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STORING BAIRE 1 FUNCTIONS

Consider the following situation. Suppose that Person A knows all about a particular function $f : [0,1] \to \mathbb{R}$, *i.e.*, he knows its value at every point of $[0,1]$. Suppose further that Person A wishes to communicate this knowledge to Person B by giving her a countable collection of ordered pairs $\{x_n, f(x_n)\}$ together with a simple algorithm so that armed with these two things, Person B can obtain $f(x)$ for any $x \in [0,1]$. For what type of functions is this possible?

The question is, of course, not well posed since we have not defined "simple" or "algorithm." Nonetheless, note that if f is continuous, the communication (or storage) problem clearly can be handled by choosing any dense sequence $\{x_n\}$ and many suitable algorithms come to mind. Recently, we have shown that any real valued Darboux Baire 1 function defined on the unit interval can be so communicated utilizing the notion of first return continuity to generate the algorithm [1]. This actually yielded a characterization of Darboux Baire 1 functions, and the proof capitalized on the Maximoff-Preiss Theorem [4] [5] and some properties of derivatives. In [2] we extend the notion of first return continuity to that of first return recoverability as follows:

By a *trajectory* we mean any sequence $\{x_n\}_{n=0}^{\infty}$ of distinct points in $[0,1]$, which is dense in $[0,1]$. Let $\{x_n\}$ be a fixed trajectory and let $y \in [0,1]$. We define what we shall mean by the *first return route to y based on the trajectory* $\{x_n\}$. If $\rho > 0$, we use $B_\rho(y)$ to denote $\{x \in [0,1] : |x-y| < \rho\}$. We let $r(B_\rho(y))$ denote the first element of the trajectory in $B_\rho(y)$. The *first return route to* y, $\mathcal{R}_y = \{y_k\}_{k=1}^{\infty}$, is defined recursively via

$$y_1 = x_0,$$

$$y_{k+1} = \begin{cases} r\left(B_{|y-y_k|}(y)\right) & \text{if } y \neq y_k \\ y_k & \text{if } y = y_k. \end{cases}$$

*Presenter

We say that $f : [0,1] \to \mathbb{R}$ is *first return recoverable with respect to* $\{x_n\}$ provided that for each $y \in [0,1]$ we have

$$\lim_{k\to\infty} f(y_k) = f(y),$$

and that f is *first return recoverable* if there exists a trajectory $\{x_n\}$ such that f is first return recoverable with respect to $\{x_n\}$.

We then give the following characterization of Baire 1 functions.

Theorem 1 *A function $f : [0,1] \to \mathbb{R}$ is first return recoverable if and only if it is Baire 1.*

In fact, the above theorem is a special case of the following result, which is presented in [3]. (By "Baire 1" in this more general setting, we mean that the inverse image of each open set is an F_σ set.) The necessary definitions are the obvious extensions of those above, but the proof is markedly more involved than the proof given for Theorem 1 in [2], principally because the order properties of the real line are no longer available.

Theorem 2 *Let X be a compact metric space and Y a separable metric space. A function $f : X \to Y$ is first return recoverable if and only if it is Baire 1.*

References

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[5] D. Preiss, *Maximoff's Theorem*, Real Anal. Exch. **5** (1979), 92–104.