A GLIMM-EFFROS DICHOTOMY FOR BOREL EQUIVALENCE RELATIONS

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A basic dichotomy on the structure of the orbit space of locally compact transformation groups and certain types of Polish transformation groups has been discovered in the 1960's by Glimm and Effros. The following result extends the Glimm-Effros dichotomy to the very general context of arbitrary Borel equivalence relations.

A Borel equivalence relation E on a Polish space X is called <u>smooth</u> if there is a Borel map f: $X \to Y$, where Y is some Polish space, such that $x \, E \, y \Leftrightarrow f(x) = f(y)$, i.e., elements of X can be classified up to E-equivalence by Borel calculable "invariants", which belong to some Polish space, thus are fairly "concrete". A typical non-smooth equivalence relation is E_0 on 2^N, where $x \, E_0 \, y \Leftrightarrow \exists m \forall n \geq m \, (x(n) = y(n))$. Finally, if E,E′ are Borel equivalence relations on X,X′ resp., we say that E is <u>embeddable</u> in E′, in symbols $E \sqsubseteq E'$, if there is a Borel injection f: $X \to X'$ such that $x \, E \, y \Leftrightarrow f(x) \, E' \, f(y)$.

<u>Theorem</u> (Harrington-Kechris-Louveau). Let E be a Borel equivalence relation on a Polish space X. Then exactly one of the following holds:

(I) E is smooth;

(II) $E_0 \sqsubseteq E$.

The proof of this result employs methods of effective descriptive set theory. Various ramifications and applications are also discussed.