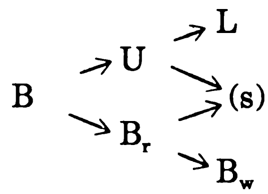
Jack B. Brown, Division of Mathematics, Parker Hall
Auburn University, AL, 36849-5310

Differentiable-, Continuous-, and Derivative-Restrictions

of Measurable Functions

"Measurability" means measurability with respect to one of the σ-algebras:



where B = the Borel sets, U = the universally measurable sets, L = the Lebesgue measurable sets, B_r represents the sets with the Baire property (restricted sense), B_w represents the sets with the Baire property (wide sense).

The best known theorem of the type we are interested in is the following:

THEOREM 1: If $f : R \to R$ is L-measurable, then

1) for every $\varepsilon > 0$, there exists M with $\lambda(M^c) < \varepsilon$ such that $f|M$ is continuous [7],

2) there exists M with $\lambda(M^c) = 0$ and a continuous (a.e. differentiable) $F : R \to R$ such that $f|M = F'|M$ [8],

3) there exists a perfect set P such that $f|P$
   i) is monotonic [5],
   ii) is C^∞ (relative to P) [6]
   iii) $= g|P$ for some C^1 $g : R \to R$ [1].

THEOREM 2: If $f : R \to R$ is B_w-measurable, then

1) there exists a co-FC set M such that $f|M$ is continuous [9],

2) there exists a smaller co-FC set M and a D^1 function F such that $f|M = F'|M$,

3) there exists a perfect set P such that $f|P$
   i) is monotonic [5],
   ii) is "D^1" (relative to P) [4],
   iii) $= f|P$ for some "C^1" $g : R \to R$ [2].

THEOREM 3: If $f : R \to R$ is (s)-measurable, then

1) there exists a perfectly dense subset M of R (every open subset of R contains a perfect subset of M) such that $f|M$ is continuous [3],

2) there exists a smaller perfectly dense subset M of R and a D^1 function $F : R \to R$ such that $f|M = F'|M$,

3) {same as 3) of Theorem 2}.

We discuss in some detail the sharpness of each of these results. The proofs of some of the new results rely heavily on the theorems of [10].

REFERENCES

[1] S. Agronsky, A. M. Bruckner, M. Laczkovich, and D. Preiss, *Convexity conditions and intersections with smooth functions*, Trans. Amer. Math. Soc. 289 (1985), 659-677.

[2] J. B. Brown, *Differentiable restrictions of real functions*, Proc. Amer. Math. Soc. 108 (1990), 391-398.

[3] J. B. Brown and K. Prikry, *Variations on Lusin's theorem*, Trans. Amer. Math. Soc. 302 (1987), 77-86.

[4] A. M. Bruckner, J. G. Ceder, and M. L. Weiss, *On the differentiability structure of real functions*, Trans. Amer. Math. Soc. 142 (1969), 1-13.

[5] F. Filipczak, *Sur les fonctions continues relativement monotones*, Fund. Math. LII (1963), 60-68.

[6] M. Laczkovich, *Differentiable restrictions of continuous functions*, Acta Math. Hungar. 44 (1984), 355-360.

[7] N. Luzin, *Sur les propriétés des fonctions mesurables*, Comp. Rend. Acad. Sci. Paris 154 (1912), 1688-1690.

[8] ________, *Sur la recherche des fonctions primitives*, Comp. Rend Acad. Sci. Paris 162 (1916), 975-978.

[9] O. Nikodym, *Sur la condition de Baire*, Bull. Acad. Polon. (1929), 591.

[10] G. Petruska and M. Laczkovich, *Baire 1 functions, approximately continuous functions, and derivatives*, Acta Math. Hungar. 25, (1974), 189-212.