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TRANSVERSALLY SYMMETRIC RIEMANNIAN FOLIATIONS

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Summary. We discuss Riemannian foliations which are transversally modeled on a Riemannian symmetric space. In particular we investigate how the transversal symmetry influences other geometric properties of the foliation and the geometry of the ambient space.

1. Introduction. A Riemannian foliation $\mathcal{F}$ is *transversally symmetric* if its transversal geometry is locally modeled on a Riemannian symmetric space. The first topic of this paper is a characterization of transversal symmetry by a condition on the canonical Levi-Civita connection ∇ of the normal bundle (Theorem 1). For a totally geodesic foliation $\mathcal{F}$ this characterization can be sharpened in the analytic case (Theorem 3), using the results of [26], [27], [28].

Next we examine the influence of the geometry of the ambient space M on the properties discussed above. A typical illustration is the following. For a space of constant curvature the total geodesic property for the leaves of $\mathcal{F}$ implies the transversal symmetry of $\mathcal{F}$ (Theorem 4). Related results are Theorem 5, Corollary 6 and Theorem 7.

Conversely, the existence of a transversally symmetric foliation has strong implications for the geometry of the ambient space (M, g). Note that throughout this paper we assume the metric g to be bundle-like for $\mathcal{F}$. A typical result is that the transversal symmetry of the foliation defined by a Killing vector field of unit length on a complete, simply connected (M, g) implies that (M, g) is a naturally reductive space (Theorem 10).

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2. Transversal symmetry. Let $\mathcal{F}$ be a *Riemannian foliation* on a Riemannian manifold (M, g). It is given by an exact sequence of vector bundles

$$0 \to L \to TM \xrightarrow{\pi} Q \to 0\,, \tag{2.1}$$

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where L is the tangent bundle and Q the normal bundle of $\mathscr{F}$. The tangent bundle TM decomposes as an orthogonal direct sum $L \oplus L^\perp \cong TM$. The assumption on g to be a *bundle-like metric* means that the induced metric g_Q on the normal bundle $Q \cong L^\perp$ satisfies the (infinitesimal) holonomy invariance condition $\theta(X)g_Q = 0$ for all $X \in \Gamma L$, where $\theta(X)$ denotes the Lie derivative with respect to X [21].

For a distinguished chart $U \subset M$ the leaves of $\mathscr{F}$ in U are given as the fibers of a *Riemannian submersion* $f: U \to V \subset N$ onto an open subset V of a model Riemannian manifold N. If $p = \dim L$, $q = \dim Q$ and $n = p + q = \dim M$, then $\dim N = q$. For overlapping charts $U_\alpha \cap U_\beta$ the corresponding local transition functions $\gamma_{\beta\alpha} = f_\beta \circ f_\alpha^{-1}$ on N are isometries. $\mathscr{F}$ is said to be transversally symmetric if N is a locally symmetric Riemannian space.

We wish to express this in terms of the canonical Levi-Civita connection ∇ of the normal bundle Q and its curvature R^∇. The connection ∇ is the unique metric and torsion free connection in Q (see e.g. [13], [15], [16], [25]). Let further D and R denote the Levi-Civita connection and curvature of the metric g on M. Finally, let A denote the O'Neill integrability tensor of type (1, 2) for $L^\perp$ [10], [19], [2], defined for a Riemannian foliation $\mathscr{F}$ by

$$A_Y Y' = \pi D_{\pi Y} \pi^\perp Y' + \pi^\perp D_{\pi Y} \pi Y' \tag{2.2}$$

for arbitrary vector fields Y, Y' and orthogonal projections $\pi: TM \to Q$, $\pi^\perp: TM \to L$. For $U, V \in \Gamma L^\perp$ we have then [2, (9.24)]

$$A_U V = \pi^\perp D_U V = \frac{1}{2} \pi^\perp [U, V]\,. \tag{2.3}$$

With these notations we have then the following result.

THEOREM 1. *Let $\mathscr{F}$ be a Riemannian foliation on (M, g), and g a bundle-like metric. The following conditions are equivalent*:

(i) *$\mathscr{F}$ is transversally symmetric*;

(ii) *the local geodesic symmetries (geodesic reflections) on the model space are isometries*;

(iii) $\nabla_U R^\nabla_{UVUV} = 0$ *for all* $U, V \in \Gamma L^\perp$;

(iv) $D_U R_{UVUV} + 2R_{UA_UVUV} = -6g((D_U A)_U V, A_U V)$ *for all* $U, V \in \Gamma L^\perp$.

All these conditions are purely local and they are automatically satisfied for a Riemannian foliation of codimension 1. Hence *a Riemannian foliation of codimension one is always transversally symmetric*. The conditions above seem to be weaker than the condition

$$\nabla R^\nabla = 0 \tag{2.4}$$

discussed in [6], since nothing is said about the vanishing of $\nabla_X R^\nabla$ and $i_X \nabla_U R^\nabla$ for $X \in \Gamma L$ and $U \in \Gamma L^\perp$.

Note that we use the sign convention

$$R^{\nabla}_{UV}=\nabla_{[U,V]}-[\nabla_U,\nabla_V]\,,$$

and we put

$$R_{UVWZ}=g(R_{UV}W,Z)\,,\qquad R^{\nabla}_{UVWZ}=g_Q(R^{\nabla}_{UV}W,Z)\,,$$

the latter being defined for $W, Z\in\Gamma Q$. In the arguments below we make extensive use of the fact that ∇ is a basic connection. This is expressed by the property [12, (2.30)]

$$i_X R^{\nabla}=0 \qquad \text{for all} \quad X\in\Gamma L\,. \tag{2.5}$$

As a consequence it suffices to evaluate $R^{\nabla}\in\Omega^2(M,\operatorname{End}(Q))$ on $U, V\in\Gamma Q$. Since locally Q is framed by projectable normal vector fields, denoted $\Gamma Q^L\subset\Gamma Q$, it is often enough to consider $R^{\nabla}(U,V)$ for $U, V\in\Gamma Q^L$. For given $\bar U\in\Gamma Q^L$ there is a unique projectable vector field $U\in\Gamma L^{\perp}$ with $\pi(U)=\bar U$ under the projection $\pi\colon TM\to Q$. We will identify U and $\bar U$.

Proof of Theorem 1. The proof is based on the relationship

$$f_*R^{\nabla}(U,V)W=R^N(f_*U,f_*V)f_*W \tag{2.6}$$

between R^{∇} and the curvature R^N of the local model in a distinguished chart, where $\mathcal{F}$ is defined via the local submersion f and $U, V, W\in\Gamma Q^L$. This is a consequence of the definition of ∇ [13, (1.3)].

Now it is classical that the local symmetry of the model space is characterized by (ii) or equivalently (see [11], [31]) by

$$\nabla^N_{\bar U}R^N_{\bar U\bar V\bar U\bar V}=0$$

for vector fields $\bar U$, $\bar V$ in the model space, and where

$$R^N_{\bar U\bar V\bar U\bar V}=g(R^N_{\bar U\bar V}\bar U,\bar V)\,.$$

For $U, V\in\Gamma L^{\perp}$ which are f-related to $\bar U$, $\bar V$, we have then

$$f_*\nabla_U R^{\nabla}_{UVUV}=\nabla^N_{\bar U}R^N_{\bar U\bar V\bar U\bar V}\,. \tag{2.7}$$

This follows from the fact that D_UV in each argument of R^{∇} can be replaced in view of (2.5) by $\pi(D_UV)=\nabla_UV$ ([13, (1.3)]).

It remains to prove the equivalence (iii)$\Leftrightarrow$(iv). First we note that by [2, (9.28f)] for $U, V\in\Gamma Q^L$

$$R_{UVUV}=R^{\nabla}_{UVUV}-3g(A_UV,A_UV)\,. \tag{2.8}$$

By [22, p. 156] we may assume $D_UU=0$. Then

$$D_UR_{UVUV}=U(R_{UVUV})-2R_{UD_UVUV}$$

and similarly

$$\nabla_U R^{\nabla}_{UVUV} = U(R^{\nabla}_{UVUV}) - 2R^{\nabla}_{U\nabla_U VUV}\,.$$

Further, it follows then by (2.8) that

$$D_U R_{UVUV} - \nabla_U R^{\nabla}_{UVUV} = -3Ug(A_U V, A_U V) - 2R_{UD_U VUV} + 2R^{\nabla}_{U\nabla_U VUV}\,.$$

Now using again [2, (9.28f)] yields

$$R_{U\nabla_U VUV} - R^{\nabla}_{U\nabla_U VUV} = -3g(A_U \nabla_U V, A_U V)$$

and thus

$$\begin{aligned} D_U R_{UVUV} - \nabla_U R^{\nabla}_{UVUV} + 2R_{UA_U VUV} &= -6g(D_U(A_U V), A_U V) + 6g(A_U \nabla_U V, A_U V) \\ &= -6g((D_U A)_U V, A_U V)\,. \end{aligned} \tag{2.9}$$

In the last equality we have used

$$g(A_U D_U V, A_U V) = g(A_U \nabla_U V, A_U V)\,,$$

which is a consequence of definition (2.2). (2.9) establishes the equivalence of (iii) and (iv), and completes the proof of Theorem 1.

Similar properties hold for *Kähler foliations*. The Riemannian foliation $\mathscr{F}$ is (transversally) Kähler (see e.g. [18]), if there exists a holonomy invariant almost complex structure $J\colon Q \to Q$, where $\dim Q = q = 2m$, satisfying the following two conditions:

$$g_Q(JU, JV) = g(U, V)\,, \qquad \nabla J = 0$$

for $U, V \in \Gamma L^{\perp}$. The basic two-form $\Phi(U, V) = g_Q(U, JV)$ is then closed. Using the result in [23] one proves then similarly as above the following result.

THEOREM 2. *Let $\mathscr{F}$ be a Kähler foliation on (M, g), and g a bundle-like metric. The following conditions are equivalent*:

(i) *$\mathscr{F}$ is transversally symmetric*;

(ii) *the geodesic reflections on the model Kähler manifold preserve the Kähler form, i.e. are symplectic*;

(iii) *the geodesic reflections on the model Kähler manifold preserve J, i.e. are holomorphic*;

(iv) $\nabla_U R^{\nabla}_{UJUUJU} = 0$ *for all* $U \in \Gamma L^{\perp}$.

Sasakian manifolds (M, g) provide examples of Kähler foliations with one-dimensional leaves. In this case the leaves are geodesics. The geodesic reflections on the model space correspond to the ϕ-geodesic symmetries on the ambient space (M, g) (see e.g. [3], [4], [7], [24]). These examples and the so called ϕ-symmetric spaces [24] may serve as a model for the theory we develop in the next section.

We finish this section by recalling that for complete and simply connected (M, g), a transversally symmetric foliation is globally given by the fibers of the developing map, a submersion to the simply connected symmetric model space [5].

3. Totally geodesic foliations. In this section we assume the foliation $\mathscr{F}$ to be in addition *totally geodesic*, i.e. all leaves are totally geodesic submanifolds with respect to a bundle-like metric g. The (*local*) *reflection* $\varphi_{\mathscr{L}}$ in each leaf $\mathscr{L}$ (or relative to $\mathscr{L}$) is defined as the local geodesic symmetry for normal geodesics to $\mathscr{L}$ in a sufficiently small tubular neighborhood of $\mathscr{L}$. For $m \in \mathscr{L}$ and p on a sufficiently short normal geodesic γ emanating from m, and parametrized by arc length, i.e. $p = \exp_m(ru) = \gamma(r)$ for some unit vector $u \in \Gamma L_m^\perp$, we have $\varphi_{\mathscr{L}}(p) = \exp_m(-ru) = \gamma(-r)$. (For more details about reflections see e.g. [8], [26], [30].)

For a Riemannian foliation it is immediate that the reflection $\varphi_{\mathscr{L}}$ sends leaves into leaves, and corresponds to a geodesic symmetry on the (local) model space for the transversal geometry at the point corresponding to the leaf $\mathscr{L}$.

It is well-known that when all the reflections are isometries, then the leaves $\mathscr{L}$ are necessarily totally geodesic. We discussed in [27] conditions to impose on the reflections in a totally geodesic submanifold, so as to guarantee that they are isometries. They involve the shape operator $T_p(m)\colon T_mG_p \to T_mG_p$ of the geodesic sphere $G_p \subset M$ with center $p = \gamma(r)$ and radius r. We have then $L_m \subset T_mG_p$, the inclusion being an identity only for $q = 1$. In [28] we discussed similar conditions using the Ricci operator $\tilde{Q}_p(m)\colon T_mG_p \to T_mG_p$ of G_p. Both results rely on a criterion, determined in [8], using the curvature R and its covariant derivatives along the submanifold.

The characterizations for transversal symmetry in Theorem 1 can be sharpened as follows.

THEOREM 3. *Let $\mathscr{F}$ be a totally geodesic and Riemannian foliation on (M, g) of codimension $q > 1$, and g a bundle-like metric. Assume all data to be analytic. The following conditions are equivalent*:

(i) *$\mathscr{F}$ is transversally symmetric*;

(ii) *$\nabla_U R^\nabla_{UVUV} = 0$ for all $U, V \in \Gamma L^\perp$*;

(iii) *$R_{UVUX} = 0$ and $D_U R_{UVUV} = 0$ for all $U, V \in \Gamma L^\perp$ and $X \in \Gamma L$*;

(iv) *the reflections φ in the leaves are isometries*;

(v) *$\varphi_*(m) \circ T_p(m) = T_{\varphi(p)}(m) \circ \varphi_*(m)$ for all $m \in M$, all unit $u \in L_m^\perp$, and all $p = \exp_m(ru)$ for all sufficiently small r*;

(vi) *same condition as in* (v), *but applied only to normal vectors $v \in L_m^\perp \cap T_mG_p$*;

(vii) *$\varphi_*(m) \circ \tilde{Q}_p(m) = \tilde{Q}_{\varphi(p)}(m) \circ \varphi_*(m)$ for all $m \in M$, all unit $u \in L_m^\perp$, and all $p = \exp_m(ru)$ for all sufficiently small r, if* $\dim M > 3$ *and* $2 \dim M = 2n \neq 3(n - q + 1)$.

PROOF. The equivalence of (i) and (ii) was proved above. Further, the equivalence of (iv) and (v) has been proved in [27], and the equivalence of (iv) and (vii) in [28]. These proofs use the analyticity assumption made.

Next, assuming totally geodesic leaves, the reflection in the leaves are isometric if and only if the geodesic reflections on the model space are (local) isometries, as is clear in a distinguished chart. This proves the equivalence of (i) and (iv).

The implication (vi) $\Rightarrow$ (iii) follows at once from the detailed computations in [27].

Now we prove the implication (iii) $\Rightarrow$ (ii). For totally geodesic $\mathscr{F}$ we have to put $T=0$ (see (4.2) below for the definition of T) in [2, (9.28e)], and this yields

$$R_{UVUX}=g((D_U A)_U V, X) .$$

By assumption this term vanishes for all $X\in \Gamma L$. In particular

$$g((D_U A)_U V, A_U V)=0$$

for U, $V\in \Gamma L^{\perp}$. Thus condition (iv) of Theorem 1 is satisfied. This completes the proof of the theorem.

4. Consequences of constant ambient curvature. In this section we apply the previous considerations to a manifold (M, g) of constant sectional curvature, and to a Kähler manifold (M, g, J) of constant holomorphic sectional curvature, i.e. to real and to complex space forms.

As already observed in [8], [26], in a space of constant curvature the reflections in totally geodesic submanifolds are isometries. Hence from this and Theorem 3 we have the following fact.

THEOREM 4. *Let $\mathscr{F}$ be a Riemannian foliation on a space (M, g) of constant curvature, and g a bundle-like metric. If $\mathscr{F}$ is totally geodesic, it is necessarily transversally symmetric.*

More generally we have the following result.

THEOREM 5. *Let $\mathscr{F}$ be a Riemannian foliation on a space (M, g) of constant curvature, and g a bundle-like metric. Then $\mathscr{F}$ is transversally symmetric if and only if*

$$g(A_U V, T_{A_U V} U)=0 \qquad \text{for all} \quad U, V\in \Gamma L^{\perp} . \tag{4.1}$$

Here T is the O'Neill tensor of type (1, 2) defined for a Riemannian foliation by

$$T_Y Y'=\pi D_{\pi^{\perp}Y}\pi^{\perp} Y'+\pi^{\perp} D_{\pi^{\perp}Y}\pi Y' \tag{4.2}$$

for arbitrary vector fields Y, Y'. Note that in particular for $X\in \Gamma L$, $U\in \Gamma L^{\perp}$ we get from (4.2)

$$T_X U=\pi^{\perp} D_X U=-W(U)X ,$$

where $W(U)\colon L\to L$ denotes the Weingarten map of $\mathscr{F}$ given for U. Thus for totally geodesic $\mathscr{F}$, condition (4.1) is satisfied, and thus Theorem 4 follows from Theorem 5.

PROOF OF THEOREM 5. By assumption $DR=0$ and for U, $V\in \Gamma L^{\perp}$ we have

$$R_{UV}U=c\{g(U, U)V-g(U, V)U\},$$

where c is the constant curvature of (M, g). Thus for $X\in\Gamma L$ it follows $R_{UVUX}=0$. In particular

$$R_{UVUA_UV}=0\,.$$

By [2, (9.28e)] we have on the other hand

$$R_{UVUA_UV}=g((D_UA)_UV, A_UV)+2g(A_UV, T_{A_UV}U)\,.$$

Then (2.9) implies

$$-\nabla_U R^{\nabla}_{UVUV}=-6g((D_UA)_UV, A_UV)=12g(A_UV, T_{A_UV}U)\,.$$

This, together with Theorem 1, completes the proof.

For a Riemannian foliation with integrable $L^{\perp}$, we have $A=0$ and hence (4.1) implies the following fact.

COROLLARY 6. *Let $\mathscr{F}$ be a Riemannian foliation on a space of constant curvature, and g a bundle-like metric. If $L^{\perp}$ is integrable, then $\mathscr{F}$ is transversally symmetric.*

Next, we combine Theorem 4 with the following recent result of Nakagawa and Takagi [17].

PROPOSITION 7. *Let $\mathscr{F}$ be a Riemannian and harmonic foliation on a compact manifold (M^n, g), $n>2$, of constant sectional curvature $c\geqq 0$. Then $\mathscr{F}$ is totally geodesic.*

The *harmonicity* condition means that all leaves are *minimal* submanifolds [13]. For codimension $q=1$ this result is a consequence of the sharper results in [14], [20] based on Ricci curvature hypotheses.

Combining this with Theorem 4 we get the following result.

THEOREM 8. *Let (M^n, g), $n>2$ be a compact Riemannian manifold of constant sectional curvature $c\geqq 0$, $\mathscr{F}$ a Riemannian foliation on M, and g a bundle-like metric. If $\mathscr{F}$ is harmonic, then $\mathscr{F}$ is transversally symmetric.*

This applies in particular to the spheres S^n, $n>2$. These foliations have been classified in [9].

Proceeding as above we finally get from Theorem 3 and the result in [8], [26] the following conclusion.

THEOREM 9. *Let (M, g, J) be a Kähler manifold of constant holomorphic sectional curvature, $\mathscr{F}$ a Riemannian and totally geodesic foliation, and g a bundle-like metric. Then the following conditions are equivalent*:

(i) *$\mathscr{F}$ is transversally symmetric;*

(ii) *$R_{UVUX}=0$ for all $U, V\in\Gamma L^{\perp}$ and all $X\in\Gamma L$;*

(iii) *every leaf* $\mathscr{L}$ *is either a holomorphic submanifold or a totally real submanifold* $\mathscr{L}$ *with* $\dim \mathscr{L}=(1/2)\dim M$.

5. Effect on the ambient metric. In this final section we will treat some aspects of the following question. How does the existence of a transversally symmetric foliation influence the geometry of the ambient space?

We consider a Riemannian flow (a Riemannian foliation with one-dimensional leaves), and we assume moreover $\mathscr{F}$ to be generated by the flow lines of a Killing vector field ξ of unit length. The leaves are then necessarily geodesics. We prove the following result.

THEOREM 10. *Let* $\mathscr{F}$ *be the Riemannian flow defined by a unit Killing vector field* ξ *on* (M, g). *If* $\mathscr{F}$ *is transversally symmetric, the space* (M, g) *is locally homogeneous. If moreover* (M, g) *is complete and simply connected, it is a naturally reductive homogeneous space.*

PROOF. By [1] and [29] we have to prove the existence of a $(1, 2)$-tensor field T (unrelated to O'Neill's tensor in section 4), such that for the new connection $\bar{D}=D-T$ we have

$$\bar{D}g=\bar{D}T=\bar{D}R=0 \tag{5.1}$$

and

$$T_X X=0 \tag{5.2}$$

for all tangent vector fields X. (5.1) guarantees the local homogeneity, while (5.2) guarantees that the homogeneous structure T is of natural reductive type. A complete and simply connected manifold with such a T is a natural reductive homogeneous space.

To prove the existence of such a T, let A^* be a tensor field defined by

$$A^*_\xi \xi=0\,, \quad A^*_U \xi=A_\xi U\,, \quad A^*_\xi U=A_U \xi, \quad A^*_U V=0 \tag{5.3}$$

for $U, V \in \Gamma L^\perp$. Then let

$$T=A-A^* \tag{5.4}$$

and define $\bar{D}$ by

$$\bar{D}=D-T\,.$$

Note that the properties of the O'Neill tensors imply (see [2, (9.21)])

$$(5.5)\qquad \begin{cases} A_U & \text{is alternating}\,, \\ A_U V = -A_V U\,, \\ A_\xi U = A_\xi \xi = 0\,, \\ A_U \xi = \pi D_U \xi\,, \\ A_U V = \pi^\perp D_U V \end{cases}$$

and hence $A_U V \sim \xi$. Hence, from (5.3), (5.4) and (5.5) we get

$$(5.6)\qquad T_\xi \xi = 0\,, \quad T_U \xi = A_U \xi\,, \quad T_\xi U = -A_U \xi\,, \quad T_U V = A_U V\,.$$

Now, from this we see at once that (5.2) is satisfied. Further, $\bar{D}g = 0$ is equivalent to

$$g(T_X Y, Z) + g(T_X Z, Y) = 0$$

and we see easily that this condition is also satisfied.

Further, when $\mathscr{F}$ is transversally symmetric and ξ is Killing, we get (see Theorem 3) first

$$R_{UVW\xi} = 0\,,$$

and then, using also the properties of the O'Neill tensors, a lengthy but straightforward computation shows that $\bar{D}R = \bar{D}T = 0$, which completes the proof.

Finally we mention the following result of Chen and the second author (see e.g. [30, p. 83]):

PROPOSITION 11. *Let (M, g) be a locally irreducible symmetric space. Then (M, g) is a space of constant curvature if it admits a curve σ such that the reflection in the curve is an isometry.*

From this, together with Theorem 4, we get at once the following property.

THEOREM 12. *Let (M, g) be a locally irreducible symmetric space, and $\mathscr{F}$ a Riemannian flow with geodesic leaves. If $\mathscr{F}$ is transversally symmetric, then (M, g) is a space of constant curvature, and conversely.*

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