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CORRECTION AND ADDITION: BUBBLING OUT OF EINSTEIN MANIFOLDS

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The purpose of this note is to correct an error in [3] which was kindly pointed out by Professors M. T. Anderson and J. Cheeger, and to make additional remarks. The reader is referred to their work [2] for the treatment of bubbling out process from a different viewpoint.

In Theorem 2, the conclusion of being a *Euclidean space* should be replaced by being *flat*.

The *non-flatness* of the bubbled out orbifold (Y, h) in Proposition 2 is important if one applies Theorem 2 in showing that the process of bubbling out terminates in finite steps. Since it is non-trivial contrary to the cases in [1], [4] and [5], we here give a proof. Suppose (Y, h) is flat. Then the ALE orbifold Y is isometric to a flat cone $\boldsymbol{R}^n/\Gamma$ which has only one singularity at the origin y_∞, and the sequence $((X_k, r_0^{-2}g_k), x_{a,k})$ smoothly converges to $((Y, h), y_\infty)$ on any compact set disjoint from y_∞. (See [1], [3], [4] and [5]. If the curvature accumulates at a point other than y_∞, then the limit orbifold Y must have a corresponding singularity.) We take a constant $K>0$ sufficiently large. Then by Proposition 3 or by its proof we have

$$\int_{D(Kr_0, K^{-1}r_\infty)} |R_{g_k}|^{n/2} \leq \frac{\varepsilon}{6}.$$

On the other hand by the definition of r_∞, it holds that for sufficiently large k

$$\int_{D(K^{-1}r_\infty, r_\infty)} |R_{g_k}|^{n/2} \leq \frac{2\varepsilon}{3}.$$

Combining the above two inequalities and the definition of r_0, we get

$$\int_{D(r_0, Kr_0)} |R_{g_k}|^{n/2} \geq \frac{\varepsilon}{6},$$

which in the limit contradicts the flatness of (Y, h);

$$\int_{D(1, K)} |R_h|^{n/2} \geq \frac{\varepsilon}{6}.$$

One way to see that the process of bubbling out terminates in finite steps is to apply Theorem 2 as remarked in the paper. The other way is to consider the contribution of the domains $D(r_0, r_\infty)$ to the curvature integral $\int|R|^{n/2}$ on the whole space. (We maintain the notation for the first bubbles also for those appearing in the repeated processes.) By the definition of r_0 and r_∞, we have

$$\int_{D(r_0, r_\infty)} |R_{g_k}|^{n/2} = \varepsilon .$$

Hence if the domains which appear in the blowing up processes are disjoint, we obtain the termination in finite steps. The only possibility for the overlapping comes from the existence of a singular point on the unit sphere $S(y_\infty, 1)$ of Y. In this case, by the choice of the center $x_{a,k}$, X_k develops a singularity also at the origin y_∞. Then we discard the previous domain $D(r_0, r_\infty)$ and consider only the domains coming from the singularities of Y. Since we have at least two singular points in Y, the contribution to the curvature integral increases at least by ε. This shows that the process of bubbling out terminates in finite steps.

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