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A short proof of a theorem of Jasinski and Weiss, Arnold W. Miller, Department of Mathematics, University of Wisconsin, Madison, WI 53706, miller@math.wisc.edu.

The following is a slight improvement of a theorem of Jasinski and Weiss, (Some remarks on Sierpiński sets, to appear). I want to thank Kechris and Cielsielski for simplifying the proof even further.

An uncountable set of reals S is a Sierpiński set iff it meets every measure zero set in a countable set.

Theorem. If S is a Sierpiński set and $\bigcup_{n<\omega} C_n$ is a union of compact sets of measure zero, then $\{x : (x + \bigcup_{n<\omega} C_n) \cap S = \emptyset\}$ is comeager.
proof:

Let the C_n be increasing and compact. Let $\bigcap_{n<\omega} U_n$ be decreasing, open, $\mu(U_n) < \frac{1}{n}$, and $\mathbf{Q} + \bigcup_{n<\omega} C_n \subseteq \bigcap_{n<\omega} U_n$.

Claim. $Y = \{x : x + \bigcup_{n<\omega} C_n \subseteq \bigcap_{n<\omega} U_n\}$ is a dense G_δ and hence comeager.
proof:

Since Y contains the rationals $\mathbf{Q}$ it is enough to see that it is a G_δ. But note that for each n

$$\{x : x + C_n \subseteq U_n\}$$

is open, since C_n is compact and U_n is open. Since

$$Y = \{x : x + \bigcup_{n<\omega} C_n \subseteq \bigcap_{n<\omega} U_n\} = \bigcap_{n<\omega} \{x : x + C_n \subseteq U_n\}$$

it is G_δ and the Claim is proved.

To prove the theorem let $X = \bigcap_{n<\omega} U_n \cap S$ and let $Z = Y \setminus (X - \bigcup_{n<\omega} C_n)$. Since X is countable and so $X - \bigcup_{n<\omega} C_n$ is meager, it is enough to see that $(z + \bigcup_{n<\omega} C_n) \cap S = \emptyset$ for every $z \in Z$. But $z \in Y$ implies $z + \bigcup_{n<\omega} C_n \subseteq \bigcap_{n<\omega} U_n$ implies $(z + \bigcup_{n<\omega} C_n) \cap S \subseteq X$ implies (if nonempty) that $z + c = x$ where $c \in \bigcup_{n<\omega} C_n$ and $x \in X$ so finally $z \in X - \bigcup_{n<\omega} C_n$.

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It remains an open question of Galvin whether the theorem is true for all measure zero sets.