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DESCRIPTIVE SET THEORETIC PHENOMENA IN ANALYSIS AND TOPOLOGY

The theorems of descriptive set theory show that there are pointsets exhibiting various phenomena, e.g., that there are universal sets. While we know that such things exist, it remains an interesting problem to find examples of the given phenomenon which arise naturally in some context. The principal purpose of this talk is to give some natural examples of three types of descriptive set theoretic phenomena, examples which occur in analysis and topology. The three phenomena are: true $\underset{\sim}{\Pi}^1_n$ sets, universal sets, and inseparable pairs.

The three theorems below constitute one example of each type. All three are about differentiation of real functions. Many more examples from other parts of analysis and topology will appear in the expository paper [B2].

Let $D_0 = \{f \in C[0,1]: f$ is differentiable$\}$.

THEOREM 1 [M]. In the space $C[0,1]$, the pointset D_0 is a true coanalytic $(= CA = \underset{\sim}{\Pi}^1_1)$ set.

For any $f \in C[0,1]$, let $R_f = \{y \in \mathbf{R}: (\exists x \in [0,1])(f$ is differentiable at x and $f'(x) = y)\}$. For any f, the set R_f is analytic $(= A = \underset{\sim}{\Sigma}^1_1)$.

THEOREM 2 [P]. Let $S \subset \mathbf{R}$ be any analytic set. There exists an $f \in C[0,1]$ such that $S = R_f$.

Hence the set $\{(f,y): y \in R_f\}$ is a universal set for $\underset{\sim}{\Sigma}^1_1$.

Let $D_1 = \{f \in C[0,1]$: There is exactly one $x \in [0,1]$ such that $f'(x)$ does not exist$\}$.

THEOREM 3 [B1]. D_0 and D_1 are a Borel-inseparable pair of coanalytic sets.

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