MARCZEWSKI SETS, MEASURE AND THE BAIRE PROPERTY

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This paper is a study of Marczewski sets, referred to as sets with property (s). This study is done by examining other set properties, such as universal measurability, then paralleling theorems about these other properties with theorems about property (s). The first section is primarily concerned with the construction of hereditarily Marczewski sets, referred to as sets with property (s^0). The two main theorems are stated here.

Theorem 1. If $\mathcal{B} = \{B_\alpha\}_{\alpha<c}$ is a collection of disjoint uncountable Borel sets then there exists a set M with property (s^0) that intersects each member of $\mathcal{B}$.

A set M is C-dense in the set B if every open set of B contains continuum many points of M. The next theorem is a variation of a theorem by Mazurkiewicz and Marczewski, where property (?) is a topological property of subsets of a Polish space.

Theorem 2. If property (?) satisfies the conditions that (1) there exists a linear set with property (?) and cardinality of the continuum, (2) property (?) is preserved under inverses of one-to-one projections and (3) property (?) is preserved under countable unions then there exists a set with property (?) that is C-dense in every nondegenerate closed connected subset of $R\times R$.

Note that property (s^0) satisfies the conditions of theorem 2 and assuming CH so do property λ' and property U_0.

The second part of this paper deals with property (s) and B-measurable functions. The two main theorems are stated here.

Theorem 3. A B-measurable function $f: X \to Y$ is bimeasurable if and only if for each $M \subseteq X$ with property (s) (property (s^0)) $f(M)$ has property (s) (property (s^0)).

Theorem 4. If $f: X \to Y$ is B-measurable and $M \subseteq Y$ has property (s) then $f^{-1}(M)$ has property (s).

References.

J. B. Brown and G. V. Cox, "Classical theory of totally imperfect spaces", Real Analysis Exchange 7 (1982), 185-232.

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