QUERIES

A special session on Classical Real Analysis was held at the 93rd Annual Meeting of the A.M.S. in San Antonio, Texas, on January 21-24, 1987. The meeting was directed by Professor Michael J. Evans, North Carolina State U., and Professor Paul D. Humke, St. Olaf College, Minnesota. More than 20 papers were presented and a number of conjectures and queries discussed. The following is a subset of that collection, incomplete and printed with the permission of the presentors. In some cases the queries editor has paraphrased the material and thus he is responsible for any misrepresentations.

Queries 180, 181, 182 Krzysztof Ostaszewski Univ. of Louisville, Kentucky.

180 Let $\mathcal{H}$ be the space of Henstock-integrable function on the unit square $[0,1] \times [0,1]$. Every function of strongly bounded variation on the unit square, and any function equivalent to such, is a multiplier for $\mathcal{H}$. Is this a complete characterization of multipliers?

181 The dual of $\mathcal{H}$ may be identified with finite Borel signed measures on $(0.1] \times (0,1]$. Is there a characterization of the dual in terms of multipliers?

182 Given a sequence $\{g_n\}$ of functions of strongly bounded variation on $[0,1] \times [0,1]$ such that for every $f \in \mathcal{H}$,

$$T(f) = \lim_{n\to\infty} \int_{[0,1]\times[0,1]} f(x,y)\, g_n(x,y)\, dxdy$$

exists, T is a continuous linear functional on $\mathcal{H}$. Is there a function of strongly bounded variation g such that

$$T(f) = \int_{[0,1]\times[0,1]} f(x,y)\, g(x,y)\, dxdy \ ?$$

Query 183 Jack Brown Auburn Univ. Auburn, Alabama

Considering the recent results of Laczkovich [Acta. Math. Hung. 44 (1984), 355-360] and of Agronsky, Bruckner, Laczkovich, and Preiss [Trans. A.M.S. 289 (1985), 659-677] it is natural to ask the following question:

If P is a perfect subset of R^2, $\lambda(P) > 0$, and $f : P \to R$ is continuous, does there necessarily exist a perfect subset Q of P such that $f{:}Q$ is differentiable, or C^∞, or extendable to a C^1 function $g : R^2 \to R$? Can you make the set Q such that $\mathrm{Tan}(Q,x)$ spans R^2 for every x in Q or such that Q satisfies the stronger condition (ii) of Theorem 4 of the recent paper by Aversa, Laczkovich, and Preiss [Comm. Mat. Univ. Car. 26.3 (1985), 597-609]?

Query 184 Jim Foran Univ. of Missouri-Kansas City, Missouri

If $A \subset [0,1]$, $U = A^c$, $|A| > 0$ $|U| > 0$, A measurable

$$F(x) = \int_0^x \chi_u(t)dt \quad \text{and} \quad F'(0) = 0,$$

must there be a point $x_0 \in [0,1]$ so that

$$\limsup_{h\to 0^+} \frac{F(x_0+h) + F(x_0-h) - 2\,F(x_0)}{h} > 0\ ?$$

(An affirmative answer would provide a monotonicity theorem for the approximate symmetric derivative.)

Query 185 Richard J. O'Malley Univ. of Wisconsin-Milwaukee

For a continuous function $f\colon [0,1] \to R$ let $\Lambda(f)$ be the set of all y such that there is an x in $[0,1]$ and $n_k \to +\infty$ such that $f^{n_k}(x) \to y$ (here f^{n_k} denotes the n_k^{th} iterate of f). It is known that $\Lambda(f)$ is a closed subset of $[0,1]$. However, reiteration of the Λ process by restricting the domain of f to Λ, i.e. $\Lambda(f|\Lambda)$ does not always yield a closed set. Problem: What is a nontrivial characterization of $\Lambda(f|\Lambda)$?

Query 186 Kevin Taylor Saint Mary's College

Question 1: Let E be a bilateral system of paths. Let F be a E-differentiable function with F_E' in the first class of Baire. If F is of Baire class 1, Darboux and Generalized Bounded Variation does the following hold: For each $x_0 \in R$ and $\epsilon > 0$, there is an interval I and a point $t \in I$ such that F is differentiable a.e. on I, F is differentiable at t, and $|F'(t)-F_E'(x_0)| < \epsilon$.

Question 2. Let E be a nonporous system of paths and F be E differentiable with F_E' in the first Baire class. If F is of Baire class I, Darboux, and Generalized Bounded Variation, does the following hold:

If $x_0 \in R$, then there is an interval I and a point $t \in I$ such that F is differentiable on I and $F'(t) = F_E'(x_0)$.

The following two notes came in response to the request made to all speakers for questions raised during the special session on real analysis. They appear without editoral scrutiny. The first is assigned Query 187 and the second, Query 188.