Tohoku Math. J.
57 (2005), 589–595

A UNICITY THEOREM FOR MOVING TARGETS COUNTING MULTIPLICITIES

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(Received December 8, 2003, revised July 27, 2004)

Abstract. R. Nevanlinna showed, in 1926, that for two nonconstant meromorphic functions on the complex plane, if they have the same inverse images counting multiplicities for four distinct complex values, then they coincide up to a Möbius transformation, and if they have the same inverse images counting multiplicities for five distinct complex values, then they are identical. H. Fujimoto, in 1975, extended Nevanlinna's result to nondegenerate holomorphic curves. This paper extends Fujimoto's uniqueness theorem to the case of moving hyperplanes in pointwise general position.

1. Introduction. In Nevanlinna theory, there is a well-known uniqueness theorem due to R. Nevanlinna. He showed, in 1926, that any two meromorphic functions sharing four distinct values must coincide up to a Möbius transformation, and if they share five distinct values, then they must be identical. In 1975, H. Fujimoto (see [F]) extended this result to holomorphic curves, and proved the following.

THEOREM A (Fujimoto [F]). *Let f and g be two non-constant holomorphic curves of the complex plane $\boldsymbol{C}$ into the complex projective n-space $\boldsymbol{P}^n(\boldsymbol{C})$. Suppose that there exist $3n+1$ hyperplanes $H_j, 1 \leq j \leq 3n+1$, in $\boldsymbol{P}^n(\boldsymbol{C})$ located in general position such that $f(\boldsymbol{C}) \not\subset H_j$, $g(\boldsymbol{C}) \not\subset H_j$ and $\nu(f, H_j) = \nu(g, H_j), 1 \leq j \leq 3n+1$, where $\nu(f, H_j)$ and $\nu(g, H_j)$ denote the pullbacks of the divisors (H_j) by f and g, respectively. Then there is a projective linear transformation L of $\boldsymbol{P}^n(\boldsymbol{C})$ such that $L(f) = g$.*

THEOREM B (Fujimoto [F]). *Let f and g be two non-constant holomorphic curves of $\boldsymbol{C}$ into $\boldsymbol{P}^n(\boldsymbol{C})$, at least one of which is non-degenerate. Suppose that there exist $3n+2$ hyperplanes $H_j, 1 \leq j \leq 3n+2$, in $\boldsymbol{P}^n(\boldsymbol{C})$ located in general position such that $f(\boldsymbol{C}) \not\subset H_j$, $g(\boldsymbol{C}) \not\subset H_j$ and $\nu(f, H_j) = \nu(g, H_j), 1 \leq j \leq 3n+2$, where $\nu(f, H_j)$ and $\nu(g, H_j)$ denote the pullbacks of the divisors (H_j) by f and g, respectively. Then $f = g$.*

Recently, there appeared two papers (see [Ye] and [Tu]) which extend the above results to moving targets. However, both papers imposed additional conditions (see the condition (ii) and the condition (iii) in Theorem C below). In particular, the condition of f and g being equal on $f^{-1}(H_j), 1 \leq j \leq q$, (see condition (iii) in Theorem C below) seems unnatural. Furthermore, as indicated in [Tu], the paper of [Ye] contains some gaps in his key argument.

2000 *Mathematics Subject Classification*. Primary 32H30; Secondary 32H25.

Key words and phrases. Holomorphic maps, unicity theorem, moving targets.

The first author is supported by National Natural Science Foundation of China (No. 10271029). The second author is supported in part by NSA under grant number MSPF-02G-175.

To state the results obtained in [Tu], we first introduce some notation. Throughout this paper, we denote by $\mathbf{f}$ a reduced representation of a holomorphic map $f : \boldsymbol{C} \to \boldsymbol{P}^n(\boldsymbol{C})$. By a moving hyperplane H in $\boldsymbol{P}^n(\boldsymbol{C})$, we mean

$$H = \left\{ [x_0 : \cdots : x_n] \,\middle|\, \sum_{i=0}^{n} a_i x_i = 0 \right\},$$

where $a_0, \ldots, a_n$ are entire functions without common zeros. So H is associated with a holomorphic map $a = [a_0 : \cdots : a_n] : \boldsymbol{C} \to \boldsymbol{P}^n(\boldsymbol{C})$. Write $\mathbf{a} = (a_0, \ldots, a_n)$. Given a holomorphic map $f : \boldsymbol{C} \to \boldsymbol{P}^n(\boldsymbol{C})$, we say that f and H (or f and a) are *free* if $\langle \mathbf{f}, \mathbf{a} \rangle \not\equiv 0$, where $\mathbf{f}$ is a reduced representation of f and $\langle\ ,\ \rangle$ is the standard inner product on $\boldsymbol{C}^{n+1}$. The moving hyperplanes $H_1, \ldots, H_q$ (or $\mathbf{a}_1, \ldots, \mathbf{a}_q$) are said to be *in general position* if $H_1(z), \ldots, H_q(z)$ are in general position for some (and hence for almost all) $z \in \boldsymbol{C}$. The main result obtained in [Tu] reads as follows.

THEOREM C (Tu [Tu]). *Let f and g be two non-constant holomorphic curves of $\boldsymbol{C}$ into $\boldsymbol{P}^n(\boldsymbol{C})$. Let $\mathcal{G} = \{H_j\}_{j=1}^q$ (or $\{a_j\}_{j=1}^q$) be a finite set of moving hyperplanes in $\boldsymbol{P}^n(\boldsymbol{C})$, located in general position. Let $\mathbf{a}_j = (a_{j,0}, \ldots, a_{j,n})$. Let $\mathcal{R}_{\mathcal{G}}$ be the smallest field containing $\boldsymbol{C}$ and all $a_{j\mu}/a_{j\nu}$ with $a_{j\nu} \not\equiv 0$, and let $\tilde{\mathcal{R}}_{\mathcal{G}}$ be the smallest field containing all meromorphic functions h such that $h^k \in \mathcal{R}_{\mathcal{G}}$. Assume that* $\max_{1\le j\le q} T_{a_j}(r) = o(T_f(r))$ *and f is linearly nondegenerate over $\mathcal{R}_{\mathcal{G}}$. Assume further that*

(i) $\langle \mathbf{f}, \mathbf{a}_j \rangle / \langle \mathbf{g}, \mathbf{a}_j \rangle$, $1 \le j \le q$, *are nowhere zero entire functions on $\boldsymbol{C}$,*

(ii) $\{z \mid \langle \mathbf{f}(z), \mathbf{a}_i(z) \rangle = 0\} \cap \{z \mid \langle \mathbf{f}(z), \mathbf{a}_j(z) \rangle = 0\} = \emptyset$ *for* $1 \le i < j \le q$,

(iii) $f = g$ *on* $\bigcup_{j=1}^q \{z \in \boldsymbol{C} \mid \langle \mathbf{f}(z), \mathbf{a}_j(z) \rangle = 0\}$.

Then the following hold:

(a) *If $q = 3n+1$, then there exists an $(n+1) \times (n+1)$ matrix L with entries in $\tilde{\mathcal{R}}_{\mathcal{G}}$ and* $\det(L) \not\equiv 0$ *such that $\mathbf{f} = L \cdot \mathbf{g}$, where $\mathbf{f}$* (*resp.* $\mathbf{g}$) *is a reduced representation of f* (*resp.* g).

(b) *If $q = 3n+2$, and, in addition, if f is linearly nondegenerate over $\tilde{\mathcal{R}}_{\mathcal{G}}$, then $f = g$.*

The purpose of this paper is to extend Fujimoto's results (Theorem A and Theorem B) to moving hyperplanes without conditions (ii) and (iii) assumed in Theorem C. Also, we do not assume that the hyperplanes are slowly moving with respect to the growth of f, i.e., we do not assume that $\max_{1\le j\le q} T_{a_j}(r) = o(T_f(r))$. Instead, we assume that the given moving hyperplanes are located in pointwise general position. Recall that moving hyperplanes H_j, $1 \le j \le q$, are said to be in *pointwise general position* (*vs.* in general position) if the hyperplanes $H_j(z)$, $1 \le j \le q$, are in general position (as a set of fixed hyperplanes) at every point $z \in \boldsymbol{C}$ (*vs.* some point $z_0 \in \boldsymbol{C}$). With this assumption, we prove the following result.

MAIN THEOREM. *Let $\mathcal{G} = \{H_j\}_{j=1}^q$ (or $\{a_j\}_{j=1}^q$) be a finite set of moving hyperplanes in $\boldsymbol{P}^n(\boldsymbol{C})$, located in pointwise general position. Let f and g be two non-constant holomorphic mappings of $\boldsymbol{C}$ into $\boldsymbol{P}^n(\boldsymbol{C})$, such that $\langle \mathbf{f}, \mathbf{a}_j \rangle \not\equiv 0$ and $\langle \mathbf{g}, \mathbf{a}_j \rangle \not\equiv 0$ for $1 \le j \le q$. Assume that $\langle \mathbf{f}, \mathbf{a}_j \rangle / \langle \mathbf{g}, \mathbf{a}_j \rangle$, $1 \le j \le q$, are nowhere zero entire functions on $\boldsymbol{C}$. Then the following hold*:

(a) *If* $q = 3n + 1$, *then there exists an* $(n + 1) \times (n + 1)$ *invertible matrix* L *with entire functions as entries, such that* $\mathbf{f} = L \cdot \mathbf{g}$, *where* $\mathbf{f}$ (*resp.* $\mathbf{g}$) *is a reduced representation of* f (*resp.* g). *In addition, the entries of* L *depend only on the given hyperplanes and can be determined effectively.*

(b) *If* $q = 3n + 2$, *and, in addition, if* f *is linearly nondegenerate over* $\tilde{\mathcal{R}}_{\mathcal{G}}$, *then* $f = g$. *Here* $\mathcal{R}_{\mathcal{G}}$ *is the smallest field containing* $\boldsymbol{C}$ *and all* $a_{j\mu}/a_{j\nu}$ *with* $a_{j\nu} \not\equiv 0$, *and* $\tilde{\mathcal{R}}_{\mathcal{G}}$ *is the smallest field containing all meromorphic functions* h *such that* $h^k \in \mathcal{R}_{\mathcal{G}}$.

Note that the main theorem can be extended easily to holomorphic maps f and g from $\boldsymbol{C}^m$ to $\boldsymbol{P}^n$.

The second author wishes to thank the Department of Mathematics, Fudan University, P.R. China for kind hospitality and support, during which part of the work on this paper took place.

2. Proof of the main theorem. To prove the main theorem, we recall the following Borel's lemma (see [Ru] or [L]).

BOREL'S LEMMA. *Let* $f_0, \dots, f_{n+1}$ *be nowhere zero entire functions such that*

$$f_0 + \cdots + f_{n+1} = 0 .$$

Let $\{0, 1, \dots, n+1\} = I_1 \cup I_2 \cdots \cup I_k$ *be the partition such that* i *and* j *are in the same class* I_l *if and only if* $f_i = c_{ij} f_j$ *for some nonzero constant* c_{ij}. *Then*

$$\sum_{i \in I_l} f_i = 0$$

for any l.

Consider the moving hyperplanes

$$H_j = \{[x_0 : \cdots : x_n] \mid a_{j0}x_0 + \cdots + a_{jn}x_n = 0\}, \quad 1 \leq j \leq q ,$$

where a_{ji}, $1 \leq j \leq q, 0 \leq i \leq n$, are entire functions without common zeros. Write $\mathbf{a}_j = (a_{j0}, \dots, a_{jn})$, and let $\mathbf{f}$ be a reduced representation of f. We first prove the part (a) in the main theorem. We assume that $q \geq 3n + 1$ in this case. Define functions

$$h_j := \langle \mathbf{f}, \mathbf{a}_j \rangle / \langle \mathbf{g}, \mathbf{a}_j \rangle , \quad 1 \leq j \leq q . \tag{2.1}$$

Then, by the assumption, h_j are nowhere zero entire functions. We need the following two claims. We note that, although the situation is different, the proofs of Claim 1, Claim 2 and the Combinatorial Lemma below follow the argument given by Fujimoto (see [F], or [F2]). We enclose the proofs in this paper for the sake of reader's convenience.

CLAIM 1. *Let* T *be any subset of* $\{1, 2, \dots, q\}$ *with* $\#T = 2n + 2$, *where* $q \geq 3n + 1$. *Then, for each* $I \subset T$ *with* $\#I = n + 1$, *there exists a set* $J \subset T$ *with* $\#J = n + 1$, $I \neq J$, *such that* h_I / h_J *is a nowhere zero entire function depending only on the given hyperplanes and can be determined effectively, where* $h_I := h_{i_0} \cdots h_{i_n}$ *for* $I = \{i_0, \dots, i_n\}$.

To prove this claim, we assume without loss of generality that $T = \{1, 2, \dots, 2n + 2\}$. Write (2.1) as

$$a_{j0} f_0 + \cdots + a_{jn} f_n = h_j (a_{j0} g_0 + \cdots + a_{jn} g_n), \quad j \in T .$$

By Cramer's rule for solving a system of linear equations, we obtain

$$\det(a_{j0}, \dots, a_{jn}, a_{j0} h_j, \dots, a_{jn} h_j \,;\, j = 1, 2, \dots, 2n + 2) = 0 .$$

Then, by the Laplace expansion theorem,

$$\sum_{J \subset T, \#J = n+1} \alpha_J A_J h_J = 0 , \tag{2.2}$$

where $J = \{i_0, \dots, i_n\}$, $\alpha_J = (-1)^{n(n+1)/2 + i_0 + \cdots + i_n}$, and

$$A_J = \det(a_{i_r, j})_{0 \le r \le n,\ 0 \le j \le n} \det(a_{i'_s, j})_{0 \le s \le n,\ 0 \le j \le n}$$

for $i'_0, \dots, i'_n \in T - J$. Since $H_1, \dots, H_q$ are in pointwise general position, any $n + 1$ vectors in $\{(a_{j0}, \dots, a_{jn}) \mid 1 \le j \le 2n + 2\}$ are linearly independent at each point $z \in \boldsymbol{C}$. Therefore, A_J is nowhere zero for all $J's$. Hence, by applying Borel's lemma, there exists constant c_{IJ} and $J \subset T$, $J \ne I$ such that $A_I h_I = c_{IJ} A_J h_J$, and hence $h_I / h_J = c_{IJ} A_J / A_I$. So we obtain Claim 1.

Our next step is to prove the following claim.

CLAIM 2. *Let $q \ge 3n + 1$. There is a subset I_0 of $\{1, 2, \dots, q\}$ with $\#I_0 = q - 2n$ such that h_i / h_j, for every $i, j \in I_0$, is a nowhere zero entire function depending only on the given hyperplanes and can be determined effectively.*

To prove Claim 2, consider the multiplicative group $\mathcal{H}^*$ of all nowhere zero entire functions. Denote by $\mathcal{T}$ the smallest subgroup of $\mathcal{H}^*$ which contains all $h \in \mathcal{H}^*$ with $h^k \in \mathcal{R}_{\mathcal{G}}$ for some positive integer k. So we have $\mathcal{H}^* \cap \mathcal{R}_{\mathcal{G}} \subset \mathcal{T} \subset \tilde{\mathcal{R}}_{\mathcal{G}}$. Then the multiplication group $\mathcal{H}^*/\mathcal{T}$ is a torsion free abelian group. We denote by $[h]$ the class in $\mathcal{H}^*/\mathcal{T}$ containing $h \in \mathcal{H}^*$. Consider the subgroup $\widetilde{\mathcal{G}}$ of $\mathcal{H}^*/\mathcal{T}$ generated by $[h_1], \dots, [h_q]$ and choose suitable functions $b_1, \dots, b_t \in \mathcal{H}^*$ such that $[b_1], \dots, [b_t] \in \mathcal{H}^*/\mathcal{T}$ give a basis of $\widetilde{\mathcal{G}}$. Then each h_j can be uniquely represented as

$$h_j = g_j b_1^{r_{j1}} \cdots b_t^{r_{jt}} , \quad 1 \le j \le q, \tag{2.6}$$

with some $g_j \in \mathcal{T}$ and integers $r_{j\tau}$, $1 \le \tau \le t$.

To proceed, we need the following combinatorial lemma.

COMBINATORIAL LEMMA. *Given integers $r_{j\tau}$, $1 \le j \le q$, $1 \le \tau \le t$, we can choose integers $\beta_1, \dots, \beta_t$ such that among the integers*

$$r_j := r_{j1} \beta_1 + \cdots + r_{jt} \beta_t , \quad 1 \le j \le q , \tag{2.7}$$

any two numbers, say, r_i and r_j are equal only if the corresponding vectors $(r_{i1}, \dots, r_{it})$ and $(r_{j1}, \dots, r_{jt})$ are equal.

The Combinatorial Lemma is the same as that in [F] page 2, (2.2), or [F2] page 118, (3.4.13). We enclose the proof here again for the sake of reader's convenience. The proof is given by induction. In fact, the lemma is trivial for the case $t = 1$. Assume that there exist $\beta_1, \dots, \beta_{t-1}$ with the property that if $r_i^* = r_j^*$ for integers $r_i^* := r_{i1}\beta_1 + \cdots + r_{i,t-1}\beta_{t-1}$, then $(r_{i1}, \dots, r_{i,t-1}) = (r_{j1}, \dots, r_{j,t-1})$. Then it is easy to show that there are only finitely many integers β_t such that $\beta_1, \beta_2, \dots, \beta_t$ do not satisfy the desired condition. Thus the lemma is proved.

We now prove Claim 2. Let $r_{j\tau}$, $1 \le j \le q$, $1 \le \tau \le t$, be the integers given by (2.6), and choose $\beta_1, \dots, \beta_t$ as in the Combinatorial Lemma. Define r_j, $1 \le j \le q$, by (2.7). After a suitable change of indices, we may assume that

$$r_1 \le \cdots \le r_q \,. \tag{2.8}$$

If $r_{n+1} = \cdots = r_{q-n}$, then $r_{n+1,\tau} = \cdots = r_{q-n,\tau}$ for $1 \le \tau \le t$. So by (2.6), $h_i/h_j = g_i/g_j \in \mathcal{T}$ for $i, j \in \{n+1, \dots, q-n\}$. Hence the proof of Claim 2 would be finished if we can show that $r_{n+1} = \cdots = r_{q-n}$. To prove this, take $T = \{1, \dots, n+1, q-n, \dots, q\}$ which contains $2n+2$ elements. Applying Claim 1 with $I = \{1, 2, \dots, n+1\}$, we see that there is a subset $J = \{i_0, \dots, i_n\}$ of T such that $\{i_0, \dots, i_n\} \ne \{1, 2, \dots, n+1\}$ and

$$\frac{h_{i_0}h_{i_1}\cdots h_{i_n}}{h_1h_2\cdots h_{n+1}} \in \mathcal{T} \,.$$

From (2.6) this implies that $b_1^{l_1}\cdots b_t^{l_t} = g b_1^{l_1'}\cdots b_t^{l_t'}$, where $l_\tau = r_{i_0\tau} + \cdots + r_{i_n\tau}$, $l_\tau' = r_{1\tau} + \cdots + r_{n+1,\tau}$, $1 \le \tau \le t$, and $g \in \mathcal{T}$. Since $b_1, \dots, b_t$ are multiplicatively independent, we have $l_\tau = l_\tau'$ for $\tau = 1, 2, \dots, t$, that is,

$$r_{i_0\tau} + \cdots + r_{i_n\tau} = r_{1\tau} + \cdots + r_{n+1\tau} \,.$$

Thus we obtain

$$\sum_{s=0}^{n} r_{i_s} = \sum_{s=0}^{n}\sum_{\tau=1}^{t} r_{i_s\tau}\beta_\tau = \sum_{i=1}^{n+1}\sum_{\tau=1}^{t} r_{i\tau}\beta_\tau = \sum_{i=1}^{n+1} r_i \,,$$

which simply means that

$$(r_{i_0} - r_1) + \cdots + (r_{i_n} - r_{n+1}) = 0 \,.$$

Since $r_{i_0} \ge r_1, \dots, r_{i_n} \ge r_{n+1}$, this is possible only when $r_{i_n} = r_{n+1}$. However, since $i_n \ge q-n$, by (2.8) we have $r_{i_n} \ge r_{q-n}$. Thus $r_{n+1} \ge r_{q-n}$. Noticing that $r_{n+1} \le r_{q-n}$, we have $r_{n+1} = r_{q-n}$. Using (2.8) again, we obtain that $r_{n+1} = \cdots = r_{q-n}$. This concludes Claim 2.

We now prove the part (a). Again we follow Fujimoto's argument (see [F] or [F2]). By Claim 2 with $q = 3n+1$ together with a suitable change of the reduced representation, we may assume, without loss of generality, that $h_1, \dots, h_{n+1}$ are the entire functions which depend only on the given hyperplanes, and can be determined effectively. We define $A := (a_{ji})_{1\le j\le n+1, 0\le i\le n}$ and H to be a diagonal matrix with diagonal entries as $h_1, \dots, h_{n+1}$. Then, by (2.1), $A\mathbf{f} = HA\mathbf{g}$. Hence $\mathbf{f} = A^{-1}HA\mathbf{g}$, and the entries of the matrix $A^{-1}HA$

depend only on the given hyperplanes and can be determined effectively. Hence the part (a) in the Main Theorem is proved.

To prove the part (b), since $q = 3n + 2$, by Claim 2 together with a suitable change of the reduced representation, we may assume, without loss of generality, that $h_1, \dots, h_{n+2}$ are the entire functions which depend only on the given hyperplanes, and can be determined effectively. Again, let $A := (a_{ji})_{1 \le j \le n+1, 0 \le i \le n}$. Then, by (2.1), $A\mathbf{f} = HA\mathbf{g}$, where H is a diagonal matrix with diagonal entries as $h_1, \dots, h_{n+1}$, and

$$(a_{n+2,0}, \dots, a_{n+2,n}) \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{pmatrix} = h_{n+2}(a_{n+2,0}, \dots, a_{n+2,n}) \begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_n \end{pmatrix} .$$

Hence

$$(a_{n+2,0}, \dots, a_{n+2,n}) \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{pmatrix} = h_{n+2}(a_{n+2,0}, \dots, a_{n+2,n}) A^{-1} H^{-1} A \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{pmatrix} .$$

Since f is linearly nondegenerate over $\tilde{\mathcal{R}}_{\mathcal{G}}$, we deduce that $(a_{n+2,0}, \dots, a_{n+2,n}) = h_{n+2}(a_{n+2,0}, \dots, a_{n+2,n}) A^{-1} H^{-1} A$. Therefore,

$$(a_{n+2,0}, \dots, a_{n+2,n}) A^{-1} \begin{pmatrix} h_1 - h_{n+2} & 0 & \cdots & 0 \\ 0 & h_2 - h_{n+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{n+1} - h_{n+2} \end{pmatrix} = \mathbf{0} .$$

Let

$$(a_{n+2,0}, \dots, a_{n+2,n}) = (b_0, \dots, b_n) A .$$

Since $\{H_j\}_{j=1}^q$ (or $\{a_j\}_{j=1}^q$) are in pointwise general position, we have $b_j \not\equiv 0$ $(j = 0, 1, \dots, n)$, and

$$(b_0, \dots, b_n) \begin{pmatrix} h_1 - h_{n+2} & 0 & \cdots & 0 \\ 0 & h_2 - h_{n+2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{n+1} - h_{n+2} \end{pmatrix} = \mathbf{0} ,$$

which implies that $h_j = h_{n+2}$ $(j = 1, \dots, n+1)$. Hence from $A\mathbf{f} = HA\mathbf{g} = AH\mathbf{g}$, we deduce $\mathbf{f} = H\mathbf{g}$, which implies that $f = g$. The proof of the Main Theorem is completed.

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