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Proofs of the Uher and Freiling Covering Theorems

In this talk we gave short proofs of two important yet rather technical results. In this note we describe both results, but do not give the proofs; they will appear elsewhere as parts of longer papers written jointly with M. Laczkovich (Eötvös Loránd University, Budapest, Hungary).

§1 Freilings Covering Theorem.

A collection of open intervals, C, is said to be a *full symmetric cover* of $\Re$ if for every real number x there is a $\delta(x) > 0$ such that $[x-t, x+t] \in C$ whenever $0 < t < \delta(x)$. If C is a full symmetric cover of $\Re$, C is said to *partition* an interval $[a, b]$ if there is a partition $a = u_0 < u_1 < \dots < u_{n+1} = b$ of $[a, b]$ with $[u_i, u_{i+1}] \in C$ for $i = 0, 1, \dots, n$. A set $E \subset \Re$ is said to be an *exceptional set for the cover* C if whenever $x, y \in \Re \setminus E$ with $x < y$, then C partitions the interval $[x, y]$.

THEOREM 1. *Let C be a full symmetric cover for $\Re$. Then:*

- *(i) There are minimal countable exceptional sets for C .*
- *(ii) No minimal exceptional set contains a subset which is bilaterally dense in itself.*

See [F, Theorem 2]. As was shown in [F], if a set, E, contains no bilaterally dense in itself subset*, then $E = E^+ \cup E^-$ where E^+ is right scattered (every non-empty subset has a right isolated point) and E^- is left scattered. As [F, Theorem 1] shows, if E has no bilaterally dense in itself subset then there is a full symmetric cover of $\Re$ for which E is a minimal countable exceptional set.

* In [F], Chris refers to this property as *splattered*

§2 Uher's Covering Theorem.

If $x, y \in \Re$ and S is a subset of $\Re^2$, we say that there is a 5-element S-chain connecting x and y if there are points $(s_i, s_i^*) \in S,\ i = 0, 1, \ldots, 4$ such that $s_0 = x,\ s_4^* = y$, and $s_i^* = s_{i+1},\ i = 0, 1, 2, 3$. Of the several possible Uher type theorems we stated but two and proved only one of these. The proof we gave is, however, representative.

THEOREM 2. *Let $E \subset \Re$ have the Baire property, let $S \subset \Re^2$ and suppose that for every $x \in E$ there is a $\delta(x) > 0$ such that either $(x - t, x + t) \in S$ for every $0 < t < \delta(x)$, or $(x + t, x - t) \in S$ for every $0 < t < \delta(x)$. Then there is an open set G such that $E \setminus G$ is of nowhere dense and for every $[x, y] \subset G$ there is a 5-element $S-$chain connecting x and y.*

THEOREM 3. *Let $E \subset \Re$ be measurable, let $S \subset \Re^2$ and suppose that for every $x \in E$ there is a $\delta(x) > 0$ such that either $(x - t, x + t) \in S$ for every $0 < t < \delta(x)$, or $(x + t, x - t) \in S$ for every $0 < t < \delta(x)$. Then for almost every $x \in E$ there is a neighbourhood U of x such that for every $y \in U$ there is a 5-element $S-$chain connecting x and y.*

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