle : H^0(T_Y) \times H^0(\Omega_Y^{p+1}(1)) \to H^0(\Omega_Y^p(1)) .$$

If there exists an $\omega \in H^0(\Omega_Y^{p+1}(1))$ which does not vanish at y, then we can find complex numbers $c_1, \cdots, c_N$ so that we have $\langle v, \omega\rangle \neq 0$ in $H^0(\Omega_Y^p(1))$ for $v = \sum_{i=1}^N c_iv_i$, a contradiction. q.e.d.

(3.6) LEMMA. *$H^0(\Omega_Y^p(1)) = 0$ for $p > q_r$, where q_r is the coefficient of α_r in $\lambda_r = \sum_{i=1}^l q_i\alpha_i$.*

PROOF. By (3.3), it suffices to show $H^0(G^i\Omega_Y^p(1)) = 0$ for all i. Let $-\lambda$ be the lowest weight of any irreducible component of $G^i(\Lambda^p\mathfrak{n}^+)$. Writing $-\lambda$ as (3.4.2), we have $n_r > q_r$. We shall prove $m_r \geqq 2$ in the expression

$$-\lambda = \sum_{j=1}^{l} m_j\lambda_j , \quad m_j \in Z , \quad 1 \leqq j \leqq l .$$

Note that we must have $m_j \leqq 0$ for $j \neq r$ and $m_r \geqq 1$ by (3.4.1) and

TABLE 3.

$\mathfrak{g}$	r	lowest weights of $G^i(\mathfrak{n}^+)$	$h^0(\Omega^1_Y(1))$
A_l	$1\leqq r\leqq l$	α_r	0
B_l	1	α_1	0
	$2\leqq r\leqq l-1$	$\alpha_r,\ \lambda_r-\lambda_{r-2}$	$\binom{2l+1}{r-2}$
C_l	$2\leqq r\leqq l-1$	$\alpha_r,\ 2\lambda_r-2\lambda_{r-1}$	0
	l	α_l	0
D_l	1	α_1	0
	$2\leqq r\leqq l-2$	$\alpha_r,\ \lambda_r-\lambda_{r-2}$	$\binom{2l}{r-2}$
	$l-1$	α_{l-1}	0
E_6	1	α_1	0
	2	$\alpha_2,\ \lambda_2$	1
	3	$\alpha_3,\ \lambda_3-\lambda_6$	27
	4	$\alpha_4,\ \lambda_4-\lambda_1-\lambda_6,\ \lambda_4-\lambda_2$	728
E_7	1	$\alpha_1,\ \lambda_1$	1
	2	$\alpha_2,\ \lambda_2-\lambda_7$	56
	3	$\alpha_3,\ \lambda_3-\lambda_6,\ \lambda_3-\lambda_1$	1672
	4	$\alpha_4,\ \lambda_4-\lambda_1-\lambda_6,\ \lambda_4-\lambda_2-\lambda_7,\ \lambda_4-\lambda_3$	201552
	5	$\alpha_5,\ \lambda_5-\lambda_1-\lambda_7,\ \lambda_5-\lambda_2$	7392
	6	$\alpha_6,\ \lambda_6-\lambda_1$	133
	7	α_7	0
E_8	1	$\alpha_1,\ \lambda_1-\lambda_8$	248
	2	$\alpha_2,\ \lambda_2-\lambda_7,\ \lambda_2-\lambda_1$	34255
	3	$\alpha_3,\ \lambda_3-\lambda_6,\ \lambda_3-\lambda_1-\lambda_8,\ \lambda_3-\lambda_2$	3376737
	4	$\begin{cases}\alpha_4,\ \lambda_4-\lambda_1-\lambda_6,\ \lambda_4-\lambda_2-\lambda_7\\ \lambda_4-\lambda_3-\lambda_8,\ \lambda_4-\lambda_1-\lambda_2,\ \lambda_4-\lambda_5\end{cases}$	8644540371
	5	$\begin{cases}\alpha_5,\ \lambda_5-\lambda_1-\lambda_7,\ \lambda_5-\lambda_2-\lambda_8\\ \lambda_5-\lambda_3,\ \lambda_5-\lambda_6\end{cases}$	88058973
	6	$\alpha_6,\ \lambda_6-\lambda_1-\lambda_8,\ \lambda_6-\lambda_2,\ \lambda_6-\lambda_7$	956877
	7	$\alpha_7,\ \lambda_7-\lambda_1,\ \lambda_7-\lambda_8$	4123
	8	$\alpha_8,\ \lambda_8$	1
F_4	1	$\alpha_1,\ \lambda_1$	1
	2	$\alpha_2,\ \lambda_2-2\lambda_4,\ \lambda_2-\lambda_1$	124
	3	$\alpha_3,\ 2\lambda_3-\lambda_1-2\lambda_4,\ \lambda_3-\lambda_4,\ 2\lambda_3-\lambda_2$	26
	4	$\alpha_4,\ 2\lambda_4-\lambda_1$	0
G_2	2	$\alpha_2,\ \lambda_2$	1

(3.4.3). If $m_r=1$, then we would have

(3.6.1) $\quad -\lambda-\lambda_r=\sum_{j\neq r} m_j\lambda_j.$

Comparing the coefficient of α_r on both sides of (3.6.1), we find that it is $n_r - q_r > 0$ on the left hand side and non-positive on the right hand side, a contradiction. Now the inequality

$$\langle \lambda + \lambda_r + \delta, \alpha_r \rangle = -m_r + 2 \leqq 0$$

shows that $\lambda + \lambda_r + \delta$ is either singular, or regular with index $\geqq 1$. Thus we have $H^0(G^i\Omega_Y^p(1)) = 0$ by (1.4). q.e.d.

(3.7) REMARK. The proof of (3.6) tells us that the obstruction for the vanishing of $H^0(\Omega_Y^p(1))$ is a component of $G^i(\Lambda^p\mathfrak{n}^+)$ with the lowest weight of the form

(3.7.1) $-\lambda = \lambda_r - \sum_{j \neq r} m_j\lambda_j,\ m_j \geqq 0.$

By (2.4), (3.6) and Table 1, we get the following:

(3.8) THEOREM. *Let $Y = (\mathfrak{g}, \alpha_r)$ be a Kähler C-space with $b_2(Y) = 1$ and X a non-singular complete intersection in Y with the ample canonical bundle. If* $\dim X \geqq [q_r] + 2$, *then the infinitesimal Torelli theorem holds for X, where $[q_r]$ is the greatest integer not exceeding q_r.*

It is relatively easy to determine the lowest weights of $G^i(\mathfrak{n}^+)$. We list them in Table 3. (See [11, p. 113] and [14, p. 379] for the case of Hermitian symmetric spaces.)

(3.9) REMARK. In practice, we determine the lowest weights of $G^i(\Lambda^p\mathfrak{n}^+)$ as follows:

(3.9.1) Determine all the weights $-\lambda$ satisfying (3.4.1), (3.4.2) and (3.4.3).

(3.9.2) Compute the dimension of the irreducible representation of U with the lowest weight $-\lambda$.

(3.9.3) Compare $\dim G^i(\Lambda^p\mathfrak{n}^+)$ and the dimensions obtained in the step (3.9.2).

Concerning Table 3, we note the following. It can be shown, according to (3.9), that each $G^i(\mathfrak{n}^+)$ is irreducible and the lowest weight of $G^1(\mathfrak{n}^+)$ is α_r. Since we have

$$(-\alpha_r + \lambda_r + \delta, \alpha_r) = 2(\lambda_r, \alpha_r) - (\alpha_r, \alpha_r) = 0$$

by the definition of λ_r, the weight $-\alpha_r + \lambda_r + \delta$ is singular and we get $H^0(G^1\Omega_Y^1(1)) = 0$ by (1.4). Let $-\lambda$ stand for one of the weights in Table 3 which is not α_r. Then it is easy to see that the weight $\lambda + \lambda_r + \delta$ is regular with index 0 (resp. singular) if the coefficient of λ_r in $-\lambda$ equals 1 (resp. 2). As a consequence, we get from (1.4) and (3.2)

$$h^0(\Omega_Y^1(1)) = \sum_{i\geq 2} h^0(G^i\Omega_Y^1(1)) = \sum_{-\lambda} \deg(\lambda + \lambda_r)\ ,$$

where $-\lambda$ runs through the weights in Table 3 which have the form (3.7.1). The right hand side can be calculated by Weyl's formula.

For example, consider the Kähler C-space $Y = (E_6, \alpha_4)$. The concrete description of the root system of type E_6 (cf. [9, p. 65]) shows that the weights of $G^i(\mathfrak{n}^+)$ are as follows:

$G^1(\mathfrak{n}^+)$: (0, 0, 0, 1, 0, 0), (0, 1, 0, 1, 0, 0), (0, 0, 1, 1, 0, 0), (0, 0, 0, 1, 1, 0), (1, 0, 1, 1, 0, 0), (0, 1, 0, 1, 1, 0), (0, 0, 1, 1, 1, 0), (0, 1, 1, 1, 0, 0), (0, 0, 0, 1, 1, 1), (1, 0, 1, 1, 1, 0), (0, 1, 0, 1, 1, 1), (0, 0, 1, 1, 1, 1), (0, 1, 1, 1, 1, 0), (1, 1, 1, 1, 0, 0), (1, 0, 1, 1, 1, 1), (0, 1, 1, 1, 1, 1), (1, 1, 1, 1, 1, 0), (1, 1, 1, 1, 1, 1).

$G^2(\mathfrak{n}^+)$: (0, 1, 1, 2, 1, 0), (0, 1, 1, 2, 1, 1), (1, 1, 1, 2, 1, 0), (1, 1, 1, 2, 1, 1), (1, 1, 2, 2, 1, 0), (0, 1, 1, 2, 2, 1), (1, 1, 1, 2, 2, 1), (1, 1, 2, 2, 1, 1), (1, 1, 2, 2, 2, 1).

$G^3(\mathfrak{n}^+)$: (1, 1, 2, 3, 2, 1), (1, 2, 2, 3, 2, 1).

Here, we abbreviate $\sum_{i=1}^{6} n_i\alpha_i$ as $(n_1, \cdots, n_6)$. By a direct calculation, one can show that the weights satisfying (3.4.3) are

$G^1(\mathfrak{n}^+)$: $(0, 0, 0, 1, 0, 0) = \alpha_4$ (18),
$G^2(\mathfrak{n}^+)$: $(0, 1, 1, 2, 1, 0) = \lambda_4 - \lambda_1 - \lambda_6$ (9),
$G^3(\mathfrak{n}^+)$: $(1, 1, 2, 3, 2, 1) = \lambda_4 - \lambda_2$ (2).

The number in a parenthesis following each weight is the dimension of the corresponding irreducible representation as in (3.9.2). Since we have $\deg(\lambda_1 + \lambda_6) = 650$ and $\deg(\lambda_2) = 78$, we conclude $h^0(\Omega_Y^1(1)) = 728$.

By (2.4), Table 1 and Table 3, we get:

(3.10) THEOREM. *Let X be a non-singular complete intersection surface with the ample canonical bundle in a Kähler C-space Y with $b_2(Y) = 1$. Then the infinitesimal Torelli theorem holds for X except possibly when $Y = (E_8, \alpha_4)$.*

(3.11) THEOREM. *Suppose Y is (C_l, α_r), (F_4, α_4) or an irreducible Hermitian symmetric space of compact type. Then the infinitesimal Torelli theorem holds for any non-singular complete intersection X in Y if K_X is ample.*

PROOF. Note that Table 3 and (3.5) show $H^0(\Omega_Y^p(1)) = 0$ for $p \geqq 1$. We also remark that if $Y = \boldsymbol{P}^N$, Q^N or (E_6, α_1), we may assume $s = 0$ in (2.4), since the hypersurface of degree 1 is $\boldsymbol{P}^{N-1}$, Q^{N-1} and (F_4, α_4), respectively ([12, p. 437]). Therefore we get the desired consequence by (2.4) and Table 1. q.e.d.

4. **Proof of the theorem (special case).** Since it seems to be difficult to write down the irreducible decomposition of $G^i(\Lambda^p\mathfrak{n}^+)$ for $p \geqq 2$, let us

restrict ourselves to the case where $\mathfrak{g}$ is one of E_6, F_4 and G_2.

Let $-\lambda$ stand for the weight satisfying the conditions in (3.4) as well as (3.7.1). We know from (3.7) that the existence of such $-\lambda$ is the obstruction for $H^0(\Omega_Y^p(1))$ to be zero. Therefore our task is reduced to determining whether the component corresponding to $-\lambda$ can really occur in the decomposition of $G^i(\Lambda^p\mathfrak{n}^+)$ and in calculating the number of components in question.

(4.1) For such $-\lambda$, we donote by $m_p(-\lambda)$ the dimension of the weight space for $-\lambda$ in $\Lambda^p\mathfrak{n}^+$. This roughly bounds the number of components and may be computed as the number of the ways in which one can express $-\lambda$ as a sum of p distinct roots of $\mathfrak{n}^+$.

We shall estimate $h^0(\Omega_Y^p(1))$ case by case. In doing so, the following refinement of (3.6) will be quite helpful: Suppose that we have determined all $-\lambda$'s. The proof of (3.6) asserts that a component of $G^i\Omega_Y^p(1))$ has global sections if and only if it corresponds to one of these weights. Thus, in particular, we would have $h^0(G^i\Omega_Y^p(1)) = 0$ if i does not equal the coefficient of α_r in any $-\lambda$. A weight $\sum n_i\alpha_i$ will be abbreviated as $(n_1, \cdots, n_l)$ in the following.

(4.2) *The case* $\mathfrak{g} = E_6$.

(4.2.1) $r = 4$. As a typical example, we first consider $Y = (E_6, \alpha_4)$ and explain how our computations go (cf. §3). By a direct calculation, we find that the possible $-\lambda$'s are

$$-\lambda = \begin{cases} \lambda_4 - \lambda_1 - \lambda_6 = (0, 1, 1, 2, 1, 0) \\ \lambda_4 - \lambda_2 = (1, 1, 2, 3, 2, 1) \\ \lambda_4 = (2, 3, 4, 6, 4, 2)\ . \end{cases}$$

Since we have $h^0(\Omega_Y^p(1)) = 0$ for $p \geqq 7$ by (3.6) and Table 1, we only have to estimate $h^0(\Omega_Y^p(1))$ for $p \leqq 6$. If $p = 1$, then we can use Table 3. Hence it suffices to consider $G^i(\Lambda^p\mathfrak{n}^+)$ for $2 \leqq p \leqq 6$ and for $i = 2, 3$ and 6, since the coefficient of α_4 in $-\lambda$ is 2, 3 and 6, respectively. For $p = 2$, we can determine the lowest weights of them completely by using (3.9).

$G^2(\Lambda^2\mathfrak{n}^+)$: $3\lambda_4-\lambda_1-2\lambda_2-\lambda_5$, $3\lambda_4-2\lambda_3-2\lambda_5$, $3\lambda_4-2\lambda_2-2\lambda_3-\lambda_6$, $\lambda_4-\lambda_1-\lambda_6$.
$G^3(\Lambda^2\mathfrak{n}^+)$: $3\lambda_4-\lambda_1-\lambda_2-\lambda_3-\lambda_5$, $2\lambda_4-\lambda_2-\lambda_5-\lambda_6$, $2\lambda_4-\lambda_1-\lambda_2-\lambda_5$, $\lambda_4-\lambda_2$.
$G^6(\Lambda^2\mathfrak{n}^+)$: λ_4.

Since $\lambda + \lambda_4 + \delta$ is dominant, we get from (1.4) and the remark mentioned just before (4.2)

$$h^0(G^2\Omega_Y^2(1)) = \deg(\lambda_1 + \lambda_6) = 650\ , \quad h^0(G^3\Omega_Y^2(1)) = \deg(\lambda_2) = 78\ ,$$
$$h^0(G^6\Omega_Y^2(1)) = \deg(0) = 1\ .$$

For $p \geqq 3$, we compute $m_p(-\lambda)$ instead of determining the lowest weights of $G^t(\Lambda^p\mathfrak{n}^+)$, since we have certain ambiguities caused by the weakness of (3.9). The result is as follows:

$$m_3(\lambda_4 - \lambda_2) = 18\ , \quad m_3(\lambda_4) = 24\ , \quad m_4(\lambda_4) = 144\ ,$$
$$m_5(\lambda_4) = 306\ , \quad m_6(\lambda_4) = 180\ .$$

Since the component with the lowest weight $-\lambda$ could appear at most $m_p(-\lambda)$ times, we obtain the rough estimates

$$h^0(G^3\Omega_Y^3(1)) \leqq 18\deg(\lambda_2) = 1404\ , \quad h^0(G^6\Omega_Y^3(1)) \leqq 24\ ,$$
$$h^0(G^6\Omega_Y^4(1)) \leqq 144\ , \quad h^0(G^6\Omega_Y^5(1)) \leqq 306\ , \quad h^0(G^6\Omega_Y^6(1)) \leqq 180\ .$$

Combining the above results and (3.3), we get

$$h^0(\Omega_Y^2(1)) \leqq h^0(G^2\Omega_Y^2(1)) + h^0(G^3\Omega_Y^2(1)) + h^0(G^6\Omega_Y^2(1)) = 729,$$
$$h^0(\Omega_Y^3(1)) \leqq h^0(G^3\Omega_Y^3(1)) + h^0(G^6\Omega_Y^3(1)) \leqq 1428\ ,$$
$$h^0(\Omega_Y^4(1)) \leqq h^0(G^6\Omega_Y^4(1)) \leqq 144\ , \quad h^0(\Omega_Y^5(1)) \leqq h^0(G^6\Omega_Y^5(1)) \leqq 306\ ,$$
$$h^0(\Omega_Y^6(1)) \leqq h^0(G^6\Omega_Y^6(1)) \leqq 180\ .$$

(4.2.2) $r = 2$. $-\lambda = \lambda_2 = (1, 2, 2, 3, 2, 1)$.
The lowest weights of $G^2(\Lambda^2\mathfrak{n}^+)$ are

$$3\lambda_2 - \lambda_3 - \lambda_4 - \lambda_5,\ \lambda_2\ .$$

Thus we have $h^0(\Omega_Y^2(1)) \leqq 1$.

(4.2.3) $r = 3$. $-\lambda = \lambda_3 - \lambda_6 = (1, 1, 2, 2, 1, 0)$.
In this case, we get $h^0(\Omega_Y^p(1)) = 0$ for $p \geqq 3$, since the coefficient of α_3 in $-\lambda$ is 2. The lowest weights of $G^2(\Lambda^2\mathfrak{n}^+)$ are

$$\lambda_3 - \lambda_6\ , \quad 3\lambda_3 - 2\lambda_4\ , \quad 3\lambda_3 - 2\lambda_1 - \lambda_5\ .$$

Thus we have $h^0(\Omega_Y^2(1)) \leqq \deg(\lambda_6) = 27$.

(4.3) *The case* $\mathfrak{g} = F_4$.

(4.3.1) $r = 1$. $-\lambda = \lambda_1 = (2, 3, 4, 2)$.
The lowest weights of $G^2(\Lambda^2\mathfrak{n}^+)$ are $2\lambda_1 - 3\lambda_3$ and λ_1. Thus we have $h^0(\Omega_Y^2(1)) \leqq 1$ in this case.

(4.3.2) $r = 2$.

$$-\lambda = \begin{cases} \lambda_2 - \lambda_1 - \lambda_4 = (0, 1, 1, 0) \\ \lambda_2 - 2\lambda_4 = (1, 2, 2, 0) \\ \lambda_2 - \lambda_3 = (1, 2, 2, 1) \\ \lambda_2 - \lambda_1 = (1, 3, 4, 2) \\ \lambda_2 - \lambda_4 = (2, 4, 5, 2) \\ \lambda_2 = (3, 6, 8, 4)\ . \end{cases}$$

We exhibit the lowest weights of $G^i(\Lambda^2\mathfrak{n}^+)$ for $i = 2, 3, 4$ and 6.

$G^2(\Lambda^2\mathfrak{n}^+)$: $3\lambda_2 - 4\lambda_3$, $3\lambda_2 - 2\lambda_1 - 2\lambda_3 - \lambda_4$, $\lambda_2 - 2\lambda_4$.
$G^3(\Lambda^2\mathfrak{n}^+)$: $3\lambda_2 - \lambda_1 - 2\lambda_3 - 2\lambda_4$, $2\lambda_2 - \lambda_1 - \lambda_3 - \lambda_4$, $\lambda_2 - \lambda_1$.
$G^4(\Lambda^2\mathfrak{n}^+)$: $2\lambda_2 - \lambda_3 - 2\lambda_4$, $3\lambda_2 - 2\lambda_1 - 2\lambda_3$, $2\lambda_2 - 2\lambda_3$.
$G^6(\Lambda^2\mathfrak{n}^+)$: λ_2.

We calculate $m_p(-\lambda)$ for $3 \leqq p \leqq 6$ and find

$$m_3(\lambda_2 - \lambda_1) = 9\ , \quad m_3(\lambda_2 - \lambda_4) = 19\ , \quad m_3(\lambda_2) = 14\ ,$$
$$m_4(\lambda_2 - \lambda_4) = 18\ , \quad m_4(\lambda_2) = 63\ , \quad m_5(\lambda_2) = 96\ , \quad m_6(\lambda_2) = 34\ .$$

From these datum, we get the estimates

$$h^0(\Omega_Y^2(1)) \leqq \deg(2\lambda_4) + \deg(\lambda_1) + 1 = 72 + 52 + 1 = 125\ ,$$
$$h^0(\Omega_Y^3(1)) \leqq 9\deg(\lambda_1) + 19\deg(\lambda_4) + 14 = 938\ ,$$
$$h^0(\Omega_Y^4(1)) \leqq 18\deg(\lambda_4) + 63 = 531\ , \quad h^0(\Omega_Y^5(1)) \leqq 96\ ,$$
$$h^0(\Omega_Y^6(1)) \leqq 34\ .$$

(4.3.3) $r = 3$.

$$-\lambda = \begin{cases} \lambda_3 - \lambda_1 = (0, 1, 2, 1) \\ \lambda_3 - \lambda_4 = (1, 2, 3, 1) \\ \lambda_3 = (2, 4, 6, 3)\ . \end{cases}$$

We exhibit the lowest weights of $G^i(\Lambda^2\mathfrak{n}^+)$ for $i = 2, 3$ and 6.

$G^2(\Lambda^2\mathfrak{n}^+)$: $2\lambda_3 - \lambda_1 - 2\lambda_4$, $3\lambda_3 - 2\lambda_2$.
$G^3(\Lambda^2\mathfrak{n}^+)$: $4\lambda_3 - \lambda_1 - \lambda_2 - 3\lambda_4$, $3\lambda_3 - \lambda_1 - \lambda_2 - \lambda_4$, $2\lambda_3 - 3\lambda_4$, $\lambda_3 - \lambda_4$.
$G^6(\Lambda^2\mathfrak{n}^+)$: $4\lambda_3 - \lambda_1 - \lambda_2 - 2\lambda_4$, $2\lambda_3 - 2\lambda_4$, λ_3.

Since we have

$$m_3(\lambda_3 - \lambda_4) = 3\ , \quad m_3(\lambda_3) = 25\ , \quad m_4(\lambda_3) = 39\ , \quad m_5(\lambda_3) = 12\ , \quad m_6(\lambda_3) = 1\ ,$$

we obtain the following estimates.

$$h^0(\Omega_Y^2(1)) \leqq \deg(\lambda_4) + 1 = 27\ , \quad h^0(\Omega_Y^3(1)) \leqq 3\deg(\lambda_4) + 25 = 103\ ,$$
$$h^0(\Omega_Y^4(1)) \leqq 39\ , \quad h^0(\Omega_Y^5(1)) \leqq 12\ , \quad h^0(\Omega_Y^6(1)) \leqq 1\ .$$

(4.4) *The case* $\mathfrak{g} = G_2$. In this case, we have $-\lambda = \lambda_2 = (3, 2)$ and the lowest weights of $G_2(\Lambda^2\mathfrak{n}^+)$ are $3\lambda_2 - 4\lambda_1$ and λ_2. Hence we have $h^0(\Omega_Y^2(1)) \leqq 1$.

Combining the above results with Table 1, (3.8) and Table 3, we get the following theorem by (2.4).

(4.5) THEOREM. *The infinitesimal Torelli theorem holds for a non-singular complete intersection with the ample canonical bundle in a Kähler C-space* $(\mathfrak{g}, \alpha_r)$ *if* $\mathfrak{g} = E_6$, F_4 *or* G_2.

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