Tôhoku Math. J.
44 (1992), 243–250

SMOOTH $SL(n, \boldsymbol{H})$, $Sp(n, \boldsymbol{C})$-ACTIONS ON $(4n-1)$-MANIFOLDS

TOMOJI TOMURO

(Received March 27, 1991, revised September 6, 1991)

Abstract. Smooth $SL(n, \boldsymbol{C})$-actions on $(2n-1)$-manifolds were classified by Uchida [12], while smooth $SL(2, \boldsymbol{H})$-actions on 7-manifolds are discussed in Abe [1]. In this paper, the classification of smooth actions of $SL(n,\boldsymbol{H})$ and $Sp(n, \boldsymbol{C})$ on simply connected closed $(4n-1)$-manifolds is carried out for $n \geqq 3$.

1. Known results. Let G be a Lie group and K a compact subgroup. A smooth G-action Φ on a smooth manifold M naturally induces a K-action $\Phi|K$ on M. For a K-action Φ_0 on M, if $\Phi|K=\Phi_0$ on M, then Φ(resp. Φ_0) is called the extension (resp. restriction) of Φ_0(resp. Φ). Let G_x, K_x, $G(x)$ and $K(x)$ denote the isotropy subgroups at x and the orbits through x with respect to Φ, Φ_0, where $\Phi|K=\Phi_0$. By definition,

$$K \cap G_x = K_x \quad \text{and} \quad G(x) \text{ is } K\text{-invariant}. \tag{1.1}$$

$$\dim G - \dim G_x = \dim G(x) \leqq \dim M. \tag{1.2}$$

Let H be the principal isotropy subgroup of the restricted K-action Φ_0. Then for any $x \in M$, we have

$$(G_x) > (H), \tag{1.3}$$

where (A) denotes the conjugacy class of A in G, and $(A_1) < (A_2)$ if there exist $B_1 \in (A_1)$ and $B_2 \in (A_2)$ with $B_1 \subset B_2$.

2. Classification of smooth $Sp(n)$-actions on $(4n-1)$-manifolds. The maximal compact subgroups of $SL(n, \boldsymbol{H})$ and $Sp(n, \boldsymbol{C})$ are both $Sp(n)$. Hence we first classify non-trivial smooth $Sp(n)$-actions.

The following results are proved by a standard method.

LEMMA 2.1 (cf. [5]). *Assume $n \geqq 3$. Let K be a closed connected subgroup of $Sp(n)$ such that* $\dim Sp(n)/K \leqq 4n-1$. *Then, up to inner automorphism of $Sp(n)$, K coincides with one of*

$$Sp(n-1), \quad U(1) \times Sp(n-1), \quad Sp(1) \times Sp(n-1) \quad \text{and} \quad Sp(n)$$

embedded in the standard way.

LEMMA 2.2. (1) *Assume $n \geqq 3$. Then there exists no non-trivial representation*

1991 *Mathematics Subject Classification.* Primary 57S20.

$Sp(n)\to O(4n-1)$, while there exists no non-trivial representation $Sp(n-1)\to O(3)$. (2) *By the identification $\boldsymbol{R}^3=\boldsymbol{H}_0$, the set of all pure quaternions, a non-trivial representation $Sp(1)\to O(3)$ is equivalent to the adjoint representation* Ad *given by*

$$\mathrm{Ad}(q)(h)=qhq^{-1} \quad \text{for} \quad q\in Sp(1)\,, \quad h\in \boldsymbol{H}_0\,. \tag{2.3}$$

REMARK 2.4. By Lemma 2.2 (1), we see that any non-trivial $Sp(n)$-action on a $(4n-1)$-manifold has no fixed points, for $n\geqq 3$.

Using the above results, we obtain the following by standard methods (cf. [8]).

THEOREM 2.5. *Assume $n\geqq 3$. Let (M, H, Φ_0) be a triple consisting of a non-trivial smooth $Sp(n)$-action Φ_0 on a simply connected closed $(4n-1)$-manifold M with the principal isotropy subgroup H. Then (M, H, Φ_0) is equivariantly diffeomorphic to one of the following triples*:

(1) $(S^{4n-1}, Sp(n-1), \Phi_1)$, $\Phi_1(k, z)=kz$.
(2) $(S^{4n-1}\times_{Sp(1)}S^3, U(1)\times Sp(n-1), \Phi_2)$, $\Phi_2(k, [z, x])=[kz, x]$.
(3) $(P_{n-1}(\boldsymbol{H})\times hS^3, Sp(1)\times Sp(n-1), \Phi_3)$, $\Phi_3(k, ([z], x))=([kz], x)$.

REMARK 2.6. The $Sp(1)$-action on S^3 in Theorem 2.5 (2) is given by $\rho(q, u+v)=u+\mathrm{Ad}(q)v$, where S^3 is a unit sphere of quaternions of modulus one, u is a real number and v is a pure quaternion, and $\mathrm{Ad}(q)$ is given in (2.3).

3. Certain subgroups of $SL(n, \boldsymbol{H})$ and $Sp(n, \boldsymbol{C})$. Let us now consider the following subgroups of $SL(n, \boldsymbol{H})$:

$$\begin{aligned}
L_{SL}&=\{(a_{ij})\in SL(n, \boldsymbol{H}):\ a_{11}=1,\ a_{21}=a_{31}=\cdots=a_{n1}=0\}\,,\\
L^*_{SL}&=\{(a_{ij})\in SL(n, \boldsymbol{H}):\ a_{11}=1,\ a_{12}=a_{13}=\cdots=a_{1n}=0\}\,,\\
N_{SL}&=\{(a_{ij})\in SL(n, \boldsymbol{H}):\ a_{21}=a_{31}=\cdots=a_{n1}=0\}\,,\\
N^*_{SL}&=\{(a_{ij})\in SL(n, \boldsymbol{H}):\ a_{12}=a_{13}=\cdots=a_{1n}=0\}\,,\\
Sp(n-1)&=Sp(n)\cap L_{SL}=Sp(n)\cap L^*_{SL}\,.
\end{aligned}$$

PROPOSITION 3.1 (cf. [7, Lemma 2.1]). *Assume $n\geqq 3$. Let P be a closed connected proper subgroup of $SL(n, \boldsymbol{H})$ such that*

$$\dim SL(n, \boldsymbol{H})/P\leqq 4n-1\,.$$

If P contains $Sp(n-1)$, then either

$$L_{SL}\subset P\subset N_{SL} \quad \text{or} \quad L^*_{SL}\subset P\subset N^*_{SL}\,.$$

Next we consider the following subspaces of $\mathfrak{sp}(n, \boldsymbol{C})$:

$$\mathfrak{sp}(n, \boldsymbol{C})=\left\{\begin{bmatrix} X & Z\\ Y & -{}^tX\end{bmatrix}:\ \begin{matrix}{}^tY=Y,\ {}^tZ=Z\\ X,\ Y,\ Z\in M_n(\boldsymbol{C})\end{matrix}\right\},\quad \mathfrak{sp}(n)=\left\{\begin{bmatrix} X & -Y^c\\ Y & X^c\end{bmatrix}:\ \begin{matrix}{}^tY=Y,\ {}^tX+X^c=0\\ X,\ Y\in M_n(\boldsymbol{C})\end{matrix}\right\},$$

$$\mathfrak{h}=\left\{\begin{bmatrix} X & Y^c \\ Y & -X \end{bmatrix} : \begin{matrix} {}^tY=Y, {}^tX+X^c=0 \\ X, Y\in M_n(\boldsymbol{C}) \end{matrix}\right\},$$

$$\mathfrak{a}=\left\{\begin{bmatrix} 0 & -{}^tV & 0 & {}^tU \\ X & 0 & U & 0 \\ 0 & {}^tY & 0 & -{}^tX \\ Y & 0 & V & 0 \end{bmatrix} : X, Y, U, V\in \boldsymbol{C}^{n-1}\right\},$$

$$\mathfrak{z}=\left\{\begin{bmatrix} x & 0 & z & 0 \\ 0 & 0 & 0 & 0 \\ y & 0 & -x & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} : x, y, z\in \boldsymbol{C}\right\},$$

$$\mathfrak{l}_{Sp}=\left\{\begin{bmatrix} 0 & * & * & * \\ 0 & X_{11} & * & X_{12} \\ 0 & 0 & 0 & 0 \\ 0 & X_{21} & * & X_{22} \end{bmatrix} : X_{ij}\in M_{n-1}(\boldsymbol{C})\right\},$$

$$\mathfrak{n}_{Sp}=\left\{\begin{bmatrix} * & * & * & * \\ 0 & X_{11} & * & X_{12} \\ 0 & 0 & * & 0 \\ 0 & X_{21} & * & X_{22} \end{bmatrix} : X_{ij}\in M_{n-1}(\boldsymbol{C})\right\},$$

$$\mathfrak{a}(a+jb, c+jd)=\left\{\begin{bmatrix} 0 & * & 0 & * \\ Xa-Y^cb & 0 & Xc-Y^cd & 0 \\ 0 & * & 0 & * \\ Ya+X^cb & 0 & Yc+X^cd & 0 \end{bmatrix} : X, Y\in \boldsymbol{C}^{n-1}\right\},$$

$$\text{for} \quad a, b, c, d\in \boldsymbol{C},$$

$$\mathfrak{l}^*_{Sp}=\{X\colon {}^tX\in \mathfrak{l}_{Sp}\}, \quad \mathfrak{n}^*_{Sp}=\{X\colon {}^tX\in \mathfrak{n}_{Sp}\}, \quad \mathfrak{sp}(n-1)=\mathfrak{sp}(n)\cap \mathfrak{l}_{Sp}.$$

Here we denote by ${}^tX$ and X^c, the transpose and the complex conjugate of a given matrix X, respectively.

Denote by $Sp(n-1)$, L_{Sp}, L^*_{Sp}, N_{Sp} and N^*_{Sp} the connected subgroups of $Sp(n, \boldsymbol{C})$ corresponding to $\mathfrak{sp}(n-1)$, $\mathfrak{l}_{Sp}$, $\mathfrak{l}^*_{Sp}$, $\mathfrak{n}_{Sp}$ and $\mathfrak{n}^*_{Sp}$, respectively. We obtain the following results:

LEMMA 3.2. *Each* $\mathrm{Ad}(Sp(n-1))$*-invariant real proper subspace of* $\mathfrak{a}$ *has the form* $\mathfrak{a}(a+jb, c+jd)$ *for some* $a, b, c, d\in \boldsymbol{C}$.

PROPOSITION 3.3 (cf. [8, Lemma 1.1]). *Assume* $n \geqq 3$. *Let* $\mathfrak{p}$ *be a proper subalgebra of* $\mathfrak{sp}(n, \boldsymbol{C})$ *such that* $\dim \mathfrak{sp}(n, \boldsymbol{C})/\mathfrak{p} \leqq 4n-1$. *If* $\mathfrak{p}$ *contains* $\mathfrak{sp}(n-1)$, *then for some complex numbers* $(e, f) \neq (0, 0)$, *we have*

$$\mathfrak{p} = \mathfrak{sp}(n-1, \boldsymbol{C}) \oplus \mathfrak{a}(e, f) \oplus (\mathfrak{p} \cap \mathfrak{z}) . \tag{3.4}$$

COROLLARY 3.5. *Assume* $n \geqq 3$. *Let* P *be a closed connected subgroup of* $Sp(n, \boldsymbol{C})$ *such that* $\dim Sp(n, \boldsymbol{C})/P \leqq 4n-1$.

(1) *If* P *contains* $U(1) \times Sp(n-1)$, *then* $L_{Sp} \subset P \subset N_{Sp}$, $L^*_{Sp} \subset P \subset N^*_{Sp}$ *or* $P = Sp(n, \boldsymbol{C})$.

(2) *If* P *contains* $Sp(1) \times Sp(n-1)$, *then* $P = Sp(n, \boldsymbol{C})$.

4. Smooth actions of $SL(n, \boldsymbol{H})$ and $Sp(n, \boldsymbol{C})$ on $(4n-1)$-manifolds. Let G be either $SL(n, \boldsymbol{H})$ or $Sp(n, \boldsymbol{C})$, and $K = Sp(n)$. If G acts smoothly and non-trivially on a $(4n-1)$-manifold M through Φ then the restricted K-action $\Phi|K$ is also non-trivial, since G is a simple Lie group. Hence, the K-action $\Phi|K$ on M is equivariantly diffeomorphic to one in Theorem 2.5.

For a given G-action Φ, we can define a new G-action Φ^* by

$$\Phi^*(g, x) = \Phi((g^*)^{-1}, x) . \tag{4.1}$$

In our cases, the restricted K-actions $\Phi^*|K$ and $\Phi|K$ coincide.

We now show the following result.

THEOREM 4.2. *Assume* $n \geqq 3$. *Then a triple* (G, M, Φ) *or* (G, M, Φ^*) *is equivariantly diffeomorphic to one of the triples given in Table* 1.

TABLE 1

$Sp(n)$-manifold	Φ for $G = SL(n, \boldsymbol{H})$	Φ for $G = Sp(n, \boldsymbol{C})$
S^{4n-1}	$z \to \|gz\|^{-1-ir} gz$	$z \to \|gz\|^{-1-ir} gz$
$S^{4n-1} \times_{Sp(1)} S^3$	$(g, [z, x]) \to [gz/\|gz\|, \phi(\log\|gz\|, x)]$	not exist
$P_{n-1}(\boldsymbol{H}) \times hS^3$	$(g, ([z], x)) \to ([gz], \phi(\log(\|gz\|/\|z\|), x))$	not exist

Exact notation is explained in the proof. The proof is separated into three parts, according to Theorem 2.5. Throughout this section, we assume $n \geqq 3$ and let $P^* = \{X : {}^tX \in P\}$ for a subgroup P of G.

I. First we consider the case $M = S^{4n-1}$ with the restricted $Sp(N)$-action Φ_0 given by $\Phi_0(k, z) = kz$. In this case, the G-action is also transitive. Thus the problem is reduced to finding a connected closed subgroup P of G satisfying

$$\dim G/P = 4n-1 \quad \text{and} \quad P \cap Sp(n) = Sp(n-1) . \tag{4.3}$$

LEMMA 4.4. *Let* P *be a connected closed subgroup of* $SL(n, \boldsymbol{H})$ *satisfying* (4.3). *Then* P *is conjugate to*

$$P_r=\left\{\begin{bmatrix}\exp(t(1+ir)) & * \\ 0 & *\end{bmatrix} : t\in \mathbf{R}\right\} \quad or \quad P_r^* \qquad for \quad r\geqq 0 .$$

PROOF. By (4.3) and Proposition 3.1, we see that $P=P(q)$ or $P=P(q)^*$, where

$$P(q)=\left\{\begin{bmatrix}\exp(tq) & * \\ 0 & *\end{bmatrix} : t\in \boldsymbol{R}\right\}$$

for a non-zero quaternion q. By the second condition of (4.3), we see that q has a non-zero real part. Then we may assume $q=1+h$, for some pure quaternion h. We see that $P(1+h)$ is conjugate to $P_{|h|}$. q.e.d.

Similarly, we can prove the following:

LEMMA 4.5. *Let P be a connected closed subgroup of $Sp(n, \boldsymbol{C})$ satisfying* (4.3). *Then P is conjugate to*

$$P_r=\left\{\begin{bmatrix}\exp(t(1+ir)) & * \\ 0 & *\end{bmatrix} : t\in \boldsymbol{R}\right\} \qquad for\ some \quad r\in \boldsymbol{R} .$$

On the other hand, an action of $G=SL(n, \boldsymbol{H})$ or $Sp(n, \boldsymbol{C})$ on S^{4n-1} is defined by $\Phi(g, z)=\| gz \|^{-1-ir}gz$. We see that the isotropy subgroup of this action is conjugate to P_r.

REMARK 4.6. As a matter of fact the actions obtained above are nothing but the twisted linear actions in [9], [10].

II. Next we consider the case $M=S^{4n-1}\times_{Sp(1)}S^3$ with the restricted $Sp(n)$-action Φ_0 given by $\Phi_0(k, [z, x])=[kz, x]$. The $Sp(1)$-action ρ on S^3 is described precisely in Remark 2.6. In fact, the action ρ on S^3 has a fixed point, and hence the $Sp(n)$-action Φ_0 on M has $Sp(1)\times Sp(n-1)$ as an isotropy subgroup. In particular, we see that the action Φ_0 on M has no extended $Sp(n, \boldsymbol{C})$-action by Corollary 3.5 (2). So we assume $G=SL(n, \boldsymbol{H})$.

Let ϕ be a smooth $\boldsymbol{R}$-action on S^3 which commutes with the $Sp(1)$-action ρ. Then we see that the $\boldsymbol{R}$-action ϕ defines a smooth $SL(n, \boldsymbol{H})$-action Φ on M given by

$$\Phi(g, [z, x])=[gz/\| gz \|, \phi(\log\| gz \|, x)] . \tag{4.7}$$

On the other hand, let an extended $SL(n, \boldsymbol{H})$-action Φ of Φ_0 be given. Then we see that

$$F(Sp(n-1), M)=F(L_{SL}, M) \quad \text{or} \quad F(Sp(n-1), M)=F(L_{SL}^*, M) ,$$

where $F(P, M)$ denotes the fixed point set of the restricted action of Φ to P. Moreover, if $F(Sp(n-1), M)=F(L_{SL}, M)$, then there exists a smooth $\boldsymbol{R}$-action ϕ on S^3 which commutes with the $Sp(1)$-action ρ, and the action Φ on M satisfies the equation (4.7) (cf. [7, Section 3]). In addition, if $F(Sp(n-1), M)=F(L_{SL}^*, M)$, then we see that $F(Sp(n-1), M)=F(L_{SL}, M)$ for the action Φ^*.

III. Finally, we consider the case $M=P_{n-1}(\boldsymbol{H})\times hS^3$ with the restricted $Sp(n)$-action Φ_0 given by $\Phi_0(k,([z],x))=([kz],x)$. As in the previous case, we see that the action Φ_0 on M has no extended $Sp(n,\boldsymbol{C})$-action. So we assume $G=SL(n,\boldsymbol{H})$.

Let ϕ be a smooth $\boldsymbol{R}$-action on a homotopy 3-sphere hS^3. Then we see that the $\boldsymbol{R}$-action ϕ defines a smooth $SL(n,\boldsymbol{H})$-action Φ on M given by

$$\Phi(g,[z,x])=([gz],\phi(\log(\|gz\|/\|z\|),x)). \tag{4.8}$$

On the other hand, let an extended $SL(n,\boldsymbol{H})$-action Φ of Φ_0 be given. Then we see that

$$F(Sp(n-1),M)=F(L_{SL},M) \quad \text{or} \quad F(Sp(n-1),M)=F(L^*_{SL},M),$$

and the set $F(Sp(n-1),M)$ is naturally diffeomorphic to the homotopy 3-sphere hS^3. Now we assume $F(Sp(n-1),M)=F(L_{SL},M)=hS^3$. Then the factor group N_{SL}/L_{SL} acts on hS^3 via the action Φ, where N_{SL}/L_{SL} is isomorphic to the group of all non-zero quaternions. Moreover, we see that the maximal compact subgroup of N_{SL}/L_{SL} acts on hS^3 trivially. Then we get a smooth R-action ϕ on hS^3, and the action Φ on M satisfies the equation (4.8). In addition, if $F(Sp(n-1),M)=F(L^*_{SL},M)$, then we see that $F(Sp(n-1),M)=F(L_{SL},M)$ for the action Φ^*.

Combining these results, we obtain the proof of Theorem 4.2.

REMARK 4.9. For $G=SL(2,\boldsymbol{H})$ and $M=S^7$ or $S^7\times_{Sp(1)}S^3$, the same results are given in [1].

5. Smooth $\boldsymbol{R}$-actions on a 3-sphere. Here we consider a smooth $\boldsymbol{R}$-action ϕ on S^3 which commutes with the $Sp(1)$-action ρ. Since $F(U(1),S^3)=S^1$ is invariant under the $\boldsymbol{R}$-action ϕ, an $\boldsymbol{R}$-action θ on S^1 can be defined naturally. The $\boldsymbol{R}$-action θ on S^1 satisfies the following conditions.

(5.1) θ commutes with the involution J on S^1 defined by $J(x,y)=(x,-y)$.

(5.2) $\phi(t,x+z)=\rho(q,\theta(t,x+i|z|))$, for some $q\in Sp(1)$, such that $z=\mathrm{Ad}(q)(i|z|)$, where x is a real number and z is a pure quaternion.

PROPOSITION 5.3. *Let θ be a smooth $\boldsymbol{R}$-action on S^1 satisfying* (5.1). *Then there exists a smooth $\boldsymbol{R}$-action ϕ on S^3 satisfying the condition* (5.2).

PROOF. Since the restricted $U(1)$-action on S^1 of ρ is trivial, we see that an abstract $\boldsymbol{R}$-action ϕ on S^3 can be defined and commutes with the $Sp(1)$-action ρ.

Finally, we show the smoothness of ϕ. Set

$$\theta(t,x+iy)=f_1(t,x,y)+if_2(t,x,y).$$

Then we see that f_1 is a smooth even function and f_2 is a smooth odd function with respect to the variable y, by (5.1). On the other hand, for $z\neq 0$,

$$\phi(t,x+z)=f_1(t,x,|z|)+(z/|z|)f_2(t,x,|z|).$$

Thus the smoothness of ϕ except at $z=0$ is clear. Since $f_2(t, x, y)$ is a smooth odd function, we see that $f_2(t, x, y)/y$ is a smooth even function with respect to the variable y. Hence $f_1(t, x, |z|)$ and $f_2(t, x, |z|)/|z|$ are both smooth at $z=0$ (cf. [2, (7.15)]). Thus the smoothness of ϕ at $z=0$ is shown. q.e.d.

EXAMPLE 5.4. For each non-zero real number r, we can define an $\boldsymbol{R}$-action θ^r on S^1 by

$$\theta^r(t, x \oplus iy) = (e^{rt}x \oplus iy)/\| e^{rt}x \oplus iy \| ,$$

which satisfies (5.1). The fixed points are $1 \oplus 0$, $-1 \oplus 0$, $0 \oplus i$ and $0 \oplus (-i)$. Let us denote S^1 with the $\boldsymbol{R}$-action θ^r by $S^1(r)$. The involutions J and J_1, defined by $J_1(x \oplus iy) = (-x \oplus iy)$, are $\boldsymbol{R}$-equivariant diffeomorphisms of $S^1(r)$. Moreover, the diffeomorphism h, defined by $h(x \oplus iy) = y \oplus ix$, is an $\boldsymbol{R}$-equivariant diffeomorphism of $S^1(r)$ to $S^1(-r)$.

We see that there exists an $\boldsymbol{R}$-equivariant homeomorphism of $S^1(r)$ to $S^1(s)$ for any non-zero real numbers r, s (cf. [11, Section 2]). Now we show the following.

PROPOSITION 5.5. *If $|r| \neq |s|$, then there is no $\boldsymbol{R}$-equivariant C^1-diffeomorphism between $S^1(r)$ and $S^1(s)$.*

PROOF. We may assume $r>0$ and $s>0$. Let f be an $\boldsymbol{R}$-equivariant C^1-diffeomorphism of $S^1(r)$ to $S^1(s)$. Then we may assume that $f(x_0 \oplus iy_0) = x_0 \oplus iy_0$, for $x_0 = y_0 = 2^{-1/2}$, and $f(1 \oplus 0) = 1 \oplus 0$ (cf. [11, Section 2]). We see that

$$x' \oplus iy' = f(x \oplus iy) = f((e^{rt}x_0 \oplus iy_0)/\| e^{rt}x_0 \oplus iy_0 \|) = (e^{st}x_0 \oplus iy_0)/\| e^{st}x_0 \oplus iy_0 \| .$$

Then

$$dx'/dx = (dx'/dt)/(dx/dt) = e^{2rt}s(x_0^2 + e^{-2rt}y_0^2)^{3/2}/e^{2st}r(x_0^2 + e^{-2st}y_0^2)^{3/2} .$$

If $\lim_{t\to\infty}(dx'/dt)/(dx/dt)$ exists, then we see $r \leqq s$. Similarly, we see $s \leqq r$. q.e.d.

ACKNOWLEDGEMENT. The author wishes to express gratitude to Professor Fuichi Uchida of Yamagata University, who not only introduced the author to this subject but also gave helpful advice and encouragement. He also wishes to express gratitude to the referees for their helpful advice.

REFERENCES

[1] T. ABE, On smooth $SL(2, \boldsymbol{H})$ actions on simply connected closed 7-manifolds, Master's thesis, Yamagata Univ. (in Japanese), 1990.

[2] T. ASOH, On smooth $SL(2, \boldsymbol{C})$ actions on 3-manifolds, Osaka J. Math. 24 (1987), 271–298.

[3] G. E. BREDON, Introduction to Compact Transformation Groups, Pure and Applied Math. 46, Academic Press, New York, London, 1972.

[4] T. BRÖCKER AND T. TOM DIECK, Representations of compact Lie groups, Graduate Texts in Math. 98, Springer-Verlag, Berlin, Heiderberg, New York, 1985.

[5] A. NAKANISHI AND F. UCHIDA, Actions of symplectic groups on certain manifolds, Tôhoku Math. J. 36 (1984), 81–89.
[6] F. UCHIDA, Classification of compact transformation groups on cohomology complex projective spaces with codimension one orbits, Japan. J. Math. 3 (1977), 141–189.
[7] F. UCHIDA, Actions of special linear groups on a product manifold, Bull. of Yamagata Univ., Nat. Sci. 10 (1982), 227–233.
[8] F. UCHIDA, On the non-existence of smooth actions of complex symplectic group on cohomology quaternion projective spaces, Hokkaido Math. J. 12 (1983), 226–236.
[9] F. UCHIDA, Real analytic actions of complex symplectic groups and other classical Lie groups on spheres, J. Math. Soc. Japan 38 (1986), 661–677.
[10] F. UCHIDA, On a method to construct analytic actions of non-compact Lie groups on a sphere, Tôhoku Math. J. 39 (1987), 61–69.
[11] F. UCHIDA, Certain aspects of twisted linear actions II, Tôhoku Math. J. 41 (1989), 561–573.
[12] F. UCHIDA, Smooth $SL(n, C)$ actions on $(2n-1)$-manifolds, Hokkaido Math. J., to appear.

SURI GIKEN CO., LTD.
4–1–9 SHINJUKU, SHINJUKU-KU
TOKYO 160
JAPAN