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ISOTONIC APPROXIMATION OF APPROXIMATELY CONTINUOUS FUNCTIONS

Let M consist of all nondecreasing functions on $[0,1]$. If f is in $L_\infty[0,1]$ and $1<p<\infty$, then M is a closed convex subset of the uniformly convex subset of the uniformly convex Banach space $L_p[0,1]$ so there is a unique best L_p-approximation, f_p, to f by elements of M, i.e.,

$$||f - f_p||_p \leq ||f - h||_p, \; h \in M.$$

If $\lim_{p\to\infty} f_p(x)$ (respectively, $\lim_{p\downarrow 1} f_p(x)$) exists almost everywhere as a bounded measurable function, then f is said to have the Polya property (respectively, Polya-one). If f has at most discontinuities of the first kind, then f has both properties [3], [1]. The purpose of this note is to present the results of our investigations of the case in which f is in bA, the set of all bounded approximately continuous functions on $[0,1]$. The theorems mentioned here are proven in [2].

THEOREM 1. Suppose $f \in bA$, $1 \leq p < \infty$ and g is a best L_p-approximation to f by elements of M. Then g is continuous.

THEOREM 2. Let $f \in bA$. Then there exists a unique best L_1-approximation f_1 of f by elements of M.

THEOREM 3. Let $f \in bA$. Then f_p converges uniformly to f_1 as p decreases to one.

So, not only does the Polya-one property hold for $f \in bA$, but the convergence is uniform. The Polya property however, may fail. In [2], an example, h, is constructed which is continuous on $[0,\frac{1}{2})$ and $[\frac{1}{2},1]$ and approximately continuous at $\frac{1}{2}$ but there exists a sequence $\{p_n\}$ with $p_n \to \infty$ such that $\{h_{p_n}\}$ diverges at every point in $(\frac{1}{2},1]$.

Of related interest is the continuity, in L_p, $1 \le p < \infty$, of the map $f \to f_p$. For $1 < p < \infty$, it is known that $f \to f_p$ is continuous if f is any bounded measurable function [4, Corollary 2]. This is not true if p = 1. Indeed, by [2], there exist functions f^n, n=1,2,... and f such that each f^n is continuous and $f^n \to f$ pointwise but $\{f_1^n\}$ is not Cauchy in L_1. We are however, able to state the following:

THEOREM 4. Let f, f^n, n=1,2,... ε bA. If $||f^n - f||_1 \to 0$, then $||f_1^n - f_1||_1 \to 0$.

REFERENCES

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