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On local characterization of almost continuous functions

The class $\mathcal{C}$ (of continuous real functions of a real variable), $\mathcal{C}on$ (of functions with connected graphs) and $\mathcal{D}$ (of Darboux functions) forming the sequence of inclusions

$$\mathcal{C} \subseteq \mathcal{C}on \subseteq \mathcal{D}$$

can be characterized locally (see [1] , [2]). The class $\mathcal{A}$ of almost continuous functions in the sense of Stallings is to be characterized locally. This is one of the approaches to that problem.

Definition 1 *A function $f : (a,b) \longrightarrow \mathbf{R}$ is said to be almost continuous at a point $x_0 \in (a,b)$ from the right side iff*

1. *$f(x) \in L^+(f, x_0)$, where $L^+(f, x_0)$ denotes the cluster set of the function f at the point x_0;*
2. *there is a positive ε such that for an arbitrary neighbourhood G of $f\big|_{[x_0,\infty)}$, arbitrary $y \in \left(\liminf_{t\to x_0^+} f(t), \limsup_{t\to x_0^+} f(t)\right)$, arbitrary neighbourhood of the point (x_0, y) and arbitrary $t \in (x_0, x_0+\varepsilon)$ there is a continuous function $g : [x_0, x_0+\varepsilon] \longrightarrow \mathbf{R}$ such that $g \subseteq G \cup U$, $g(x_0) = y$, $g(t) = f(t)$.*

Similarly, we define almost continuity at x_0 from the left and we say that f is almost continuous at x_0 if it almost continuous at both sides.

This definition is good enough to get the following properties:

Property 1 *A function $f : [a,b] \longrightarrow \mathbf{R}$ is almost continuous if and only if it is almost continuous at each point of $[a,b]$. (The interval $[a,b]$ can be replaced by open interval (a,b)).*

Property 2 *The set of all points of almost continuity of any real function of a real variable is of the type $\mathcal{G}_\delta$.*

Property 3 *If f is continuous at x_0, then it is almost continuous at x_0; if f is almost continuous at x_0, then it is connected at x_0.*

References

[1] Bruckner A.M., Ceder J.G., Darboux Continuity, Jber. Deutsch. Math. Ver. 67 (1965), 93-117

[2] Garret B.D., Kellum K.R., Characterization of connected functions, ibid 73 (1971), 131-137.