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On Residual Subsets of Darboux Baire Class 1 Functions

We denote by $b\mathcal{DB}_1$ the class of bounded real valued functions defined on the interval $I = [0,1]$ which are Darboux and in Baire class 1. The symbol(s) $\mathcal{C}$, $b\mathcal{D}$usc, $b\mathcal{D}$ lsc, $b\Delta$, and $b\mathcal{M}_i$, $i = 2,\ldots,5$, denotes that subfamily of $b\mathcal{DB}_1$ all of whose members are continuous, upper semi-continuous, lower semi-continuous, derivatives, and in Zahorski class $\mathcal{M}_i$, $i = 2,\ldots,5$, respectively. Each of these families is a Banach space with norm $\|f\| = \sup|f|$.

Lebesgue's measure on I is denoted by m, and for any function f, C(f) (resp. A(f)) denotes the continuity (resp. approximate continuity) points of f.

We say that a property P is typical in a family Φ of functions if P holds for a residual subset of Φ.

The following table contains a list of properties and the families in which they are typical. If a property is typical in a given family we indicate this by a "Y", otherwise we write an "N". Blanks in the table indicate problems currently under investigation.

REFRENCES

1. Ceder, J.G., Pearson, T.L., A survey of Darboux Baire 1 functions, Real Anal. Exchange 9 (1983-84), 179-194.

2. Mustafa, Ibrahim, Ph.D. On upper semi-continuous Darboux functions Thesis, University of California at Santa Barbara, in preparation.

3. Petruska, Gy., An extention of the Darboux property and some typical properties of Baire-1 functions, Real Anal. Exchange 8 (1982-83), 62-64.

	$\mathcal{C}$	$b\mathcal{D}usc$	$b\mathcal{D}lsc$	$b\Delta$	$b\mathcal{M}_i$ $i=2,\dots,15$	$b\mathcal{D}\mathcal{B}_1$
1. $m(f(A(f))) = 0$	N	Y	Y			Y
2. $m(\mathrm{cl}\ f(C(f))) = 0$	N	Y	Y			Y
3. $m(\mathrm{cl}\ f(A(f))) = 0$	N	Y	Y			Y
4. $m(C(f)) = 0$	N	Y	Y	Y	Y	Y
5. card $f(C(f)) = 2^{\aleph_0}$	Y	Y	Y	Y	Y	Y
6. $I - C(f)$ is dense in I	N	Y	Y	Y	Y	Y
7. For all y, $f^{-1}(y)$ is nowhere dense and null	Y	Y	Y	Y	Y	Y
8. The function $f(x) + rx$ is nowhere monotonic	Y	Y	Y	Y	Y	Y
9. f has $+\infty$ and $-\infty$ as derived numbers at each point	Y	Y	Y	Y	Y	Y
10. f has an infinite derived number on both the right and left at each point	Y	Y	Y	Y	Y	Y
11. Each real number is a derived number at each point	Y	Y	Y	Y	Y	Y
12. There exists a residual set $E \subsetneq I$ such that for each $x \in E$ each real number is a derived number from both the right and left at each point					Y	Y
13. There exists a residual set $E \subsetneq I$ such that the intersection of the line $y = ax + b$ with the graph of f is a dense in itself boundry set whenever a is rational and $b \in E$					Y	Y
14. The set of all (a,b) such that $y = ax + b$ fails to intersect the graph of f in a dense in itself set is null and of first category					Y	Y
15. f attains its maximum on each open/closed subinterval of I at exactly one point				Y	Y	Y

The address by Jim Foran, Transformations of Functions, appears in the Inroads section of this issue of the Exchange.