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CURVATURE PINCHING THEOREM FOR MINIMAL SURFACES WITH CONSTANT KAEHLER ANGLE IN COMPLEX PROJECTIVE SPACES, II

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Abstract. We consider minimal surfaces with constant Kaehler angle in complex projective spaces. By using J-invariant higher order osculating spaces and pinched Gaussian curvature, we give characterization theorems for these minimal surfaces.

This is a continuation of our paper [12]. For each integer p with $0 \leq p \leq n$, it is known that there exists a full isometric minimal immersion $\varphi_{n,p}$: $S^2(K_{n,p}) \rightarrow P^n(C)$ of a 2-dimensional sphere of constant Gaussian curvature $K_{n,p} = 4\rho/(n+2p(n-p))$ into the complex projective n-space with the Fubini-Study metric of constant holomorphic sectional curvature 4ρ (cf. [1] and [2]). In [12], using J-invariant first order osculating spaces, we gave characterization theorems for immersions $\varphi_{n,p}$ for $p \leq 3$. The purpose of this paper is to generalize these to the case of $\varphi_{n,p}$ for $p \geq 4$ (cf. Section 4). To study the problem, we use J-invariant higher order osculating spaces to find some scalars defined globally on M, and calculate their Laplacians (cf. Section 6). In this paper, we use the same terminology and notation as in [12] unless otherwise stated.

4. *J*-invariant higher order osculating spaces and the main theorems. Let X be a Kaehler manifold of complex dimension n of constant holomorphic sectional curvature 4ρ and $x: M \rightarrow X$ an isometric immersion of an oriented 2-dimensional Riemannian manifold M into X. Let $C(s)$ be a smooth curve in M through a point $p = C(0)$ of M with parameter s proportional to the arc length. We denote by D^kC/ds^k the k-th covariant derivative along $C(s)$ in X. Let $T_p^{(k)}(C)$ be a subspace of $T_p(X)$ spanned by $\{DC/ds, JDC/ds, \ldots, D^kC/ds^k, JD^kC/ds^k\}$ at $s=0$, where J is the complex structure of X. $T_p^{(k)}$ is defined to be the subspace spanned by all $T_p^{(k)}(C)$ for curves C lying on M through p and is called the *J-invariant k-th osculating space* of M at p. We then have $T_p(M) \subset T_p^{(1)} \subset \cdots \subset T_p^{(m)} \subset T_p(X)$. Let $O_p^{(k+1)}$ be the orthogonal complement of $T_p^{(k)}$ in $T_p^{(k+1)}$ and N_p^m the orthogonal complement of $T_p^{(m)}$ in $T_p(X)$, so that we have $T_p^{(k+1)} = T_p^{(k)} + O_p^{(k+1)}$ and $T_p(X) = T_p^{(m)} + N_p^m$. We put $O_p^1 = T_p^{(1)}$. Note that we have

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$0 \leq \dim(O_p^k) \leq 4$ and, if $\dim(O_p^k)=0$ for some k, then we have $\dim(O_p^r)=0$ for all $r \geq k$.

A point $p \in M$ is called a *J-regular point of order m* if the J-invariant m-th osculating space $T_{p'}^{(m)}$ exists in a neighbourhood U of p and if each $O_{p'}^k$ is of dimension 4 for any $p' \in U$ and $k=1, 2, \ldots, m$. We denote $O^k = \bigcup_{p \in M} O_p^k$. We say that $x(M)$ is a *J-regular manifold* if each O^k is of constant rank on M for any k. Note that $\mathrm{rank}(O^1)=4$ if and only if x is neither holomorphic nor anti-holomorphic.

Let $p \in M$ be a J-regular point of order m. Then we have an orthogonal decomposition of $T_p(X)$ such that $T_p(X) = O_p^1 + \cdots + O_p^m + N_p^m$. Now we define a J-canonical basis in O_p^k as follows: Let $\{\tilde{e}_1, \tilde{e}_2\}$ be an orthonormal local frame of M and $\{\tilde{e}_{4k-3}, \tilde{e}_{4k-2}\}$ an orthonormal system of normal vector fields along M such that it belongs to O_p^k at p ($k \geq 2$). We put $\cos(\alpha) = \langle J\tilde{e}_1, \tilde{e}_2 \rangle$ and $\cos(\alpha_k) = \langle J\tilde{e}_{4k-3}, \tilde{e}_{4k-2} \rangle$. Then we have $\cos(\alpha) \neq \pm 1$, $\cos(\alpha_k) \neq \pm 1$. Hence we can define local normal vector fields $\tilde{e}_{4k-1}, \tilde{e}_{4k}$ along M such that $\{\tilde{e}_{4k-3}, \tilde{e}_{4k-2}, \tilde{e}_{4k-1}, \tilde{e}_{4k}\}$ at p is an orthonormal basis of O_p^k in the following way:

$$\tilde{e}_{4k-1} = -\cot(\alpha_k)\tilde{e}_{4k-3} - \mathrm{cosec}(\alpha_k)J\tilde{e}_{4k-2}\,,$$
$$\tilde{e}_{4k} = \mathrm{cosec}(\alpha_k)J\tilde{e}_{4k-3} - \cot(\alpha_k)\tilde{e}_{4k-2}\,.$$

By using them, we define local vector fields $e_{4k-3}, e_{4k-2}, e_{4k-1}$ and e_{4k}, $k=1, 2, \ldots, m$, in a neighbourhood of p as follows:

$$
\begin{aligned}
e_{4k-3} &= \cos\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k-3} + \sin\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k-1}\,, \\
e_{4k-2} &= \cos\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k-2} + \sin\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k}\,, \\
e_{4k-1} &= \sin\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k-3} - \cos\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k-1}\,, \\
e_{4k} &= -\sin\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k-2} + \cos\left(\frac{\alpha_k}{2}\right)\tilde{e}_{4k}\,,
\end{aligned}
\tag{4.1}
$$

where $\alpha_1 = \alpha$. Then $\{e_{4k-3}, e_{4k-2}, e_{4k-1}, e_{4k}\}$ at p is a J-canonical basis of O_p^k, that is, an orthonormal basis of O_p^k with $Je_{4k-3} = e_{4k-2}$ and $Je_{4k-1} = e_{4k}$. Let $\{e_{4m+1}, \ldots, e_n\}$ be an orthonormal system of normal vector fields along M such that it is a J-canonical basis of N_p^m at p.

We denote the coframe fields dual to these frames by $\{\tilde{\theta}_{4k-3}, \tilde{\theta}_{4k-2}, \tilde{\theta}_{4k-1}, \tilde{\theta}_{4k}\}$, $\{\theta_{4k-3}, \theta_{4k-2}, \theta_{4k-1}, \theta_{4k}\}$ and $\{\theta_{4m+1}, \ldots, \theta_n\}$, respectively. For $\alpha \geq 2m+1$, we put $\tilde{e}_{2\alpha-1} = e_{2\alpha-1}$ and $\tilde{e}_{2\alpha} = e_{2\alpha}$, so that we have $\tilde{\theta}_{2\alpha-1} = \theta_{2\alpha-1}$ and $\tilde{\theta}_{2\alpha} = \theta_{2\alpha}$. If we put $\omega_\alpha = \theta_{2\alpha-1} + i\theta_{2\alpha}$ where $i^2 = -1$, then $\{\omega_\alpha\}$ is a local field of unitary coframes on X and we have, by (4.1):

$$\tilde{\theta}_{4k-3}+i\tilde{\theta}_{4k-2}=\cos\left(\frac{\alpha_k}{2}\right)\omega_{2k-1}+\sin\left(\frac{\alpha_k}{2}\right)\bar{\omega}_{2k}\,,$$

$$\tilde{\theta}_{4k-1}+i\tilde{\theta}_{4k}=\sin\left(\frac{\alpha_k}{2}\right)\omega_{2k-1}-\cos\left(\frac{\alpha_k}{2}\right)\bar{\omega}_{2k}\,,\qquad (k=1,\dots,m)\,,\tag{4.2}$$

$$\tilde{\theta}_{2\alpha-1}+i\tilde{\theta}_{2\alpha}=\theta_{2\alpha-1}+i\theta_{2\alpha}=\omega_\alpha\,,\qquad (\alpha=2m+1,\dots,n)\,.$$

Now we introduce inductively the higher order fundamental forms $\{h_{\lambda_k i_1\cdots i_k}\}$ of M in X. Let $\{\tilde{\theta}_{AB}\}$ be the Riemannian connection form of X with respect to the canonical 1-form $\{\tilde{\theta}_A\}$, and $\{\omega_{\alpha\beta}\}$ the unitary connection form of X with respect to $\{\omega_\alpha\}$. We shall use the following ranges of indices:

$$\begin{aligned}&1\le A, B,\dots\le 2n\,,\quad 1\le i,j,\dots\le 2\,,\quad 3\le\lambda_0,\mu_0,\dots\le 4\,,\\ &4k-3\le\lambda_k,\mu_k,\dots\le 4k\,,\quad 4k+1\le s_k, t_k,\dots\le 2n\,,\\ &2k+1\le\alpha_k,\beta_k,\dots\le n\,,\qquad \text{for}\quad k=1,2,\dots,m\,,\\ &4m+1\le\alpha,\beta,\dots\le 2n\,,\quad 2m+1\le\lambda,\mu,\dots\le n\,.\end{aligned}\tag{4.3}$$

We denote the restriction of forms on X to M by the same letters. We then have

$$\begin{aligned}&\tilde{\theta}_{\lambda_0}=\tilde{\theta}_{\lambda_k}=0\,,\qquad (k=2)\,,\\ &\tilde{\theta}_{\lambda_k\lambda_{l+2}}=0\,,\qquad k=1,2,\dots,m-2\,;\quad l=k,\dots,m-2\,,\\ &\tilde{\theta}_{\lambda_k\alpha}=0\,,\qquad k=1,\dots,m-1\,.\end{aligned}\tag{4.4}$$

By the exterior differentiation of (4.4) and the Riemannian structure equations, we get

$$\begin{aligned}&\sum_i\tilde{\theta}_i\wedge\tilde{\theta}_{i\lambda_0}=\sum_i\tilde{\theta}_i\wedge\tilde{\theta}_{i\lambda_2}=0\,,\\ &\sum_{\lambda_{k+1}}\tilde{\theta}_{\lambda_k\lambda_{k+1}}\wedge\tilde{\theta}_{\lambda_{k+1}\lambda_{k+2}}=0\,,\qquad (k=2,\dots,m-2)\,,\\ &\sum_{\lambda_m}\tilde{\theta}_{\lambda_{m-1}\lambda_m}\wedge\tilde{\theta}_{\lambda_m\alpha}=0\,.\end{aligned}\tag{4.5}$$

From these we get inductively the quantities $h_{\lambda_k i_1\cdots i_k}$ in the following way:

$$\begin{aligned}&\tilde{\theta}_{i\lambda_0}=\sum_j h_{\lambda_0 ij}\tilde{\theta}_j\,,\qquad \tilde{\theta}_{i\lambda_2}=\sum_j h_{\lambda_2 ij}\tilde{\theta}_j\,,\\ &\sum_{\lambda_k}h_{\lambda_k i_1\cdots i_k}\tilde{\theta}_{\lambda_k\lambda_{k+1}}=\sum_{i_{k+1}}h_{\lambda_{k+1}i_1\cdots i_k i_{k+1}}\tilde{\theta}_{i_{k+1}}\,,\\ &\sum_{\lambda_m}h_{\lambda_m i_1\cdots i_m}\tilde{\theta}_{\lambda_m\alpha}=\sum_{i_{m+1}}h_{\alpha i_1\cdots i_{m+1}}\tilde{\theta}_{i_{m+1}}\,.\end{aligned}\tag{4.6}$$

Then they have the following properties:

$$(4.7)\qquad \begin{aligned} &(1)\quad h_{\lambda_k i_1 \cdots i_k} \text{ are symmetric in the set of indices } i_1, i_2, \dots, i_k, \\ &(2)\quad \sum_i h_{\lambda_k i_1 \cdots i \cdots i \cdots i_k} = 0\,, \\ &(3)\quad \langle \tilde{e}_{\lambda_k}, D^k x\rangle = \sum h_{\lambda_k i_1 \cdots i_k} \tilde{\theta}_{i_1} \cdots \tilde{\theta}_{i_k}\,. \end{aligned}$$

The vector-valued symmetric k-form $\sum h_{\lambda_k i_1 \cdots i_k} \tilde{\theta}_{i_1} \cdots \tilde{\theta}_{i_k} \tilde{e}_{\lambda_k}$ is called the *k-th fundamental form* of M in X.

We introduce the following notation for brevity: $1[k] = 1 \cdots 1$ (k times) and put $V_1^{(k)} = \sum_{\lambda_k} h_{\lambda_k 1[k-1]1} \tilde{e}_{\lambda_k}$, $V_2^{(k)} = \sum_{\lambda_k} h_{\lambda_k 1[k-1]2} \tilde{e}_{\lambda_k}$, which are elements of O_p^k at p for $k = 2, 3, \dots, m$. Define also $V_1^{(m+1)} = \sum_\alpha h_{\alpha 1[m]1} \tilde{e}_\alpha$ and $V_2^{(m+1)} = \sum_\alpha h_{\alpha 1[m]2} \tilde{e}_\alpha$, which are called the *$(m+1)$-th normal vectors* at a J-regular point of order m.

Now we can state the main theorems in this paper.

THEOREM 4.1. *Let X be a Kaehler manifold of complex dimension n of positive constant holomorphic sectional curvature 4ρ and M a complete connected Riemannian 2-manifold. Let $x: M \to X$ be a full isometric minimal immersion with constant Kaehler angle α, which is neither holomorphic, anti-holomorphic nor totally real. Suppose there exists an integer m such that each point of M is J-regular of order $(m+1)$ and that the Gaussian curvature K of M satisfies $K \geq 2\{1 - (2m+3)\cos(\alpha)\}\rho/(m+1)(m+2) > 0$ on M. Then K is constant on M. Moreover, x is locally congruent to $\varphi_{n,m+1}$.*

THEOREM 4.2. *Let $x: M \to X$ be as in Theorem 4.1, and $s = [n/2 - 1] - 1$ ($[a]$ means the integer part of a). Further assume that M is a J-regular manifold. If K satisfies $K \geq 2\{1 - (2s+3)\cos(\alpha)\}\rho/(s+1)(s+2) \geq 0$, then K is constant on M so that x is locally congruent to either $\varphi_{n,1}, \dots, \varphi_{n,s}$ or $\varphi_{n,s+1}$.*

This generalizes Theorem 3.10 in [12].

5. A J-regular point of order m. In this section, adopting the normalized k-th normal vectors as a basis of each O_p^k for $k = 2, \dots, m$, we calculate the $(m+1)$-th fundamental forms and the $(m+1)$-th normal vectors in terms of some complex-valued smooth functions defined locally on M and study their properties. In [12], we have treated the case $m = 2$. Let M be a complete connected 2-dimensional Riemannian manifold such that the Gaussian curvature K of M satisfies $K \geq \delta > 0$ for some positive number δ and $x: M \to X$ an isometric minimal immersion with constant Kaehler angle α. We assume that every point p of M is J-regular of order m (≥ 3) and that the k-th normal vectors $V_1^{(k)}$ and $V_2^{(k)}$ are perpendicular to each other and of the same non-zero length for $k = 3, \dots, m$. Normalizing these vectors, we adopt them as a basis of O_p^k, so that we have $\tilde{e}_{4k-3} = V_1^{(k)}/\|V_1^{(k)}\|$, $\tilde{e}_{4k-2} = V_2^{(k)}/\|V_2^{(k)}\|$ and $\cos(\alpha_k) = \langle J\tilde{e}_{4k-3}, \tilde{e}_{4k-2}\rangle \neq \pm 1$ on M. Then with respect to these frames we assume

$$
(5.1)\qquad \begin{aligned}
&h_{4k-3,1[k-1]1}=-h_{4k-2,1[k-1]2}\,,\\
&h_{4k-3,1[k-1]2}=h_{4k-2,1[k-1]1}=h_{t_k,1[k-1]1}=h_{t_k,1[k-1]2}=0\,, \qquad (t_k\geq 4k-1)\,.
\end{aligned}
$$

We put

$$
(5.2)\qquad \begin{aligned}
&c_{2k-1,2[k-2]}=-\cos\left(\frac{\alpha_k}{2}\right)h_{4k-3,1[k-1]1}\,,\\
&a_{2k,1[k-2]}=-\sin\left(\frac{\alpha_k}{2}\right)h_{4k-3,1[k-1]1}\,,\\
&c_{2k,2[k-2]}=a_{2k-1,1[k-2]}=c_{\lambda_k,2[k-2]}=a_{\lambda_k,1[k-2]}=0\,, \qquad (\lambda_k\geq 2k+1)\,,
\end{aligned}
$$

where $c_{2k-1,2[k-1]}$ and others are real-valued smooth functions locally defined on M. We assume that they satisfy the following:

$$
(5.3)\qquad \begin{aligned}
&c_{2k-3,2[k-3]}\omega_{2k-1,2k-3}=-c_{2k-1,2[k-2]}\bar{\phi}\,,\\
&a_{2k-2,1[k-3]}\omega_{2k,2k-2}=-a_{2k,1[k-2]}\phi\,,\\
&\omega_{2k,2k-3}=\omega_{2k-1,2k-2}=\omega_{\lambda_k,2k-3}=\omega_{\lambda_k,2k-2}=0 \qquad (\lambda_k\geq 2k+1)\,,\\
&dc_{2k-1,2[k-2]}+ikc_{2k-1,2[k-2]}\tilde{\theta}_{12}-c_{2k-1,2[k-2]}\omega_{2k-1,2k-1}=c_{2k-1,2[k-1]}\bar{\phi}\,,\\
&da_{2k,1[k-2]}-ika_{2k,1[k-2]}\tilde{\theta}_{12}-a_{2k,1[k-2]}\omega_{2k,2k}=a_{2k,1[k-1]}\phi\,,\\
&c_{2k-1,2[k-2]}\omega_{2k,2k-1}=-c_{2k,2[k-1]}\bar{\phi}\,,\\
&a_{2k,1[k-2]}\omega_{2k-1,2k}=-a_{2k-1,1[k-1]}\phi\,,\\
&c_{2k-1,2[k-2]}\omega_{\lambda_k,2k-1}=-c_{\lambda_k,2[k-1]}\bar{\phi}\,,\\
&a_{2k,1[k-2]}\omega_{\lambda_k,2k}=-a_{\lambda_k,1[k-1]}\phi\,, \qquad (\lambda_k\geq 2k+1)\,, \qquad \text{for} \quad k=3,\dots,m\,.
\end{aligned}
$$

By (5.3), we have

$$
(5.4)\qquad \begin{aligned}
\Delta(c_{2k-1,2[k-2]})^2&=2kK(c_{2k-1,2[k-2]})^2+4\{c^2_{2k-1,2[k-1]}+c^2_{2k+1,2[k-1]}\}\\
&\quad+4\rho c^2_{2k-1,2[k-2]}\cos(\alpha)-4(c_{2k-1,2[k-2]})^4/(c_{2k-3,2[k-3]})^2\,,\\
&\qquad\qquad \text{for} \quad k=3,\dots,m-1\,,\\
\Delta(c_{2m-1,2[m-2]})^2&=2mK(c_{2m-1,2[m-2]})^2+4\sum_{\lambda\geq 2m-1}|c_{\lambda,2[m-1]}|^2\\
&\quad+4\rho(c_{2m-1,2[m-2]})^2\cos(\alpha)-4(c_{2m-1,2[m-2]})^4/(c_{2m-3,2[m-3]})^2\,,\\
\Delta(a_{2m,1[m-2]})^2&=2mK(a_{2m,1[m-2]})^2+4\sum_{\lambda\geq 2m-1}|a_{\lambda,1[m-1]}|^2\\
&\quad-4\rho(a_{2m,1[m-2]})^2\cos(\alpha)-4(a_{2m,1[m-2]})^4/(a_{2m-2,1[m-3]})^2\,.
\end{aligned}
$$

Now, we calculate the $(m+1)$-th fundamental forms and the $(m+1)$-th normal vectors. Using the third equality in (4.6) and (5.1), we have, for $\lambda \geq 2m+1$,

$$
(5.5)\qquad
\begin{aligned}
&h_{4m-3,1[m]}\tilde{\theta}_{4m-3,2\lambda-1}=h_{2\lambda-1,1[m]1}\tilde{\theta}_1+h_{2\lambda-1,1[m]2}\tilde{\theta}_2\,,\\
&h_{4m-3,1[m]}\tilde{\theta}_{4m-3,2\lambda}=h_{2\lambda,1[m]1}\tilde{\theta}_1+h_{2\lambda,1[m]2}\tilde{\theta}_2\,.
\end{aligned}
$$

By taking the exterior derivatives of (4.2) and using the structure equation of X, we get, for $k=1, 2, \dots, m$:

$$
(5.6)\qquad
\begin{aligned}
&\tilde{\theta}_{4k-3,2\lambda-1}+i\tilde{\theta}_{4k-2,2\lambda-1}=\cos\left(\frac{\alpha_k}{2}\right)\omega_{2k-1,\lambda}+\sin\left(\frac{\alpha_k}{2}\right)\bar{\omega}_{2k,\lambda}\,,\\
&\tilde{\theta}_{4k-3,2\lambda}+i\tilde{\theta}_{4k-2,2\lambda}=i\left\{\cos\left(\frac{\alpha_k}{2}\right)\omega_{2k-1,\lambda}-\sin\left(\frac{\alpha_k}{2}\right)\bar{\omega}_{2k,\lambda}\right\},\\
&\tilde{\theta}_{4k-1,2\lambda-1}+i\tilde{\theta}_{4k,2\lambda-1}=\sin\left(\frac{\alpha_k}{2}\right)\omega_{2k-1,\lambda}+\cos\left(\frac{\alpha_k}{2}\right)\bar{\omega}_{2k,\lambda}\,,\\
&\tilde{\theta}_{4k-1,2\lambda}+i\tilde{\theta}_{4k,2\lambda}=i\left\{\sin\left(\frac{\alpha_k}{2}\right)\omega_{2k-1,\lambda}+\cos\left(\frac{\alpha_k}{2}\right)\bar{\omega}_{2k,\lambda}\right\}.
\end{aligned}
$$

Substituting (5.1), (5.2), the eighth and the ninth equalities in (5.3) and (5.6) into (5.5), we have

$$
(5.7)\qquad
\begin{aligned}
&h_{2\lambda-1,1[m]1}=-\frac{1}{2}(a_{\lambda,1[m-1]}+\bar{a}_{\lambda,1[m-1]}+c_{\lambda,2[m-1]}+\bar{c}_{\lambda,2[m-1]})\,,\\
&h_{2\lambda-1,1[m]2}=-\frac{i}{2}(a_{\lambda,1[m-1]}-\bar{a}_{\lambda,1[m-1]}-c_{\lambda,2[m-1]}+\bar{c}_{\lambda,2[m-1]})\,,\\
&h_{2\lambda,1[m]1}=\frac{i}{2}(a_{\lambda,1[m-1]}-\bar{a}_{\lambda,1[m-1]}+c_{\lambda,2[m-1]}-\bar{c}_{\lambda,2[m-1]})\,,\\
&h_{2\lambda,1[m]2}=-\frac{1}{2}(a_{\lambda,1[m-1]}+\bar{a}_{\lambda,1[m-1]}-c_{\lambda,2[m-1]}-\bar{c}_{\lambda,2[m-1]})\,.
\end{aligned}
$$

By taking the exterior derivatives of the sixth through the ninth equalities in (5.3), we have

$$
\begin{aligned}
&dc_{2m,2[m-1]}+(m+1)ic_{2m,2[m-1]}\tilde{\theta}_{12}-c_{2m,2[m-1]}\omega_{2m,2m}=c_{2m,2[m]}\bar{\phi}\,,\\
&dc_{\lambda,2[m-1]}+(m+1)ic_{\lambda,2[m-1]}\tilde{\theta}_{12}-\sum_{\mu}c_{\mu,2[m-1]}\omega_{\lambda\mu}=c_{\lambda,2[m-1]1}\phi+c_{\lambda,2[m]}\bar{\phi}\\
&\qquad\text{with}\quad c_{\lambda,2[m-1]1}=-a_{\lambda,1[m-1]}c_{2m,2[m-1]}/a_{2m,1[m-2]}\,,
\end{aligned}
$$

$$
(5.8)\qquad da_{2m-1,1[m-1]}-(m+1)ia_{2m-1,1[m-1]}\tilde{\theta}_{12}-a_{2m-1,1[m-1]}\omega_{2m-1,2m-1}=a_{2m-1,1[m]}\phi\,,
$$

$$da_{\lambda,1[m-1]}-(m+1)ia_{\lambda,1[m-1]}\tilde{\theta}_{12}-\sum_{\mu}a_{\mu,1[m-1]}\omega_{\lambda\mu}=a_{\lambda,1[m]}\phi+a_{\lambda,1[m-1]2}\bar{\phi}$$

$$\text{with}\quad a_{\lambda,1[m-1]2}=-c_{\lambda,2[m-1]}a_{2m-1,1[m-1]}/c_{2m-1,2[m-2]}\,,$$

where $c_{2m,2[m]}$, $c_{\lambda,2[m]}$, $a_{2m-1,1[m]}$ and $a_{\lambda,1[m]}$ are complex-valued smooth functions defined locally on M.

PROPOSITION 5.1. *Let M be a complete connected 2-dimensional Riemannian manifold such that the Gaussian curvature K of M satisfies $K\geq\delta>0$ for some positive number δ. Let $x\colon M\to X$ be an isometric minimal immersion with constant Kaehler angle α, which is neither holomorphic, anti-holomorphic nor totally real. We assume that each point of M is J-regular of order m, and all formulas in Section 5 are valid on M. Then we have $|c_{2m,2[m-1]}|^2=0$.*

PROOF. Using the first equality in (5.8), we have

$$d|c_{2m,2[m-1]}|^2=c_{2m,2[m-1]}\bar{c}_{2m,2[m]}\phi+\bar{c}_{2m,2[m-1]}c_{2m,2[m]}\bar{\phi}\,,$$

$$\begin{aligned}\Delta|c_{2m,2[m-1]}|^2&=2(m+1)K|c_{2m,2[m-1]}|^2+4|c_{2m,2[m]}|^2\\&\quad+4|c_{2m,2[m-1]}|^2\Big\{a^2_{2m,1[m-2]}/a^2_{2m-2,1[m-3]}-|c_{2m,2[m-1]}|^2/c^2_{2m-1,2[m-2]}\\&\quad-\sum_{\mu\geq 2m+1}|a_{\mu,1[m-1]}|^2/a^2_{2m,1[m-2]}+\rho\cos(\alpha)\Big\}\,.\end{aligned}$$

Combining the third equality in (5.4) with the above equality, we have

$$\begin{aligned}\Delta(a^2_{2m,1[m-2]}|c_{2m,2[m-1]}|^2)&=2(2m+1)Ka^2_{2m,1[m-2]}|c_{2m,2[m-1]}|^2\\&\quad+4|a_{2m,1[m-2]}c_{2m,2[m]}+a_{2m,1[m-1]}c_{2m,2[m-1]}|^2\,.\end{aligned}$$

By assumption, we see that M is compact and $a_{2m,1[m-2]}\neq 0$ on M. Hence, using the above equality, we have $c_{2m,2[m-1]}=0$. q.e.d.

The $(m+1)$-th normal vectors $V_1^{(m+1)}$ and $V_2^{(m+1)}$ of N_p^m at p are given as follows: For $\lambda\geq 2m+1$

$$V_1^{(m+1)}=\sum_{\lambda}(h_{2\lambda-1,1[m]1}e_{2\lambda-1}+h_{2\lambda,1[m]1}e_{2\lambda})\,,$$

$$V_2^{(m+1)}=\sum_{\lambda}(h_{2\lambda-1,1[m]2}e_{2\lambda-1}+h_{2\lambda,1[m]2}e_{2\lambda})\,.$$

We put $\Omega_{(m+1)}=\{p\in M;\,V_1^{(m+1)}(p)=0$ or $V_2^{(m+1)}(p)=0\}$ and $\cos(\alpha_{m+1})=\langle JV_1^{(m+1)}/\|JV_1^{(m+1)}\|,\,V_2^{(m+1)}/\|V_2^{(m+1)}\|\rangle$. Then, using (5.7), we have $\sum_\lambda(a_{\lambda,1[m-1]}-c_{\lambda,2[m-1]})^2=0$ or $\sum_\lambda(a_{\lambda,1[m-1]}+c_{\lambda,2[m-1]})^2=0$ at $p\in\Omega_{(m+1)}$ and $\cos(\alpha_{m+1})=\sum_\lambda\{|a_{\lambda,1[m-1]}|^2-|c_{\lambda,2[m-1]}|^2\}/\{|a_{\lambda,1[m-1]}|^2+|c_{\lambda,2[m-1]}|^2\}$. Also, using the third equality in (4.7), we see that $O_p^{(m+1)}$ is spanned by $V_1^{(m+1)}$, $V_2^{(m+1)}$, $JV_1^{(m+1)}$ and $JV_2^{(m+1)}$ at p. Hence, if we assume that each point of M is J-regular of order $(m+1)$, then $\Omega_{(m+1)}=\varnothing$ and $\cos(\alpha_{(m+1)})\neq 0,\ \pm 1$.

Next we define $H^{(m+1)}_{2\lambda-1}$ and $H^{(m+1)}_{2\lambda}$ by

$$V_1^{(m+1)}+iV_2^{(m+1)}=\sum_\lambda (H^{(m+1)}_{2\lambda-1}e_{2\lambda-1}+H^{(m+1)}_{2\lambda}e_{2\lambda})$$

and we put

$$H^{(m+1)}=\sum_\lambda \{(H^{(m+1)}_{2\lambda-1})^2+(H^{(m+1)}_{2\lambda})^2\}\,.$$

Using (5.7), we have $H^{(m+1)}=4\sum_\lambda \bar{a}_{\lambda,1[m-1]}c_{\lambda,2[m-1]}$. Note that $|H^{(m+1)}|^2$ is a globally defined smooth function on M. The geometric meaning of $|H^{(m+1)}|^2$ follows from the identity $|H^{(m+1)}|^2=(\|V_1^{(m+1)}\|^2-\|V_2^{(m+1)}\|^2)^2+4\langle V_1^{(m+1)}, V_2^{(m+1)}\rangle^2$.

PROPOSITION 5.2. *In addition to the assumption in Proposition* 5.1, *we assume that each point of M is J-regular of order* $(m+1)$. *Then we have* $H^{(m+1)}=0$ *on* M.

PROOF. Using Proposition 5.1 and (5.8), we have

$$\begin{aligned}
&dH^{(m+1)}+2(m+1)iH^{(m+1)}\tilde{\theta}_{12}=4\sum_{\lambda\geq 2m+1}(\bar{a}_{\lambda,1[m]}c_{\lambda,2[m-1]}+\bar{a}_{\lambda,1[m-1]}c_{\lambda,2[m]})\bar{\phi}\,,\\
(5.9)\qquad &\\
&\Delta|H^{(m+1)}|^2=2\left\{2(m+1)K|H^{(m+1)}|^2+2\left|\sum_\lambda(\bar{a}_{\lambda,1[m]}c_{\lambda,2[m-1]}+\bar{a}_{\lambda,1[m-1]}c_{\lambda,2[m]})\right|^2\right\},
\end{aligned}$$

from which we have $H^{(m+1)}=0$. q.e.d.

LEMMA 5.3.

$$\begin{aligned}
\Delta\Big(\sum_\lambda|c_{\lambda,2[m-1]}|^2\Big)&=2(m+1)K\Big(\sum_\lambda|c_{\lambda,2[m-1]}|^2\Big)+4\sum_\lambda|c_{\lambda,2[m]}|^2\\
&\quad-4\Big(\sum_\lambda|c_{\lambda,2[m-1]}|^2\Big)^2\Big/c^2_{2m-1,2[m-1]}+4\rho\cos(\alpha)\Big(\sum_\lambda|c_{\lambda,2[m-1]}|^2\Big).\\
\Delta\Big(\sum_\lambda|a_{\lambda,1[m-1]}|^2\Big)&=2(m+1)K\Big(\sum_\lambda|a_{\lambda,1[m-1]}|^2\Big)+4\sum_\lambda|a_{\lambda,1[m]}|^2\\
&\quad-4\Big(\sum_\lambda|a_{\lambda,1[m-1]}|^2\Big)^2\Big/a^2_{2m,1[m-1]}-4\rho\cos(\alpha)\Big(\sum_\lambda|a_{\lambda,1[m-1]}|^2\Big).
\end{aligned}$$

PROOF. Using Proposition 5.1 and the second equality in (5.8), we have

$$d\Big(\sum_\lambda|c_{\lambda,2[m-1]}|^2\Big)=\sum_\lambda\{c_{\lambda,2[m-1]}\bar{c}_{\lambda,2[m]}\phi+\bar{c}_{\lambda,2[m-1]}c_{\lambda,2[m]}\bar{\phi}\}\,,$$

which implies

$$d^c\left(\sum_\lambda |c_{\lambda,2[m-1]}|^2\right)=i\sum_\lambda\{-c_{\lambda,2[m-1]}\bar{c}_{\lambda,2[m]}\phi+\bar{c}_{\lambda,2[m-1]}c_{\lambda,2[m]}\bar{\phi}\}$$

$$=i\sum_\lambda\{-c_{\lambda,2[m-1]}d\bar{c}_{\lambda,2[m-1]}+\bar{c}_{\lambda,2[m-1]}dc_{\lambda,2[m-1]}$$

$$+2i(m+1)|c_{\lambda,2[m-1]}|^2\tilde{\theta}_{12}-2\bar{c}_{\lambda,2[m-1]}c_{\mu,2[m-1]}\omega_{\lambda\mu}\}\,.$$

By a direct calculation of $dd^c(\sum|c_{\lambda,2[m-1]}|^2)$ we get the first formula of Lemma 5.3. In a similar way, by the fourth equality in (5.8), we can prove the formula for $\Delta(\sum|a_{\lambda,1[m-1]}|^2)$. q.e.d.

6. Proofs of Theorems. We assume that $p\in M$ is a J-regular point of order $(m+1)$. By Proposition 5.2, we have that $V_1^{(m+1)}$ and $V_2^{(m+1)}$ are perpendicular to each other and of the same length. Normalizing these vectors we adopt them as a basis of $O_{p'}^{(m+1)}$ in a neighbourhood of p, so that we have $\tilde{e}_{4m+1}=V_1^{(m+1)}/\|V_1^{(m+1)}\|$ and $\tilde{e}_{4m+2}=V_2^{(m+1)}/\|V_2^{(m+1)}\|$ and $\cos(\alpha_{m+1})=\langle J\tilde{e}_{4m+1},\tilde{e}_{4m+2}\rangle\neq\pm 1$. With respect to these new frames, we have

$$\begin{aligned}&h_{4m+1,1[m]1}=-h_{4m+2,1[m]2}(\neq 0)\,,\\ &h_{4m+1,1[m]2}=h_{4m+2,1[m]1}=h_{\lambda,1[m]1}=h_{\lambda,1[m]2}=0\,,\qquad(\lambda\geq 4m+3)\,.\end{aligned}\tag{6.1}$$

Substituting (6.1) into (5.5), we have

$$\begin{aligned}&h_{4m-3,1[m]}(\tilde{\theta}_{4m-3,4m+1}+i\tilde{\theta}_{4m-2,4m+1})=h_{4m+1,1[m]1}\phi\,,\\ &h_{4m-3,1[m]}(\tilde{\theta}_{4m-3,4m+2}+i\tilde{\theta}_{4m-2,4m+2})=-h_{4m+2,1[m]2}\phi\,,\\ &h_{4m-3,1[m]}(\tilde{\theta}_{4m-3,4m+3}+i\tilde{\theta}_{4m-2,4m+3})=0\,,\\ &h_{4m-3,1[m]}(\tilde{\theta}_{4m-3,4m+4}+i\tilde{\theta}_{4m-2,4m+4})=0\,,\\ &h_{4m-3,1[m]}(\tilde{\theta}_{4m-3,2\alpha-1}+i\tilde{\theta}_{4m-2,2\alpha-1})=0\,,\\ &h_{4m-3,1[m]}(\tilde{\theta}_{4m-3,2\alpha}+i\tilde{\theta}_{4m-2,2\alpha})=0\,,\qquad(\alpha\geq 2m+3)\,.\end{aligned}\tag{6.2}$$

On the other hand, by taking the exterior derivatives of (4.2) for $k=1,2,\dots,(m+1)$ and using the structure equations for X, we have, for $k,l=1,2,\dots,(m+1)$,

$$\tilde{\theta}_{4k-3,4l-3}+i\tilde{\theta}_{4k-2,4l-3}$$

$$=\cos\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l-1}+\cos\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l}$$

$$+\sin\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l-1}+\sin\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l}\,,$$

$$\tilde{\theta}_{4k-3,4l-2}+i\tilde{\theta}_{4k-2,4l-2}$$
$$=i\left\{\cos\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l-1}-\cos\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l}\right.$$
$$\left.-\sin\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l-1}+\sin\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l}\right\},$$

(6.3)
$$\tilde{\theta}_{4k-3,4l-1}+i\tilde{\theta}_{4k-2,4l-1}$$
$$=\cos\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l-1}-\cos\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l}$$
$$+\sin\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l-1}-\sin\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l},$$
$$\tilde{\theta}_{4k-3,4l}+i\tilde{\theta}_{4k-2,4l}$$
$$=i\left\{\cos\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l-1}+\cos\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\omega_{2k-1,2l}\right.$$
$$\left.-\sin\left(\frac{\alpha_k}{2}\right)\sin\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l-1}-\sin\left(\frac{\alpha_k}{2}\right)\cos\left(\frac{\alpha_l}{2}\right)\bar{\omega}_{2k,2l}\right\}.$$

In the first and second equalities in (2.2) and the eighth and ninth equalities in (5.3) we put $k=m$. Then we have $h_{4m-3,1[m]}=-\sec(\alpha_m/2)c_{2m-1,2[m-2]}=-\operatorname{cosec}(\alpha_m/2)a_{2m,1[m-2]}$, $c_{2m-1,2[m-2]}\omega_{2m-1,\lambda}=\bar{c}_{\lambda,2[m-1]}\phi$ and $a_{2m,1[m-2]}\omega_{2m,\lambda}=\bar{a}_{\lambda,1[m-1]}\bar{\phi}$ for $\lambda\geq 2m+1$, respectively. Substituting these equalities and (6.3) into (6.2), we get

$$\cos\left(\frac{\alpha_{m+1}}{2}\right)(\bar{c}_{2m+1,2[m-1]}+a_{2m+1,1[m-1]})$$
$$+\sin\left(\frac{\alpha_{m+1}}{2}\right)(\bar{c}_{2m+2,2[m-1]}+a_{2m+2,1[m-1]})=-h_{4m+1,1[m]1},$$
$$\cos\left(\frac{\alpha_{m+1}}{2}\right)(\bar{c}_{2m+1,2[m-1]}-a_{2m+1,1[m-1]})$$
$$-\sin\left(\frac{\alpha_{m+1}}{2}\right)(\bar{c}_{2m+2,2[m-1]}-a_{2m+2,1[m-1]})=h_{4m+2,1[m]2},$$

(6.4)
$$-\sin\left(\frac{\alpha_{m+1}}{2}\right)(\bar{c}_{2m+1,2[m-1]}+a_{2m+1,1[m-1]})$$
$$+\cos\left(\frac{\alpha_{m+1}}{2}\right)(\bar{c}_{2m+1,2[m-1]}+a_{2m+2,1[m-1]})=0,$$

$$\sin\left(\frac{\alpha_{m+1}}{2}\right)(-\bar{c}_{2m+1,2[m-1]}+a_{2m+1,1[m-1]})$$
$$+\cos\left(\frac{\alpha_{m+1}}{2}\right)(-\bar{c}_{2m+2,2[m-1]}+a_{2m+2,1[m-1]})=0\,,$$
$$\bar{c}_{\lambda,2[m-1]}-a_{\lambda,1[m-1]}=0\,,$$
$$\bar{c}_{\lambda,2[m-1]}+a_{\lambda,1[m-1]}=0\,.$$

Solving the above equations, we have

$$\bar{c}_{2m+1,2[m-1]}=\cot\left(\frac{\alpha_{m+1}}{2}\right)a_{2m+2,1[m-1]}\,,$$
$$a_{2m+1,1[m-1]}=\cot\left(\frac{\alpha_{m+1}}{2}\right)\bar{c}_{2m+2,2[m-1]}\,,$$
$$\bar{c}_{\lambda,2[m-1]}=a_{\lambda,1[m-1]}=0\,.$$

Moreover, since $H^{(m+1)}=0$, we see that $c_{2m+1,2[m-1]}$ is real-valued and $c_{2m+2,2[m-1]}=0$. Summarizing these results, we have

$$\begin{aligned}
&h_{4m+1,1[m]1}=-h_{4m+3,1[m]2}=-\sec\left(\frac{\alpha_{m+1}}{2}\right)c_{2m+1,2[m-1]}\,,\\
&h_{4m+1,1[m]2}=h_{4m+2,1[m]1}=h_{t,1[m]1}=h_{t,1[m]2}=0\,,\qquad (t\geq 4m+3)\,,\\
&c_{2m+1,2[m-1]}=\cot\left(\frac{\alpha_{m+1}}{2}\right)a_{2m+1,1[m-1]}\,,\\
&c_{2m+2,2[m-1]}=a_{2m+1,1[m-1]}=c_{\lambda,2[m-1]}=a_{\lambda,1[m-1]}=0\,,\qquad (\lambda\geq 2m+3)\,.
\end{aligned}\tag{6.5}$$

Now substituting (6.5) into the eighth and ninth equalities in (5.3), we have

$$\begin{aligned}
&c_{2m-1,2[m-2]}\omega_{2m+1,2m-1}=-c_{2m+1,2[m-1]}\bar{\phi}\,,\\
&a_{2m,1[m-2]}\omega_{2m+2,2m}=-a_{2m+2,1[m-1]}\phi\,,\\
&\omega_{2m+2,2m-1}=\omega_{2m+1,2m}=\omega_{\alpha,2m-1}=\omega_{\alpha,2m}=0\,,\qquad (\alpha\geq 2m+3)\,.
\end{aligned}\tag{6.6}$$

Moreover, by (5.8), we have

$$\begin{aligned}
&dc_{2m+1,2[m-1]}+i(m+1)c_{2m+1,2[m-1]}\tilde{\theta}_{12}-c_{2m+1,2[m-1]}\omega_{2m+1,2m+1}=c_{2m+1,2[m]}\bar{\phi}\,,\\
&da_{2m+2,1[m-1]}-i(m+1)a_{2m+2,1[m-1]}\tilde{\theta}_{12}-a_{2m+2,1[m-1]}\omega_{2m+2,2m+2}=a_{2m+2,1[m]}\phi\,,\\
&c_{2m+1,2[m-1]}\omega_{2m+2,2m+1}=-c_{2m+2,2[m]}\bar{\phi}\,,\\
&a_{2m+2,1[m-1]}\omega_{2m+1,2m+2}=-a_{2m+1,1[m]}\phi\,,
\end{aligned}\tag{6.7}$$

$$c_{2m+1,2[m-1]}\omega_{\lambda,2m+1}=-c_{\lambda,2[m]}\bar{\phi}\,,$$

$$a_{2m+2,1[m-1]}\omega_{\lambda,2m+2}=-a_{\lambda,1[m]}\phi\,,\qquad(\lambda\geq 2m+3)\,.$$

Hence, (6.5), (6.6), (6.7) and Lemma 5.3 show that (5.2), (5.3) and (5.4) are valid for $k=(m+1)$.

We define smooth functions on M by

$$\mathscr{C}_k^2=c_3^2c_{5,2}^2\cdots c_{2k-1,2[k-2]}^2\,,\qquad k=2,3,\dots,m\,. \tag{6.8}$$

Note that these functions are scalar invariants of x, which can be seen in a way similar to that in [12, p. 372]. Using (5.2) and (5.3), we get $d\mathscr{C}_k^2=\mathscr{C}_k(A_k\phi+\bar{A}_k\bar{\phi})$, where A_k satisfies $\bar{A}_k=\mathscr{C}_{k-1}c_{2k-1,2[k-1]}+\bar{A}_{k-1}c_{2k-1,2[k-2]}$ for $k=3,\dots,m$ and $\bar{A}_2=c_{3,2}$. Hence, using (5.4) and Lemma 5.3, we have:

LEMMA 6.1.

$$\Delta\mathscr{C}_m^2=2\mathscr{C}_m^2\{m(m+1)K/2-\rho+(2m+1)\rho\cos(\alpha)\}$$
$$+4|A_m|^2+4\mathscr{C}_{m-1}^2\sum_\lambda|c_{\lambda,2[m-1]}|^2\,, \tag{6.9}$$

$$\Delta\left(\mathscr{C}_m^2\sum_\lambda|c_{\lambda,2[m-1]}|^2\right)=2\mathscr{C}_m^2\sum_\lambda|c_{\lambda,2[m-1]}|^2\{(m+1)(m+2)K/2$$
$$-\rho+(2m+3)\rho\cos(\alpha)\}+4\sum_\lambda|\mathscr{C}_m c_{\lambda,2[m]}+\bar{A}_m c_{\lambda,2[m-1]}|^2\,. \tag{6.10}$$

Note that (6.10) coinsides with (3.8) in [12] for $m=2$.

Now we give the proofs of the main theorems.

PROOF OF THEOREM 4.1. By (6.10) and the assumption, $\mathscr{C}_m^2\sum_\lambda|c_{\lambda,2[m-1]}|^2$ is a non-zero subharmonic function on a compact manifold M, which is constant on M. This shows that $K=2\{1-(2m+3)\cos(\alpha)\}\rho/(m+1)(m+2)$. Hence, by Ohnita's theorem [10], we get Theorem 4.1. q.e.d.

COROLLARY 6.2. *Let $x\colon M\to X$ be as in Theorem* 4.1. *If M is a J-regular manifold and the Gaussian curvature K satisfies* $2\{1-(2m+1)\cos(\alpha)\}/m(m+1)>K\geq 2\{1-(2m+3)\cos(\alpha)\}\rho/(m+1)(m+2)\geq 0$ *on M, then we have* $K=2\{1-(2m+3)\cos(\alpha)\}\rho/(m+1)(m+2)$.

PROOF. By the J-regularity of M and the assumption, we have $\sum|c_{\lambda,2[m-1]}|^2\neq 0$ on M. Hence, each point of M is J-regular of order $(m+1)$. By Theorem 4.1, we are done. q.e.d.

Proof of THEOREM 4.2. We may assume that each point of M is J-regular of order s. If $\sum|c_{\lambda,2[s-1]}|^2\neq 0$ at a point p of M, then we get $\sum|c_{\lambda,2[s-1]}|^2\neq 0$ on M. Hence, each point of M is J-regular of order $(s+1)$. By Theorem 4.1, we see that x is locally congruent to $\varphi_{n,s+1}$. If $\sum|c_{\lambda,2[s-1]}|^2=0$ on M, then, by (6.9), we see that x is locally congruent to $\varphi_{n,s}$. q.e.d.

REFERENCES
(continuation of those in [12])

[11] T. OGATA, U. Simon's conjecture on minimal surfaces in a sphere, Bull. Yamagata Univ. 11 (1987), 345–350.

[12] T. OGATA, Curvature pinching theorem for minimal surfaces with constant Kaehler angle in complex projective spaces, Tôhoku Math. J. 43 (1991), 361–374.

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