P. Keller, Mathematics Dept., U. C. Santa Barbara, CA 93106

Countable Collections of ω-limit sets for Darboux Baire 1 Functions

In [BCP] Ceder proved that any nonempty closed subset F of $I = [0,1]$ is an ω-limit set for some $\mathcal{DB}_1$ function $f:I\to I$. Given any collection $\mathfrak{F}$ of such nonempty closed sets, is there an $f\in \mathcal{DB}_1$ with $\Lambda(f)\supseteq\mathfrak{F}$? ($\Lambda(f)$ denotes the collection of ω-limit sets of f.) To answer this question for some countable collections of nonempty closed sets, the following lemma is useful.

LEMMA (Bruckner). *Consider $g:D\to D$, where countable $D\subset I$. For each $n\in \mathbb{N}$, let $D_n=\{x\in D:osc_g(x)\geq 1/n\}$. clD_n is countable for each $n \Rightarrow$ there is an extension of g to some $f:I\to I$ with $f\in \mathcal{DB}_1$.*

THEOREM 1. *Suppose $\mathfrak{F}$ is a finite collection of nonempty closed subsets of I. Then there is an $f\in \mathcal{DB}_1$ with $\Lambda(f)\supseteq\mathfrak{F}$.*

THEOREM 2. *Suppose $\mathfrak{F}$ is a countable collection of closed subintervals of I. Then there is an $f\in \mathcal{DB}_1$ with $\Lambda(f)\supseteq\mathfrak{F}$.*

THEOREM 3. *Suppose, for some $\varepsilon>0$, $\mathfrak{F}$ is a countable collection of closed subsets of I with each $F\in\mathfrak{F}$ being a union of intervals of diameter ε. Then there is an $f\in \mathcal{DB}_1$ with $\Lambda(f)\supseteq\mathfrak{F}$.*

Common to the proofs of each theorem is the construction, for each $F_i\in\mathfrak{F}$, of an orbit $\{x_{i,j}\}_{j=0}^{\infty}$, with closure F_i, where, for $N=\text{card}(\mathfrak{F})$, $D=\cup_{i=1}^{N}\{x_{i,j}\}_{j=0}^{\infty}$. Each orbit is constructed by stepping along I near F_i, the size of the i^{th} step close to something like $1/(i+j)$, taking care to keep each orbit disjoint from the other orbits. Large steps spanning complementary open intervals of the F_i are strategically placed so clD_n remains countable. For $g(x_{i,j})=x_{i,j+1}$ for each $x_{i,j}\in D$, the lemma gives its extension to some $f\in \mathcal{DB}_1$.

[BCP] A. M. Bruckner, J. G. Ceder and T. L. Pearson, *On ω-limit sets for various classes of functions*, Real Analysis Exchange (to appear).