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APPROXIMATE HIGH ORDER SMOOTHNESS

This talk was based on joint work with Zoltán Buczolich and Paul Humke.

A substantial amount is known concerning the properties of smooth functions. For example see [8], [5], and [6]. Recently, Dutta [2] introduced the notion of high order smoothness and established a number of interesting results analogous to those for smooth functions. (See also [3] and [4].) Also, the notion of approximate smoothness generalizes that of smoothness and has been found to have a number of similarities. (For example, see [9] and [7].) In [1] we attempt to combine these latter two concepts in the obvious manner to arrive at the notion of approximate high order smoothness and show that results analogous to those of Dutta carry over to this setting. Central to this endeavor is the following result.

Theorem 1 *If f is approximately continuous and approximately m-smooth, then $D_{ap}^{m-2} f$ (the $(m-2)^{nd}$ approximate symmetric derivative of f) is a Baire*1 function and f is continuous on an open dense set.*

Examples of the type of results that this yields are the following.

Theorem 2 *If f is approximately continuous and approximately m-smooth, then there is an open dense set on which f has a continuous $(m-2)^{nd}$ ordinary derivative.*

Theorem 3 *If f is approximately continuous and approximately m-smooth, then the $(m-1)^{st}$ approximate Peano derivative exists finitely at each point of a set having the power of the continuum in every interval.*

Concerning Theorem 1, it is interesting to note that if m is even then the original function f is a Baire*1 function, but in the odd case this need not be so as the next result illustrates.

Theorem 4 *There is an approximately continuous function f which is approximately m-smooth for every odd natural mumber greater than one, but f is not a Baire*1 function.*

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