

(ε, η) -Approximating Partitions

If δ is a positive function on a closed interval $[a, b]$, a δ -fine partition of $[a, b]$ is a collection $\{(I_1, x_1), \dots, (I_p, x_p)\}$ where I_1, \dots, I_p are nonoverlapping closed intervals with union $[a, b]$ such that $x_i \in I_i$ and $\text{diam}(I_i) < \delta(x_i)$ for $i = 1, \dots, p$. Cousin's lemma assures that such a δ -fine partition always exists—a fact that is essential to the definition of the Generalized Riemann Integral. Recently, Washek F. Pfeffer has given a Riemann type definition of an integral in \mathbf{R}^m which is coordinate free and for which a general divergence theorem holds (see [Pfeffer, *A Riemann Type Definition of a Variational Integral*, to appear]). In this definition, the simple δ -fine partitions of the Generalized Riemann Integral are replaced by a partitioning concept which is more general in several directions. Among the generalizations: intervals are replaced by sets in \mathbf{R}^m with finite perimeters in the sense of De Giorgi; the partitioning points are not allowed to lie in a prespecified exceptional set; and the partition is only required to cover “most” of the set being partitioned. The purpose of this talk was to show how to prove a Cousin-like existence lemma for these more general partitions. The proof uses ideas from [Howard, *Analyticity of almost everywhere differentiable functions*, Proc. American Math. Soc., at press].