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UNIFORM GENERALIZED ABSOLUTE CONTINUITY

Let H be the space of all Henstock integrable functions on a given interval [a,b] of the real line $\mathbb{R}$ and let $\mathcal{H}$ be its completion with respect to the norm

$$\| f \|_H = \sup_{x \in [a,b]} \left| \int_a^x f(t)dt \right| .$$

Theorem 1 *Given* $\{f_n \in H\}$,

- *if* $\{\int_a^x f_n\}$ *is equicontinuous, then* $\{f_n\}$ *is relatively weakly compact in* $\mathcal{H}$,
- *if* $\{\int_a^x f_n\}$ *is uniformly-ACG*$_*$, *then* $\{f_n\}$ *is relatively weakly compact in* $\mathcal{H}$ *and its closure belongs to* H,
- *if* $\{\int_a^x f_n\}$ *is uniformly-ACG*$_*$ *and* $f_n(x) \to f(x)$ *a.e. in* $[a,b]$, *then* $f \in H$ *and* $\{f_n\}$ *is weakly convergent to* f.

Theorem 2 *([1], n.2) There is in* H *a sequence* $\{f_n\}$ *which is weakly convergent to 0 and such that* $\{\int_a^x f_n\}$ *is neither uniformly-ACG*$_*$ *nor equicontinuous.*

Theorem 3 *If* $\{f_n \in H\}$ *is weakly convergent to* $f \in H$, *then* $\{\int_a^x f_n\}$ *is pointwisely convergent to* $\int_a^x f$ *and equicontinuous outside a (possibly empty) nowhere dense set.*

References

[1] B. Bongiorno, L. Di Piazza, and V. Skvortzov, *Uniform generalized absolute continuity and convergence of H-integrals* (submitted.)

[2] B. Bongiorno and L. Di Piazza, *Weak convergence in the BV-predual* (in preparation.)