A RIEMANN INTEGRAL AND THE DIVERGENCE THEOREM

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By integration, we want to obtain the flux of a vector field from its divergence. The Denjoy-Perron integral does this in dimension one, but its higher dimensional generalizations, including the most recent ones, do not; a notable exception is (M), however, the integral defined there is deficient in other respects - see (M, Theorem 2). Elaborating on the ideas of Henstock and Kurzweil, we define an m-dimensional Riemann type integral which has all the standard properties expected of integrals, and for which the following theorem holds.

Theorem. Let A be an interval, and let v be a continuous vector field on A, which is differentiable in the interior of A. Then the div v is integrable over A, and the integral is equal to the flux of v from A.

This is accomplished by employing partitions with a weak Vitali property.

(M) J. Mawhin, Generalized Riemann integrals and the divergence theorem for differentiable vector fields, pp. 704-714 in E.B. Christoffel, Birkhäser Verlag, Basel, 1981.