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# A Note on Blumberg Pairs

(Abstract)

If $X$ and $Y$ are topological spaces, we say that $(X,Y)$ is a Blumberg pair ("BP") if it is true that for every $f : X \to Y$, there exists a dense subset $D$ of $X$ such that $f|D$ is continuous. $w(X)$, $d(X)$, and $c(X)$ denote the weight, density character, and cellularity (or Souslin number) of $X$. For topological spaces $X$ and arbitrary cardinal numbers, we review the known relationships and establish new relationships between the following:

(1) $(X,Y)$ is a BP for every space $Y$ with $\text{card}(Y) = m$,

(2) no open set in $X$ is the union of $m$ or fewer nowhere dense sets,

(3) the intersection of $m$ or fewer dense open sets in $X$ is dense in $X$,

(4) $(X,Y)$ is a BP for every discrete space $Y$ with $\text{card}(Y) = m$,

(5) for every decomposition $P$ of $X$ of cardinality $\leq m$, there exists a dense subset $D$ of $X$ such that for every $A \in P$, $A \cap D$ is open relative to $D$,

(6) $(X,R)$ is a BP (where $R$ is the reals),

(7) $(X,2^m)$ is a BP,

(8) $(X,Y)$ is a BP for every space $Y$ with $w(Y) \leq m$,

(9) for every cover $P$ of $X$ of cardinality $\leq m$, there exists a dense subset $D$ of $X$ such that for every $A \in P$, $A \cap D$ is open relative to $D$.

We discuss the applicability of (3) for metric and non-metric $X$, and what this implies about (4) for metric and non-metric pairs $(X,Y)$ when $m$ is greater than $c$, the cardinality of $R$. We consider the effect of $w(Y)$, $d(Y)$, and $c(Y)$ on the question of whether $(R,Y)$ is a BP for arbitrary spaces $Y$.